Computer algebra independent integration tests

1-Algebraic-functions/1.1-Binomial-products/1.1.2-Quadratic/1.1.2.5-a+b-x^2-^p-c+d-x^2-^q-e+f-x^2-^r

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [115]. This is test number [22].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (115)	% 0. (0)
Mathematica	% 99.13 (114)	% 0.87 (1)
Maple	% 91.3 (105)	% 8.7 (10)
Maxima	% 13.04 (15)	% 86.96 (100)
Fricas	% 26.09 (30)	% 73.91 (85)
Sympy	% 23.48 (27)	% 76.52 (88)
Giac	% 29.57 (34)	% 70.43 (81)

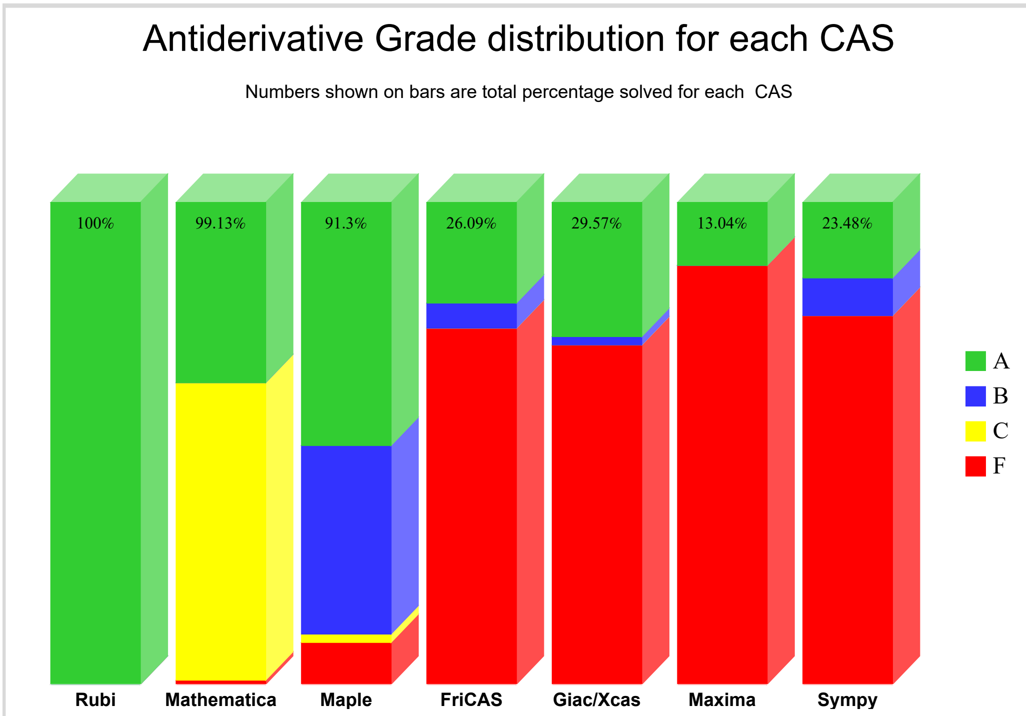
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

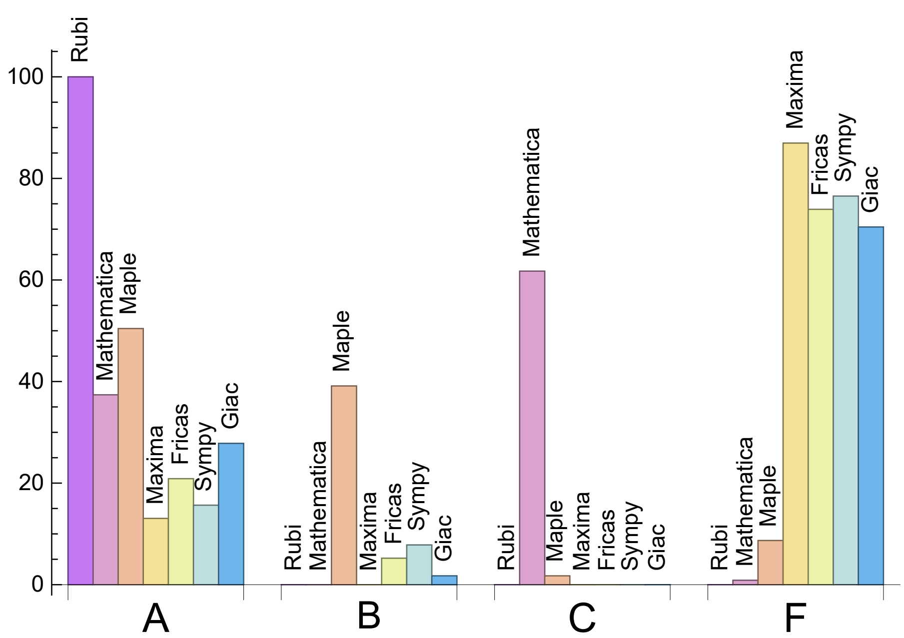
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Mathematica	37.39	0.	61.74	0.87
Maple	50.43	39.13	1.74	8.7
Maxima	13.04	0.	0.	86.96
Fricas	20.87	5.22	0.	73.91
Sympy	15.65	7.83	0.	76.52
Giac	27.83	1.74	0.	70.43

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.27	283.78	0.96	242.	1.
Mathematica	0.84	261.14	0.92	212.	0.93
Maple	0.03	867.32	2.73	397.	1.78
Maxima	0.67	150.27	0.93	135.	1.37
Fricas	3.87	1093.07	6.38	856.5	5.78
Sympy	7.79	253.33	1.44	216.	1.36
Giac	1.61	259.24	1.52	244.	1.68

1.4 list of integrals that has no closed form antiderivative

{103, 107, 110, 112, 115}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {63}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in-s

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

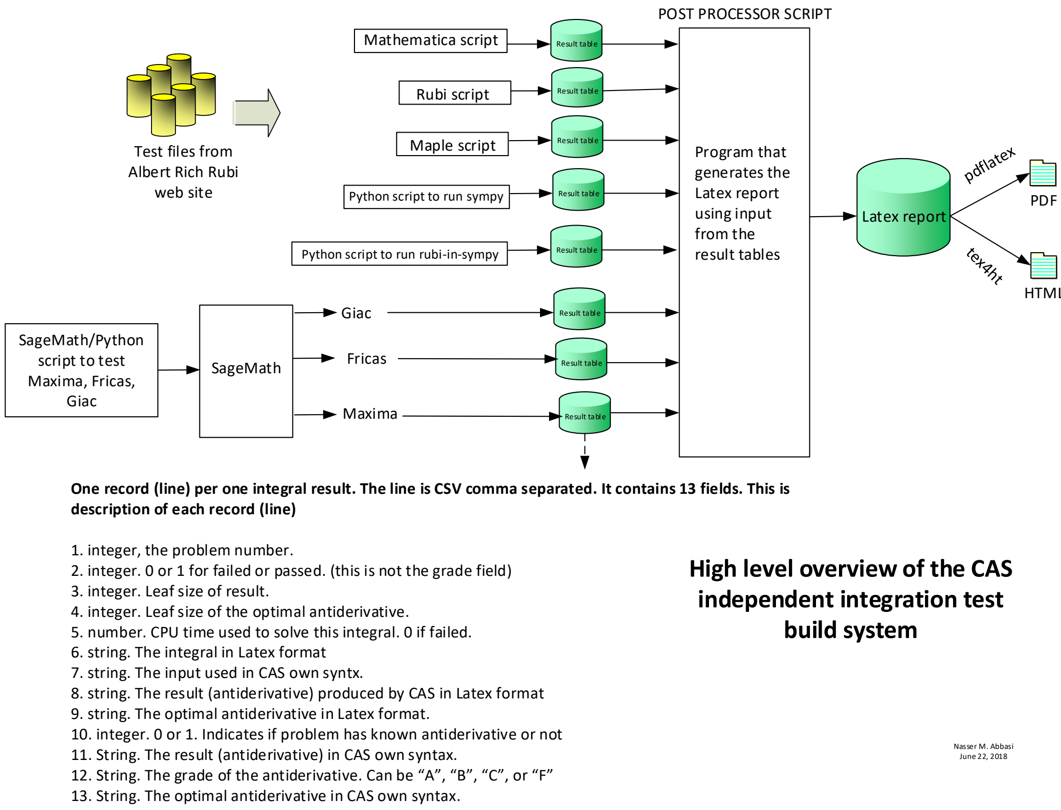
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.


POST PROCESSOR SCRIPT
Mathematica script
Rubi script
Maple script
Python script to run sympy
Python script to run rubi-in-sympy
Test files from
Albert Rich Rubi
web site
Result table
Program that
generates the
Latex report
using input
from the
result tables
Latex report
pdflatex
tex4ht
PDF
HTML
SageMath/Python
script to test
Maxima, Fricas,
Giac
SageMath
Giac
Fricas
Maxima
One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)
1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. The optimal antiderivative in CAS own syntax.
High level overview of the CAS independent integration test build system
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Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 57, 58, 59, 60, 61, 62, 96, 97, 98, 103, 104, 105, 106, 107, 109, 110, 111, 112, 113, 114, 115 }

B grade: { }

C grade: { 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 99, 100, 101, 102 }

F grade: { 108}

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 15, 16, 17, 18, 19, 25, 26, 31, 37, 38, 39, 43, 44, 45, 46, 48, 49, 50, 51, 52, 53, 66, 67, 68, 73, 77, 78, 79, 80, 81, 83, 85, 86, 87, 91, 92, 94, 95, 96, 97, 100, 103, 107, 110, 112, 115 }

B grade: { 13, 14, 20, 21, 22, 23, 24, 27, 28, 29, 30, 32, 33, 34, 35, 36, 40, 41, 42, 47, 54, 57, 58, 59, 60, 61, 62, 63, 64, 65, 69, 70, 71, 72, 74, 75, 76, 82, 84, 88, 93, 98, 99, 101, 102 }

C grade: { 89, 90 }

F grade: { 55, 56, 104, 105, 106, 108, 109, 111, 113, 114 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 9, 10, 11, 16, 17, 18, 103, 107, 110, 112, 115

B grade: { }

C grade: { }

F grade: { 5, 6, 7, 8, 12, 13, 14, 15, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 104, 105, 106, 108, 109, 111, 113, 114 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 14, 16, 17, 18, 19, 20, 21, 57, 58, 59, 110, 115 }

B grade: { 8, 15, 22, 60, 61, 98 }

C grade: { }

F grade: { 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 111, 112, 113, 114 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 6, 8, 9, 10, 11, 16, 17, 18, 97, 103, 107, 110, 112, 115 }

B grade: { 5, 7, 12, 13, 14, 15, 19, 20, 21 }

C grade: { }

F grade: { 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 98, 99, 100, 101, 102, 104, 105, 106, 108, 109, 111, 113, 114 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 57, 58, 59, 60, 61, 103, 107, 110, 112, 115 }

B grade: { 63, 98 }

C grade: { }

F grade: { 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 62, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 99, 100, 101, 102, 104, 105, 106, 108, 109, 111, 113, 114 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	172	176	236	531	236	284
normalized size	1	1.	1.	1.02	1.37	3.09	1.37	1.65
time (sec)	N/A	0.192	0.087	0.002	0.986	1.25	0.089	1.21

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	130	135	181	398	173	217
normalized size	1	1.	1.	1.04	1.39	3.06	1.33	1.67
time (sec)	N/A	0.135	0.066	0.001	0.996	1.247	0.081	1.16

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	96	94	126	282	121	154
normalized size	1	1.	1.02	1.	1.34	3.	1.29	1.64
time (sec)	N/A	0.083	0.036	0.	0.99	1.245	0.072	1.216

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	56	53	70	163	63	89
normalized size	1	1.	1.	0.95	1.25	2.91	1.12	1.59
time (sec)	N/A	0.037	0.012	0.	1.026	1.228	0.063	1.15

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	72	119	0	423	206	108
normalized size	1	1.	0.89	1.47	0.	5.22	2.54	1.33
time (sec)	N/A	0.08	0.053	0.006	0.	1.486	0.837	1.196

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	95	163	0	660	190	128
normalized size	1	1.	0.88	1.51	0.	6.11	1.76	1.19
time (sec)	N/A	0.085	0.067	0.008	0.	1.465	1.583	1.147

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	130	175	0	1002	246	182
normalized size	1	1.	1.	1.35	0.	7.71	1.89	1.4
time (sec)	N/A	0.108	0.08	0.009	0.	1.506	3.642	1.158

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	171	210	0	1355	313	248
normalized size	1	1.	1.	1.23	0.	7.92	1.83	1.45
time (sec)	N/A	0.163	0.102	0.01	0.	1.559	8.424	1.2

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	226	226	226	237	319	676	304	382
normalized size	1	1.	1.	1.05	1.41	2.99	1.35	1.69
time (sec)	N/A	0.214	0.081	0.001	1.036	1.258	0.098	1.176

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	158	169	227	479	216	273
normalized size	1	1.	1.	1.07	1.44	3.03	1.37	1.73
time (sec)	N/A	0.167	0.06	0.	1.015	1.276	0.087	1.145

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	96	101	135	282	121	162
normalized size	1	1.	1.02	1.07	1.44	3.	1.29	1.72
time (sec)	N/A	0.081	0.029	0.002	1.016	1.262	0.074	1.216

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	115	243	0	775	343	240
normalized size	1	1.	0.81	1.71	0.	5.46	2.42	1.69
time (sec)	N/A	0.208	0.064	0.004	0.	1.481	1.334	1.173

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	134	299	0	1150	479	263
normalized size	1	1.	0.82	1.82	0.	7.01	2.92	1.6
time (sec)	N/A	0.232	0.093	0.009	0.	1.591	3.647	1.161

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	207	183	397	0	1613	400	321
normalized size	1	1.	0.88	1.92	0.	7.79	1.93	1.55
time (sec)	N/A	0.239	0.127	0.011	0.	1.586	18.923	1.2

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	240	242	360	0	2117	486	420
normalized size	1	1.	1.01	1.5	0.	8.82	2.02	1.75
time (sec)	N/A	0.28	0.16	0.01	0.	1.606	96.292	1.158

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	310	310	310	339	440	938	423	541
normalized size	1	1.	1.	1.09	1.42	3.03	1.36	1.75
time (sec)	N/A	0.387	0.114	0.	0.986	1.276	0.113	1.142

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	226	226	226	244	323	676	304	390
normalized size	1	1.	1.	1.08	1.43	2.99	1.35	1.73
time (sec)	N/A	0.216	0.083	0.	1.011	1.247	0.098	1.16

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	130	149	197	398	173	234
normalized size	1	1.	1.	1.15	1.52	3.06	1.33	1.8
time (sec)	N/A	0.135	0.05	0.002	0.996	1.238	0.085	1.139

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	179	401	0	1216	508	414
normalized size	1	1.	0.79	1.77	0.	5.36	2.24	1.82
time (sec)	N/A	0.371	0.093	0.005	0.	1.551	2.001	1.148

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	242	176	475	0	1701	654	433
normalized size	1	1.	0.73	1.96	0.	7.03	2.7	1.79
time (sec)	N/A	0.399	0.128	0.013	0.	1.552	7.281	1.22

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	291	291	219	589	0	2291	862	501
normalized size	1	1.	0.75	2.02	0.	7.87	2.96	1.72
time (sec)	N/A	0.416	0.157	0.013	0.	1.646	59.853	1.187

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	348	348	295	735	0	2913	0	603
normalized size	1	1.	0.85	2.11	0.	8.37	0.	1.73
time (sec)	N/A	0.45	0.21	0.013	0.	1.711	0.	1.208

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	544	544	373	1332	0	0	0	0
normalized size	1	1.	0.69	2.45	0.	0.	0.	0.
time (sec)	N/A	0.681	1.135	0.039	0.	0.	0.	0.

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	381	381	267	865	0	0	0	0
normalized size	1	1.	0.7	2.27	0.	0.	0.	0.
time (sec)	N/A	0.378	0.752	0.015	0.	0.	0.	0.

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	283	283	212	394	0	0	0	0
normalized size	1	1.	0.75	1.39	0.	0.	0.	0.
time (sec)	N/A	0.181	0.395	0.017	0.	0.	0.	0.

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	271	192	328	0	0	0	0
normalized size	1	1.	0.71	1.21	0.	0.	0.	0.
time (sec)	N/A	0.171	0.349	0.038	0.	0.	0.	0.

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	274	274	297	1236	0	0	0	0
normalized size	1	1.	1.08	4.51	0.	0.	0.	0.
time (sec)	N/A	0.205	0.973	0.042	0.	0.	0.	0.

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	385	385	379	2856	0	0	0	0
normalized size	1	1.	0.98	7.42	0.	0.	0.	0.
time (sec)	N/A	0.38	1.269	0.055	0.	0.	0.	0.

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	543	543	372	1332	0	0	0	0
normalized size	1	1.	0.69	2.45	0.	0.	0.	0.
time (sec)	N/A	0.645	1.141	0.016	0.	0.	0.	0.

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	400	400	275	870	0	0	0	0
normalized size	1	1.	0.69	2.17	0.	0.	0.	0.
time (sec)	N/A	0.439	0.844	0.018	0.	0.	0.	0.

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	369	369	248	671	0	0	0	0
normalized size	1	1.	0.67	1.82	0.	0.	0.	0.
time (sec)	N/A	0.4	0.706	0.025	0.	0.	0.	0.

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	373	373	296	1231	0	0	0	0
normalized size	1	1.	0.79	3.3	0.	0.	0.	0.
time (sec)	N/A	0.394	1.002	0.028	0.	0.	0.	0.

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	376	376	382	2860	0	0	0	0
normalized size	1	1.	1.02	7.61	0.	0.	0.	0.
time (sec)	N/A	0.423	1.3	0.037	0.	0.	0.	0.

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	531	531	545	5113	0	0	0	0
normalized size	1	1.	1.03	9.63	0.	0.	0.	0.
time (sec)	N/A	0.615	1.835	0.069	0.	0.	0.	0.

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	551	551	386	1386	0	0	0	0
normalized size	1	1.	0.7	2.52	0.	0.	0.	0.
time (sec)	N/A	0.628	1.18	0.025	0.	0.	0.	0.

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	396	396	279	924	0	0	0	0
normalized size	1	1.	0.7	2.33	0.	0.	0.	0.
time (sec)	N/A	0.448	0.823	0.02	0.	0.	0.	0.

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	282	282	215	501	0	0	0	0
normalized size	1	1.	0.76	1.78	0.	0.	0.	0.
time (sec)	N/A	0.184	0.424	0.016	0.	0.	0.	0.

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	206	131	158	0	0	0	0
normalized size	1	1.	0.64	0.77	0.	0.	0.	0.
time (sec)	N/A	0.102	0.161	0.018	0.	0.	0.	0.

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	206	334	0	0	0	0
normalized size	1	1.	0.99	1.6	0.	0.	0.	0.
time (sec)	N/A	0.085	0.637	0.024	0.	0.	0.	0.

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	284	284	302	1352	0	0	0	0
normalized size	1	1.	1.06	4.76	0.	0.	0.	0.
time (sec)	N/A	0.211	1.082	0.031	0.	0.	0.	0.

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	401	401	393	3039	0	0	0	0
normalized size	1	1.	0.98	7.58	0.	0.	0.	0.
time (sec)	N/A	0.413	1.291	0.042	0.	0.	0.	0.

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	501	501	369	1169	0	0	0	0
normalized size	1	1.	0.74	2.33	0.	0.	0.	0.
time (sec)	N/A	0.599	1.161	0.046	0.	0.	0.	0.

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	358	358	260	750	0	0	0	0
normalized size	1	1.	0.73	2.09	0.	0.	0.	0.
time (sec)	N/A	0.363	0.765	0.025	0.	0.	0.	0.

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	258	258	208	393	0	0	0	0
normalized size	1	1.	0.81	1.52	0.	0.	0.	0.
time (sec)	N/A	0.162	0.382	0.023	0.	0.	0.	0.

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	212	349	0	0	0	0
normalized size	1	1.	1.01	1.67	0.	0.	0.	0.
time (sec)	N/A	0.085	0.374	0.025	0.	0.	0.	0.

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	272	272	262	581	0	0	0	0
normalized size	1	1.	0.96	2.14	0.	0.	0.	0.
time (sec)	N/A	0.221	0.72	0.029	0.	0.	0.	0.

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	375	375	428	1742	0	0	0	0
normalized size	1	1.	1.14	4.65	0.	0.	0.	0.
time (sec)	N/A	0.392	2.065	0.038	0.	0.	0.	0.

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	212	349	0	0	0	0
normalized size	1	1.	1.01	1.67	0.	0.	0.	0.
time (sec)	N/A	0.086	0.409	0.036	0.	0.	0.	0.

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	247	220	359	0	0	0	0
normalized size	1	1.	0.89	1.45	0.	0.	0.	0.
time (sec)	N/A	0.24	0.694	0.048	0.	0.	0.	0.

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	213	345	0	0	0	0
normalized size	1	1.	0.9	1.46	0.	0.	0.	0.
time (sec)	N/A	0.218	0.432	0.042	0.	0.	0.	0.

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	242	221	354	0	0	0	0
normalized size	1	1.	0.91	1.46	0.	0.	0.	0.
time (sec)	N/A	0.237	0.466	0.048	0.	0.	0.	0.

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	81	105	0	0	0	0
normalized size	1	1.	0.42	0.55	0.	0.	0.	0.
time (sec)	N/A	0.096	0.142	0.037	0.	0.	0.	0.

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	262	262	142	367	0	0	0	0
normalized size	1	1.	0.54	1.4	0.	0.	0.	0.
time (sec)	N/A	0.168	0.202	0.02	0.	0.	0.	0.

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	356	356	186	775	0	0	0	0
normalized size	1	1.	0.52	2.18	0.	0.	0.	0.
time (sec)	N/A	0.322	0.324	0.023	0.	0.	0.	0.

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	104	0	0	0	0	0
normalized size	1	1.	0.92	0.	0.	0.	0.	0.
time (sec)	N/A	0.204	0.383	0.236	0.	0.	0.	0.

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	526	526	203	0	0	0	0	0
normalized size	1	1.	0.39	0.	0.	0.	0.	0.
time (sec)	N/A	0.63	0.402	0.154	0.	0.	0.	0.

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	124	1942	0	1701	0	227
normalized size	1	1.	0.97	15.17	0.	13.29	0.	1.77
time (sec)	N/A	0.142	0.245	0.042	0.	7.519	0.	1.472

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	304	304	265	1541	0	3518	0	392
normalized size	1	1.	0.87	5.07	0.	11.57	0.	1.29
time (sec)	N/A	0.299	0.343	0.039	0.	43.415	0.	1.755

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	150	1052	0	2346	0	248
normalized size	1	1.	0.9	6.34	0.	14.13	0.	1.49
time (sec)	N/A	0.105	0.306	0.015	0.	11.074	0.	1.65

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	88	646	0	1596	0	159
normalized size	1	1.	0.97	7.1	0.	17.54	0.	1.75
time (sec)	N/A	0.048	0.11	0.009	0.	7.24	0.	1.665

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	306	0	513	0	100
normalized size	1	1.	1.	6.24	0.	10.47	0.	2.04
time (sec)	N/A	0.02	0.011	0.009	0.	1.705	0.	1.542

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	113	782	0	0	0	0
normalized size	1	1.	0.93	6.41	0.	0.	0.	0.
time (sec)	N/A	0.111	0.177	0.037	0.	0.	0.	0.

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	203	203	531	1865	0	0	0	647
normalized size	1	1.	2.62	9.19	0.	0.	0.	3.19
time (sec)	N/A	0.265	2.558	0.056	0.	0.	0.	16.932

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	608	776	456	1891	0	0	0	0
normalized size	1	1.28	0.75	3.11	0.	0.	0.	0.
time (sec)	N/A	0.763	2.638	0.036	0.	0.	0.	0.

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	400	400	346	1059	0	0	0	0
normalized size	1	1.	0.86	2.65	0.	0.	0.	0.
time (sec)	N/A	0.289	1.49	0.015	0.	0.	0.	0.

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	321	321	184	340	0	0	0	0
normalized size	1	1.	0.57	1.06	0.	0.	0.	0.
time (sec)	N/A	0.191	0.366	0.013	0.	0.	0.	0.

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	143	191	0	0	0	0
normalized size	1	1.	1.4	1.87	0.	0.	0.	0.
time (sec)	N/A	0.034	0.231	0.017	0.	0.	0.	0.

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	347	390	0	0	0	0
normalized size	1	1.	1.66	1.87	0.	0.	0.	0.
time (sec)	N/A	0.105	0.773	0.024	0.	0.	0.	0.

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	401	401	427	2068	0	0	0	0
normalized size	1	1.	1.06	5.16	0.	0.	0.	0.
time (sec)	N/A	0.367	3.437	0.036	0.	0.	0.	0.

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	630	630	584	6245	0	0	0	0
normalized size	1	1.	0.93	9.91	0.	0.	0.	0.
time (sec)	N/A	0.719	3.079	0.063	0.	0.	0.	0.

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	659	784	445	1939	0	0	0	0
normalized size	1	1.19	0.68	2.94	0.	0.	0.	0.
time (sec)	N/A	0.753	2.495	0.023	0.	0.	0.	0.

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	403	403	739	1028	0	0	0	0
normalized size	1	1.	1.83	2.55	0.	0.	0.	0.
time (sec)	N/A	0.304	1.104	0.017	0.	0.	0.	0.

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	328	328	184	300	0	0	0	0
normalized size	1	1.	0.56	0.91	0.	0.	0.	0.
time (sec)	N/A	0.191	0.356	0.018	0.	0.	0.	0.

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	492	630	0	0	0	0
normalized size	1	1.	2.2	2.81	0.	0.	0.	0.
time (sec)	N/A	0.112	1.277	0.03	0.	0.	0.	0.

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	391	391	999	1876	0	0	0	0
normalized size	1	1.	2.55	4.8	0.	0.	0.	0.
time (sec)	N/A	0.404	1.813	0.036	0.	0.	0.	0.

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	639	639	570	6211	0	0	0	0
normalized size	1	1.	0.89	9.72	0.	0.	0.	0.
time (sec)	N/A	0.779	2.891	0.062	0.	0.	0.	0.

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	621	621	350	988	0	0	0	0
normalized size	1	1.	0.56	1.59	0.	0.	0.	0.
time (sec)	N/A	0.463	1.389	0.026	0.	0.	0.	0.

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	319	319	197	341	0	0	0	0
normalized size	1	1.	0.62	1.07	0.	0.	0.	0.
time (sec)	N/A	0.176	0.371	0.021	0.	0.	0.	0.

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	143	191	0	0	0	0
normalized size	1	1.	1.4	1.87	0.	0.	0.	0.
time (sec)	N/A	0.033	0.23	0.018	0.	0.	0.	0.

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	101	118	0	0	0	0
normalized size	1	1.	1.01	1.18	0.	0.	0.	0.
time (sec)	N/A	0.121	0.221	0.021	0.	0.	0.	0.

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	344	344	365	413	0	0	0	0
normalized size	1	1.	1.06	1.2	0.	0.	0.	0.
time (sec)	N/A	0.22	0.787	0.03	0.	0.	0.	0.

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	435	435	433	2062	0	0	0	0
normalized size	1	1.	1.	4.74	0.	0.	0.	0.
time (sec)	N/A	0.554	3.422	0.043	0.	0.	0.	0.

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	980	980	352	1063	0	0	0	0
normalized size	1	1.	0.36	1.08	0.	0.	0.	0.
time (sec)	N/A	1.113	1.567	0.032	0.	0.	0.	0.

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	304	594	0	0	0	0
normalized size	1	1.	1.36	2.66	0.	0.	0.	0.
time (sec)	N/A	0.12	1.042	0.027	0.	0.	0.	0.

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	207	285	0	0	0	0
normalized size	1	1.	0.99	1.36	0.	0.	0.	0.
time (sec)	N/A	0.105	0.466	0.025	0.	0.	0.	0.

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	344	344	221	303	0	0	0	0
normalized size	1	1.	0.64	0.88	0.	0.	0.	0.
time (sec)	N/A	0.222	0.619	0.026	0.	0.	0.	0.

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	539	539	418	956	0	0	0	0
normalized size	1	1.	0.78	1.77	0.	0.	0.	0.
time (sec)	N/A	0.479	3.856	0.035	0.	0.	0.	0.

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	814	814	1645	4115	0	0	0	0
normalized size	1	1.	2.02	5.06	0.	0.	0.	0.
time (sec)	N/A	0.958	5.942	0.058	0.	0.	0.	0.

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	239	204	370	0	0	0	0
normalized size	1	0.99	0.84	1.53	0.	0.	0.	0.
time (sec)	N/A	0.147	0.356	0.035	0.	0.	0.	0.

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	71	120	0	0	0	0
normalized size	1	1.	0.37	0.62	0.	0.	0.	0.
time (sec)	N/A	0.092	0.192	0.008	0.	0.	0.	0.

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	50	64	0	0	0	0
normalized size	1	1.	0.86	1.1	0.	0.	0.	0.
time (sec)	N/A	0.022	0.173	0.013	0.	0.	0.	0.

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	122	147	0	0	0	0
normalized size	1	1.	1.01	1.21	0.	0.	0.	0.
time (sec)	N/A	0.06	0.346	0.033	0.	0.	0.	0.

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	357	477	0	0	0	0
normalized size	1	1.	1.66	2.22	0.	0.	0.	0.
time (sec)	N/A	0.145	0.378	0.033	0.	0.	0.	0.

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	298	134	293	0	0	0	0
normalized size	1	1.	0.45	0.98	0.	0.	0.	0.
time (sec)	N/A	0.179	0.302	0.027	0.	0.	0.	0.

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	94	133	0	0	0	0
normalized size	1	1.	1.01	1.43	0.	0.	0.	0.
time (sec)	N/A	0.036	0.198	0.02	0.	0.	0.	0.

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	53	0	0	0	0
normalized size	1	1.	1.	1.08	0.	0.	0.	0.
time (sec)	N/A	0.042	0.017	0.019	0.	0.	0.	0.

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	37	35	0	0	19	0
normalized size	1	1.	1.03	0.97	0.	0.	0.53	0.
time (sec)	N/A	0.025	0.042	0.026	0.	0.	5.787	0.

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	191	1622	0	1079	0	454
normalized size	1	1.	1.69	14.35	0.	9.55	0.	4.02
time (sec)	N/A	0.119	0.578	0.03	0.	13.927	0.	3.946

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	359	359	422	793	0	0	0	0
normalized size	1	1.	1.18	2.21	0.	0.	0.	0.
time (sec)	N/A	0.324	2.412	0.043	0.	0.	0.	0.

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	381	381	401	765	0	0	0	0
normalized size	1	1.	1.05	2.01	0.	0.	0.	0.
time (sec)	N/A	0.291	1.993	0.036	0.	0.	0.	0.

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	426	426	617	1105	0	0	0	0
normalized size	1	1.	1.45	2.59	0.	0.	0.	0.
time (sec)	N/A	0.373	5.699	0.046	0.	0.	0.	0.

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	485	485	587	1078	0	0	0	0
normalized size	1	1.	1.21	2.22	0.	0.	0.	0.
time (sec)	N/A	0.347	2.795	0.028	0.	0.	0.	0.

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	36	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.057	0.741	0.071	0.	0.	0.	0.

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	545	545	503	0	0	0	0	0
normalized size	1	1.	0.92	0.	0.	0.	0.	0.
time (sec)	N/A	0.487	1.351	0.066	0.	0.	0.	0.

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	162	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.134	0.1	0.062	0.	0.	0.	0.

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	148	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.085	0.082	0.076	0.	0.	0.	0.

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	36	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.061	0.756	0.073	0.	0.	0.	0.

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	484	484	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.753	0.667	0.06	0.	0.	0.	0.

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	319	319	148	0	0	0	0	0
normalized size	1	1.	0.46	0.	0.	0.	0.	0.
time (sec)	N/A	0.467	0.099	0.069	0.	0.	0.	0.

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	36	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.06	0.968	0.069	0.	0.	0.	0.

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	541	541	512	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.436	1.623	0.059	0.	0.	0.	0.

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	36	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.056	0.17	0.068	0.	0.	0.	0.

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	159	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.135	0.092	0.062	0.	0.	0.	0.

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	148	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.08	0.083	0.067	0.	0.	0.	0.

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	36	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.055	0.873	0.07	0.	0.	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [64] had the largest ratio of [0.2812]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	1	1.	24	0.042
2	A	2	1	1.	24	0.042
3	A	2	1	1.	24	0.042
4	A	2	1	1.	22	0.045
5	A	3	3	1.	24	0.125
6	A	3	3	1.	24	0.125
7	A	3	3	1.	24	0.125
8	A	4	4	1.	24	0.167
9	A	2	1	1.	26	0.038
10	A	2	1	1.	26	0.038
11	A	2	1	1.	24	0.042
12	A	4	3	1.	26	0.115
13	A	4	4	1.	26	0.154

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
14	A	4	3	1.	26	0.115
15	A	4	3	1.	26	0.115
16	A	2	1	1.	26	0.038
17	A	2	1	1.	26	0.038
18	A	2	1	1.	24	0.042
19	A	5	3	1.	26	0.115
20	A	5	4	1.	26	0.154
21	A	5	4	1.	26	0.154
22	A	5	3	1.	26	0.115
23	A	7	5	1.	30	0.167
24	A	6	5	1.	30	0.167
25	A	5	5	1.	30	0.167
26	A	5	5	1.	30	0.167
27	A	4	4	1.	30	0.133
28	A	5	5	1.	30	0.167
29	A	7	5	1.	30	0.167
30	A	6	5	1.	30	0.167
31	A	6	6	1.	30	0.2
32	A	6	5	1.	30	0.167
33	A	5	4	1.	30	0.133
34	A	6	5	1.	30	0.167
35	A	7	5	1.	30	0.167
36	A	6	5	1.	30	0.167
37	A	5	5	1.	30	0.167
38	A	4	4	1.	30	0.133
39	A	3	3	1.	30	0.1
40	A	4	4	1.	30	0.133
41	A	5	4	1.	30	0.133
42	A	7	6	1.	30	0.2
43	A	6	6	1.	30	0.2
44	A	5	5	1.	30	0.167
45	A	3	3	1.	30	0.1
46	A	4	4	1.	30	0.133
47	A	5	4	1.	30	0.133
48	A	3	3	1.	30	0.1
49	A	8	7	1.	31	0.226
50	A	8	7	1.	31	0.226
51	A	8	7	1.	32	0.219
52	A	4	4	1.	30	0.133
53	A	5	5	1.	30	0.167
54	A	6	5	1.	30	0.167
55	A	2	2	1.	87	0.023
56	A	5	5	1.	81	0.062
57	A	6	6	1.	28	0.214

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
58	A	14	8	1.	30	0.267
59	A	9	7	1.	30	0.233
60	A	5	5	1.	28	0.179
61	A	2	2	1.	21	0.095
62	A	5	3	1.	30	0.1
63	A	7	5	1.	30	0.167
64	A	14	9	1.28	32	0.281
65	A	7	7	1.	32	0.219
66	A	6	6	1.	32	0.188
67	A	1	1	1.	32	0.031
68	A	3	3	1.	32	0.094
69	A	6	6	1.	32	0.188
70	A	9	8	1.	32	0.25
71	A	14	9	1.19	32	0.281
72	A	7	7	1.	32	0.219
73	A	6	6	1.	32	0.188
74	A	3	3	1.	32	0.094
75	A	6	6	1.	32	0.188
76	A	9	8	1.	32	0.25
77	A	12	8	1.	32	0.25
78	A	6	6	1.	32	0.188
79	A	1	1	1.	32	0.031
80	A	3	2	1.	32	0.062
81	A	5	5	1.	32	0.156
82	A	8	7	1.	32	0.219
83	A	14	9	1.	32	0.281
84	A	3	3	1.	32	0.094
85	A	3	3	1.	32	0.094
86	A	5	5	1.	32	0.156
87	A	8	7	1.	32	0.219
88	A	11	6	1.	32	0.188
89	A	7	7	0.99	28	0.25
90	A	6	6	1.	28	0.214
91	A	1	1	1.	28	0.036
92	A	3	3	1.	28	0.107
93	A	6	6	1.	28	0.214
94	A	6	6	1.	32	0.188
95	A	1	1	1.	32	0.031
96	A	1	1	1.	32	0.031
97	A	3	3	1.	30	0.1
98	A	4	4	1.	28	0.143
99	A	11	9	1.	33	0.273
100	A	8	7	1.	32	0.219
101	A	11	9	1.	33	0.273

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
102	A	8	7	1.	32	0.219
103	A	0	0	0.	0	0.
104	A	7	7	1.	34	0.206
105	A	2	2	1.	34	0.059
106	A	2	2	1.	34	0.059
107	A	0	0	0.	0	0.
108	A	8	8	1.	34	0.235
109	A	5	5	1.	34	0.147
110	A	0	0	0.	0	0.
111	A	7	7	1.	34	0.206
112	A	0	0	0.	0	0.
113	A	2	2	1.	34	0.059
114	A	2	2	1.	34	0.059
115	A	0	0	0.	0	0.

Chapter 3

Listing of integrals

3.1 $\int \left(a+bx^2\right)\left(c+dx^2\right)\left(e+fx^2\right)^4\,dx$

Optimal. Leaf size=172

$$\frac{1}{5}e^2x^5(2af(3cf+2de)+be(4cf+de))+\frac{1}{3}e^3x^3(4acf+ade+bce)+\frac{1}{11}f^3x^{11}(adf+bcf+4bde)+\frac{1}{9}f^2x^9(af(cf+4d$$

```
[Out] a*c*e^4*x + (e^3*(b*c*e + a*d*e + 4*a*c*f)*x^3)/3 + (e^2*(2*a*f*(2*d*e + 3*
c*f) + b*e*(d*e + 4*c*f))*x^5)/5 + (2*e*f*(a*f*(3*d*e + 2*c*f) + b*e*(2*d*e
 + 3*c*f))*x^7)/7 + (f^2*(a*f*(4*d*e + c*f) + 2*b*e*(3*d*e + 2*c*f))*x^9)/9
 + (f^3*(4*b*d*e + b*c*f + a*d*f)*x^11)/11 + (b*d*f^4*x^13)/13
```

Rubi [A] time = 0.192214, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {521}

$$\frac{1}{5}e^2x^5(2af(3cf+2de)+be(4cf+de))+\frac{1}{3}e^3x^3(4acf+ade+bce)+\frac{1}{11}f^3x^{11}(adf+bcf+4bde)+\frac{1}{9}f^2x^9(af(cf+4d$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)*(c + d*x^2)*(e + f*x^2)^4,x]
```

```
[Out] a*c*e^4*x + (e^3*(b*c*e + a*d*e + 4*a*c*f)*x^3)/3 + (e^2*(2*a*f*(2*d*e + 3*
c*f) + b*e*(d*e + 4*c*f))*x^5)/5 + (2*e*f*(a*f*(3*d*e + 2*c*f) + b*e*(2*d*e
 + 3*c*f))*x^7)/7 + (f^2*(a*f*(4*d*e + c*f) + 2*b*e*(3*d*e + 2*c*f))*x^9)/9
 + (f^3*(4*b*d*e + b*c*f + a*d*f)*x^11)/11 + (b*d*f^4*x^13)/13
```

Rule 521

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c +
d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[p
, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rubi steps

$$\int \left(a+bx^2\right)\left(c+dx^2\right)\left(e+fx^2\right)^4\,dx=\int \left(ace^4+e^3(bce+ade+4acf)x^2+e^2(2af(2de+3cf)+be(de+4cf))x^4+2e$$

$$=ace^4x+\frac{1}{3}e^3(bce+ade+4acf)x^3+\frac{1}{5}e^2(2af(2de+3cf)+be(de+4cf))x^5+\frac{2}{7}$$

Mathematica [A] time = 0.087498, size = 172, normalized size = 1.

$$\frac{1}{5}e^2x^5(2af(3cf+2de)+be(4cf+de))+\frac{1}{3}e^3x^3(4acf+ade+bce)+\frac{1}{11}f^3x^{11}(adf+bcf+4bde)+\frac{1}{9}f^2x^9(af(cf+4de)+$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)*(c + d*x^2)*(e + f*x^2)^4,x]
```

```
[Out] a*c*e^4*x + (e^3*(b*c*e + a*d*e + 4*a*c*f)*x^3)/3 + (e^2*(2*a*f*(2*d*e + 3*
c*f) + b*e*(d*e + 4*c*f))*x^5)/5 + (2*e*f*(a*f*(3*d*e + 2*c*f) + b*e*(2*d*e
 + 3*c*f))*x^7)/7 + (f^2*(a*f*(4*d*e + c*f) + 2*b*e*(3*d*e + 2*c*f))*x^9)/9
 + (f^3*(4*b*d*e + b*c*f + a*d*f)*x^11)/11 + (b*d*f^4*x^13)/13
```

Maple [A] time = 0.002, size = 176, normalized size = 1.

$$\frac{bdf^4x^{13}}{13}+\frac{\left((ad+bc)f^4+4bdef^3\right)x^{11}}{11}+\frac{\left(acf^4+4(ad+bc)ef^3+6bde^2f^2\right)x^9}{9}+\frac{\left(4acef^3+6(ad+bc)e^2f^2+4bde^3f\right.}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x^2+c)*(f*x^2+e)^4,x)
```

```
[Out] 1/13*b*d*f^4*x^13+1/11*((a*d+b*c)*f^4+4*b*d*e*f^3)*x^11+1/9*(a*c*f^4+4*(a*d
+b*c)*e*f^3+6*b*d*e^2*f^2)*x^9+1/7*(4*a*c*e*f^3+6*(a*d+b*c)*e^2*f^2+4*b*d*e
^3*f)*x^7+1/5*(6*a*c*e^2*f^2+4*(a*d+b*c)*e^3*f+b*d*e^4)*x^5+1/3*(4*a*c*e^3*
f+(a*d+b*c)*e^4)*x^3+a*c*e^4*x
```

Maxima [A] time = 0.986467, size = 236, normalized size = 1.37

$$\frac{1}{13}bdf^4x^{13}+\frac{1}{11}\left(4bdef^3+(bc+ad)f^4\right)x^{11}+\frac{1}{9}\left(6bde^2f^2+acf^4+4(bc+ad)ef^3\right)x^9+\frac{2}{7}\left(2bde^3f+2acef^3+3(bc+a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)*(f*x^2+e)^4,x, algorithm="maxima")
```

```
[Out] 1/13*b*d*f^4*x^13 + 1/11*(4*b*d*e*f^3 + (b*c + a*d)*f^4)*x^11 + 1/9*(6*b*d*
e^2*f^2 + a*c*f^4 + 4*(b*c + a*d)*e*f^3)*x^9 + 2/7*(2*b*d*e^3*f + 2*a*c*e*f
^3 + 3*(b*c + a*d)*e^2*f^2)*x^7 + a*c*e^4*x + 1/5*(b*d*e^4 + 6*a*c*e^2*f^2
+ 4*(b*c + a*d)*e^3*f)*x^5 + 1/3*(4*a*c*e^3*f + (b*c + a*d)*e^4)*x^3
```

Fricas [A] time = 1.24982, size = 531, normalized size = 3.09

$$\frac{1}{13}x^{13}f^4db+\frac{4}{11}x^{11}f^3edb+\frac{1}{11}x^{11}f^4cb+\frac{1}{11}x^{11}f^4da+\frac{2}{3}x^9f^2e^2db+\frac{4}{9}x^9f^3ecb+\frac{4}{9}x^9f^3eda+\frac{1}{9}x^9f^4ca+\frac{4}{7}x^7fe^3db+\frac{6}{7}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)*(f*x^2+e)^4,x, algorithm="fricas")
```

```
[Out] 1/13*x^13*f^4*d*b + 4/11*x^11*f^3*e*d*b + 1/11*x^11*f^4*c*b + 1/11*x^11*f^4
*d*a + 2/3*x^9*f^2*e^2*d*b + 4/9*x^9*f^3*e*c*b + 4/9*x^9*f^3*e*d*a + 1/9*x^
```

```
9*f^4*c*a + 4/7*x^7*f*e^3*d*b + 6/7*x^7*f^2*e^2*c*b + 6/7*x^7*f^2*e^2*d*a +
 4/7*x^7*f^3*e*c*a + 1/5*x^5*e^4*d*b + 4/5*x^5*f*e^3*c*b + 4/5*x^5*f*e^3*d*
a + 6/5*x^5*f^2*e^2*c*a + 1/3*x^3*e^4*c*b + 1/3*x^3*e^4*d*a + 4/3*x^3*f*e^3
*c*a + x*e^4*c*a
```

Sympy [A] time = 0.088928, size = 236, normalized size = 1.37

$$ace^4x + \frac{bdf^4x^{13}}{13} + x^{11}\left(\frac{adf^4}{11} + \frac{bcf^4}{11} + \frac{4bdef^3}{11}\right) + x^9\left(\frac{acf^4}{9} + \frac{4adef^3}{9} + \frac{4bcef^3}{9} + \frac{2bde^2f^2}{3}\right) + x^7\left(\frac{4acef^3}{7} + \frac{6ade^2f}{7}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+c)*(f*x**2+e)**4,x)
```

```
[Out] a*c*e**4*x + b*d*f**4*x**13/13 + x**11*(a*d*f**4/11 + b*c*f**4/11 + 4*b*d*e
*f**3/11) + x**9*(a*c*f**4/9 + 4*a*d*e*f**3/9 + 4*b*c*e*f**3/9 + 2*b*d*e**2
*f**2/3) + x**7*(4*a*c*e*f**3/7 + 6*a*d*e**2*f**2/7 + 6*b*c*e**2*f**2/7 + 4
*b*d*e**3*f/7) + x**5*(6*a*c*e**2*f**2/5 + 4*a*d*e**3*f/5 + 4*b*c*e**3*f/5
+ b*d*e**4/5) + x**3*(4*a*c*e**3*f/3 + a*d*e**4/3 + b*c*e**4/3)
```

Giac [A] time = 1.20963, size = 284, normalized size = 1.65

$$\frac{1}{13}bdf^4x^{13} + \frac{1}{11}bcf^4x^{11} + \frac{1}{11}adf^4x^{11} + \frac{4}{11}bdf^3x^{11}e + \frac{1}{9}acf^4x^9 + \frac{4}{9}bcf^3x^9e + \frac{4}{9}adf^3x^9e + \frac{2}{3}bdf^2x^9e^2 + \frac{4}{7}acf^3x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)*(f*x^2+e)^4,x, algorithm="giac")
```

```
[Out] 1/13*b*d*f^4*x^13 + 1/11*b*c*f^4*x^11 + 1/11*a*d*f^4*x^11 + 4/11*b*d*f^3*x^
11*e + 1/9*a*c*f^4*x^9 + 4/9*b*c*f^3*x^9*e + 4/9*a*d*f^3*x^9*e + 2/3*b*d*f^
2*x^9*e^2 + 4/7*a*c*f^3*x^7*e + 6/7*b*c*f^2*x^7*e^2 + 6/7*a*d*f^2*x^7*e^2 +
 4/7*b*d*f*x^7*e^3 + 6/5*a*c*f^2*x^5*e^2 + 4/5*b*c*f*x^5*e^3 + 4/5*a*d*f*x^
5*e^3 + 1/5*b*d*x^5*e^4 + 4/3*a*c*f*x^3*e^3 + 1/3*b*c*x^3*e^4 + 1/3*a*d*x^3
*e^4 + a*c*x*e^4
```

3.2 $\int (a+bx^2)(c+dx^2)(e+fx^2)^3\,dx$

Optimal. Leaf size=130

$$\frac{1}{3}e^2x^3(3acf+ade+bce)+\frac{1}{9}f^2x^9(adf+bcf+3bde)+\frac{1}{7}fx^7(af(cf+3de)+3be(cf+de))+\frac{1}{5}ex^5(3af(cf+de)+be(3c$$

```
[Out] a*c*e^3*x + (e^2*(b*c*e + a*d*e + 3*a*c*f)*x^3)/3 + (e*(3*a*f*(d*e + c*f) +
 b*e*(d*e + 3*c*f))*x^5)/5 + (f*(3*b*e*(d*e + c*f) + a*f*(3*d*e + c*f))*x^7
)/7 + (f^2*(3*b*d*e + b*c*f + a*d*f)*x^9)/9 + (b*d*f^3*x^11)/11
```

Rubi [A] time = 0.13458, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {521}

$$\frac{1}{3}e^2x^3(3acf+ade+bce)+\frac{1}{9}f^2x^9(adf+bcf+3bde)+\frac{1}{7}fx^7(af(cf+3de)+3be(cf+de))+\frac{1}{5}ex^5(3af(cf+de)+be(3c$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)*(c + d*x^2)*(e + f*x^2)^3,x]
```

```
[Out] a*c*e^3*x + (e^2*(b*c*e + a*d*e + 3*a*c*f)*x^3)/3 + (e*(3*a*f*(d*e + c*f) +
 b*e*(d*e + 3*c*f))*x^5)/5 + (f*(3*b*e*(d*e + c*f) + a*f*(3*d*e + c*f))*x^7
)/7 + (f^2*(3*b*d*e + b*c*f + a*d*f)*x^9)/9 + (b*d*f^3*x^11)/11
```

Rule 521

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c +
d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[p
, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rubi steps

$$\int (a+bx^2)(c+dx^2)(e+fx^2)^3\,dx = \int \left(ace^3+e^2(bce+ade+3acf)x^2+e(3af(de+cf)+be(de+3cf))x^4+f(3be(de$$
$$= ace^3x+\frac{1}{3}e^2(bce+ade+3acf)x^3+\frac{1}{5}e(3af(de+cf)+be(de+3cf))x^5+\frac{1}{7}f(3be($$

Mathematica [A] time = 0.0656983, size = 130, normalized size = 1.

$$\frac{1}{3}e^2x^3(3acf+ade+bce)+\frac{1}{9}f^2x^9(adf+bcf+3bde)+\frac{1}{7}fx^7(af(cf+3de)+3be(cf+de))+\frac{1}{5}ex^5(3af(cf+de)+be(3c$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)*(c + d*x^2)*(e + f*x^2)^3,x]
```

```
[Out] a*c*e^3*x + (e^2*(b*c*e + a*d*e + 3*a*c*f)*x^3)/3 + (e*(3*a*f*(d*e + c*f) +
 b*e*(d*e + 3*c*f))*x^5)/5 + (f*(3*b*e*(d*e + c*f) + a*f*(3*d*e + c*f))*x^7
)/7 + (f^2*(3*b*d*e + b*c*f + a*d*f)*x^9)/9 + (b*d*f^3*x^11)/11
```

Maple [A] time = 0.001, size = 135, normalized size = 1.

$$\frac{bdf^3x^{11}}{11}+\frac{\left((ad+bc)\,f^3+3\,bdef^2\right)x^9}{9}+\frac{\left(acf^3+3\,(ad+bc)\,ef^2+3\,bde^2f\right)x^7}{7}+\frac{\left(3\,acef^2+3\,(ad+bc)\,e^2f+bde^3\right)x}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x^2+c)*(f*x^2+e)^3,x)
```

```
[Out] 1/11*b*d*f^3*x^11+1/9*((a*d+b*c)*f^3+3*b*d*e*f^2)*x^9+1/7*(a*c*f^3+3*(a*d+b
*c)*e*f^2+3*b*d*e^2*f)*x^7+1/5*(3*a*c*e*f^2+3*(a*d+b*c)*e^2*f+b*d*e^3)*x^5+
1/3*(3*a*c*e^2*f+(a*d+b*c)*e^3)*x^3+a*c*e^3*x
```

Maxima [A] time = 0.995561, size = 181, normalized size = 1.39

$$\frac{1}{11}\,bdf^3x^{11}+\frac{1}{9}\left(3\,bdef^2+(bc+ad)f^3\right)x^9+\frac{1}{7}\left(3\,bde^2f+acf^3+3\,(bc+ad)ef^2\right)x^7+ace^3x+\frac{1}{5}\left(bde^3+3\,acef^2+3\,(b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)*(f*x^2+e)^3,x, algorithm="maxima")
```

```
[Out] 1/11*b*d*f^3*x^11 + 1/9*(3*b*d*e*f^2 + (b*c + a*d)*f^3)*x^9 + 1/7*(3*b*d*e^
2*f + a*c*f^3 + 3*(b*c + a*d)*e*f^2)*x^7 + a*c*e^3*x + 1/5*(b*d*e^3 + 3*a*c
*e*f^2 + 3*(b*c + a*d)*e^2*f)*x^5 + 1/3*(3*a*c*e^2*f + (b*c + a*d)*e^3)*x^3
```

Fricas [A] time = 1.24731, size = 398, normalized size = 3.06

$$\frac{1}{11}x^{11}f^3db+\frac{1}{3}x^9f^2edb+\frac{1}{9}x^9f^3cb+\frac{1}{9}x^9f^3da+\frac{3}{7}x^7fe^2db+\frac{3}{7}x^7f^2ecb+\frac{3}{7}x^7f^2eda+\frac{1}{7}x^7f^3ca+\frac{1}{5}x^5e^3db+\frac{3}{5}x^5fe^2c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)*(f*x^2+e)^3,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*f^3*d*b + 1/3*x^9*f^2*e*d*b + 1/9*x^9*f^3*c*b + 1/9*x^9*f^3*d*a +
 3/7*x^7*f*e^2*d*b + 3/7*x^7*f^2*e*c*b + 3/7*x^7*f^2*e*d*a + 1/7*x^7*f^3*c*
a + 1/5*x^5*e^3*d*b + 3/5*x^5*f*e^2*c*b + 3/5*x^5*f*e^2*d*a + 3/5*x^5*f^2*e
*c*a + 1/3*x^3*e^3*c*b + 1/3*x^3*e^3*d*a + x^3*f*e^2*c*a + x*e^3*c*a
```

Sympy [A] time = 0.080539, size = 173, normalized size = 1.33

$$ace^3x+\frac{bdf^3x^{11}}{11}+x^9\left(\frac{adf^3}{9}+\frac{bcf^3}{9}+\frac{bdef^2}{3}\right)+x^7\left(\frac{acf^3}{7}+\frac{3adef^2}{7}+\frac{3bcef^2}{7}+\frac{3bde^2f}{7}\right)+x^5\left(\frac{3acef^2}{5}+\frac{3ade^2f}{5}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+c)*(f*x**2+e)**3,x)
```

```
[Out] a*c*e**3*x + b*d*f**3*x**11/11 + x**9*(a*d*f**3/9 + b*c*f**3/9 + b*d*e*f**2
/3) + x**7*(a*c*f**3/7 + 3*a*d*e*f**2/7 + 3*b*c*e*f**2/7 + 3*b*d*e**2*f/7)
+ x**5*(3*a*c*e*f**2/5 + 3*a*d*e**2*f/5 + 3*b*c*e**2*f/5 + b*d*e**3/5) + x*
```

```
*3*(a*c*e**2*f + a*d*e**3/3 + b*c*e**3/3)
```

Giac [A] time = 1.16037, size = 217, normalized size = 1.67

$$\frac{1}{11}bdf^3x^{11} + \frac{1}{9}bcf^3x^9 + \frac{1}{9}adf^3x^9 + \frac{1}{3}bdf^2x^9e + \frac{1}{7}acf^3x^7 + \frac{3}{7}bcf^2x^7e + \frac{3}{7}adf^2x^7e + \frac{3}{7}bdfx^7e^2 + \frac{3}{5}acf^2x^5e + \frac{3}{5}bcf$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)*(f*x^2+e)^3,x, algorithm="giac")
```

```
[Out] 1/11*b*d*f^3*x^11 + 1/9*b*c*f^3*x^9 + 1/9*a*d*f^3*x^9 + 1/3*b*d*f^2*x^9*e +
 1/7*a*c*f^3*x^7 + 3/7*b*c*f^2*x^7*e + 3/7*a*d*f^2*x^7*e + 3/7*b*d*f*x^7*e^
2 + 3/5*a*c*f^2*x^5*e + 3/5*b*c*f*x^5*e^2 + 3/5*a*d*f*x^5*e^2 + 1/5*b*d*x^5
*e^3 + a*c*f*x^3*e^2 + 1/3*b*c*x^3*e^3 + 1/3*a*d*x^3*e^3 + a*c*x*e^3
```

3.3 $\int \left(a+bx^2\right)\left(c+dx^2\right)\left(e+fx^2\right)^2\,dx$

Optimal. Leaf size=94

$$\frac{1}{7}fx^7(adf+bcf+2bde)+\frac{1}{5}x^5(af(cf+2de)+be(2cf+de))+\frac{1}{3}ex^3(2acf+ade+bce)+ace^2x+\frac{1}{9}bdf^2x^9$$

```
[Out] a*c*e^2*x + (e*(b*c*e + a*d*e + 2*a*c*f)*x^3)/3 + ((a*f*(2*d*e + c*f) + b*e
*(d*e + 2*c*f))*x^5)/5 + (f*(2*b*d*e + b*c*f + a*d*f)*x^7)/7 + (b*d*f^2*x^9
)/9
```

Rubi [A] time = 0.0826775, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {521}

$$\frac{1}{7}fx^7(adf+bcf+2bde)+\frac{1}{5}x^5(af(cf+2de)+be(2cf+de))+\frac{1}{3}ex^3(2acf+ade+bce)+ace^2x+\frac{1}{9}bdf^2x^9$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)*(c + d*x^2)*(e + f*x^2)^2,x]
```

```
[Out] a*c*e^2*x + (e*(b*c*e + a*d*e + 2*a*c*f)*x^3)/3 + ((a*f*(2*d*e + c*f) + b*e
*(d*e + 2*c*f))*x^5)/5 + (f*(2*b*d*e + b*c*f + a*d*f)*x^7)/7 + (b*d*f^2*x^9
)/9
```

Rule 521

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c +
d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[p
, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rubi steps

$$\int \left(a+bx^2\right)\left(c+dx^2\right)\left(e+fx^2\right)^2\,dx=\int \left(ace^2+e(bce+ade+2acf)x^2+(af(2de+cf)+be(de+2cf))x^4+f(2bde\right.$$

$$=ace^2x+\frac{1}{3}e(bce+ade+2acf)x^3+\frac{1}{5}(af(2de+cf)+be(de+2cf))x^5+\frac{1}{7}f(2bc$$

Mathematica [A] time = 0.0358652, size = 96, normalized size = 1.02

$$\frac{1}{5}x^5\left(acf^2+2adef+2bcef+bde^2\right)+\frac{1}{7}fx^7(adf+bcf+2bde)+\frac{1}{3}ex^3(2acf+ade+bce)+ace^2x+\frac{1}{9}bdf^2x^9$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)*(c + d*x^2)*(e + f*x^2)^2,x]
```

```
[Out] a*c*e^2*x + (e*(b*c*e + a*d*e + 2*a*c*f)*x^3)/3 + ((b*d*e^2 + 2*b*c*e*f + 2
*a*d*e*f + a*c*f^2)*x^5)/5 + (f*(2*b*d*e + b*c*f + a*d*f)*x^7)/7 + (b*d*f^2
*x^9)/9
```

Maple [A] time = 0., size = 94, normalized size = 1.

$$\frac{bdf^2x^9}{9}+\frac{\left((ad+bc)\,f^2+2\,bdef\right)x^7}{7}+\frac{\left(acf^2+2\,(ad+bc)\,ef+bde^2\right)x^5}{5}+\frac{\left(2\,acef+(ad+bc)\,e^2\right)x^3}{3}+ace^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x^2+c)*(f*x^2+e)^2,x)
```

```
[Out] 1/9*b*d*f^2*x^9+1/7*((a*d+b*c)*f^2+2*b*d*e*f)*x^7+1/5*(a*c*f^2+2*(a*d+b*c)*
e*f+b*d*e^2)*x^5+1/3*(2*a*c*e*f+(a*d+b*c)*e^2)*x^3+a*c*e^2*x
```

Maxima [A] time = 0.990341, size = 126, normalized size = 1.34

$$\frac{1}{9}\,bdf^2x^9+\frac{1}{7}\left(2\,bdef+(bc+ad)f^2\right)x^7+\frac{1}{5}\left(bde^2+acf^2+2\,(bc+ad)ef\right)x^5+ace^2x+\frac{1}{3}\left(2\,acef+(bc+ad)e^2\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)*(f*x^2+e)^2,x, algorithm="maxima")
```

```
[Out] 1/9*b*d*f^2*x^9 + 1/7*(2*b*d*e*f + (b*c + a*d)*f^2)*x^7 + 1/5*(b*d*e^2 + a*
c*f^2 + 2*(b*c + a*d)*e*f)*x^5 + a*c*e^2*x + 1/3*(2*a*c*e*f + (b*c + a*d)*e
^2)*x^3
```

Fricas [A] time = 1.24458, size = 282, normalized size = 3.

$$\frac{1}{9}x^9f^2db+\frac{2}{7}x^7fedb+\frac{1}{7}x^7f^2cb+\frac{1}{7}x^7f^2da+\frac{1}{5}x^5e^2db+\frac{2}{5}x^5fecb+\frac{2}{5}x^5feda+\frac{1}{5}x^5f^2ca+\frac{1}{3}x^3e^2cb+\frac{1}{3}x^3e^2da+\frac{2}{3}x^3fe$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)*(f*x^2+e)^2,x, algorithm="fricas")
```

```
[Out] 1/9*x^9*f^2*d*b + 2/7*x^7*f*e*d*b + 1/7*x^7*f^2*c*b + 1/7*x^7*f^2*d*a + 1/5
*x^5*e^2*d*b + 2/5*x^5*f*e*c*b + 2/5*x^5*f*e*d*a + 1/5*x^5*f^2*c*a + 1/3*x^
3*e^2*c*b + 1/3*x^3*e^2*d*a + 2/3*x^3*f*e*c*a + x*e^2*c*a
```

Sympy [A] time = 0.072183, size = 121, normalized size = 1.29

$$ace^2x+\frac{bdf^2x^9}{9}+x^7\left(\frac{adf^2}{7}+\frac{bcf^2}{7}+\frac{2bdef}{7}\right)+x^5\left(\frac{acf^2}{5}+\frac{2adef}{5}+\frac{2bcef}{5}+\frac{bde^2}{5}\right)+x^3\left(\frac{2acef}{3}+\frac{ade^2}{3}+\frac{bce^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+c)*(f*x**2+e)**2,x)
```

```
[Out] a*c*e**2*x + b*d*f**2*x**9/9 + x**7*(a*d*f**2/7 + b*c*f**2/7 + 2*b*d*e*f/7)
 + x**5*(a*c*f**2/5 + 2*a*d*e*f/5 + 2*b*c*e*f/5 + b*d*e**2/5) + x**3*(2*a*c
*e*f/3 + a*d*e**2/3 + b*c*e**2/3)
```

Giac [A] time = 1.21591, size = 154, normalized size = 1.64

$$\frac{1}{9}bdf^2x^9 + \frac{1}{7}bcf^2x^7 + \frac{1}{7}adf^2x^7 + \frac{2}{7}bdfx^7e + \frac{1}{5}acf^2x^5 + \frac{2}{5}bcfx^5e + \frac{2}{5}adfx^5e + \frac{1}{5}bdx^5e^2 + \frac{2}{3}acfx^3e + \frac{1}{3}bcx^3e^2 +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)*(f*x^2+e)^2,x, algorithm="giac")
```

```
[Out] 1/9*b*d*f^2*x^9 + 1/7*b*c*f^2*x^7 + 1/7*a*d*f^2*x^7 + 2/7*b*d*f*x^7*e + 1/5
*a*c*f^2*x^5 + 2/5*b*c*f*x^5*e + 2/5*a*d*f*x^5*e + 1/5*b*d*x^5*e^2 + 2/3*a*
c*f*x^3*e + 1/3*b*c*x^3*e^2 + 1/3*a*d*x^3*e^2 + a*c*x*e^2
```

3.4 $\int (a+bx^2)(c+dx^2)(e+fx^2)\,dx$

Optimal. Leaf size=56

$$\frac{1}{5}x^5(adf+bcf+bde)+\frac{1}{3}x^3(acf+ade+bce)+acex+\frac{1}{7}bdfx^7$$

```
[Out] a*c*e*x + ((b*c*e + a*d*e + a*c*f)*x^3)/3 + ((b*d*e + b*c*f + a*d*f)*x^5)/5
 + (b*d*f*x^7)/7
```

Rubi [A] time = 0.0370527, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {521}

$$\frac{1}{5}x^5(adf+bcf+bde)+\frac{1}{3}x^3(acf+ade+bce)+acex+\frac{1}{7}bdfx^7$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)*(c + d*x^2)*(e + f*x^2),x]
```

```
[Out] a*c*e*x + ((b*c*e + a*d*e + a*c*f)*x^3)/3 + ((b*d*e + b*c*f + a*d*f)*x^5)/5
 + (b*d*f*x^7)/7
```

Rule 521

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c +
d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[p
, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rubi steps

$$\begin{aligned}\int (a+bx^2)(c+dx^2)(e+fx^2)\,dx &= \int \left(ace+(bce+ade+acf)x^2+(bde+bcf+adf)x^4+bdfx^6\right)dx\\ &= acex+\frac{1}{3}(bce+ade+acf)x^3+\frac{1}{5}(bde+bcf+adf)x^5+\frac{1}{7}bdfx^7\end{aligned}$$

Mathematica [A] time = 0.0124191, size = 56, normalized size = 1.

$$\frac{1}{5}x^5(adf+bcf+bde)+\frac{1}{3}x^3(acf+ade+bce)+acex+\frac{1}{7}bdfx^7$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)*(c + d*x^2)*(e + f*x^2),x]
```

```
[Out] a*c*e*x + ((b*c*e + a*d*e + a*c*f)*x^3)/3 + ((b*d*e + b*c*f + a*d*f)*x^5)/5
 + (b*d*f*x^7)/7
```

Maple [A] time = 0., size = 53, normalized size = 1.

$$\frac{bdfx^7}{7}+\frac{\left((ad+bc)f+bde\right)x^5}{5}+\frac{\left(acf+(ad+bc)e\right)x^3}{3}+acex$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x^2+c)*(f*x^2+e),x)
```

```
[Out] 1/7*b*d*f*x^7+1/5*((a*d+b*c)*f+b*d*e)*x^5+1/3*(a*c*f+(a*d+b*c)*e)*x^3+a*c*e
*x
```

Maxima [A] time = 1.02574, size = 70, normalized size = 1.25

$$\frac{1}{7}\,bdfx^{7}+\frac{1}{5}\left(bde+(bc+ad)f\right)x^{5}+acex+\frac{1}{3}\left(acf+(bc+ad)e\right)x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)*(f*x^2+e),x, algorithm="maxima")
```

```
[Out] 1/7*b*d*f*x^7 + 1/5*(b*d*e + (b*c + a*d)*f)*x^5 + a*c*e*x + 1/3*(a*c*f + (b
*c + a*d)*e)*x^3
```

Fricas [A] time = 1.22783, size = 163, normalized size = 2.91

$$\frac{1}{7}x^{7}fdb+\frac{1}{5}x^{5}edb+\frac{1}{5}x^{5}fcb+\frac{1}{5}x^{5}fda+\frac{1}{3}x^{3}ecb+\frac{1}{3}x^{3}eda+\frac{1}{3}x^{3}fca+xeca$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)*(f*x^2+e),x, algorithm="fricas")
```

```
[Out] 1/7*x^7*f*d*b + 1/5*x^5*e*d*b + 1/5*x^5*f*c*b + 1/5*x^5*f*d*a + 1/3*x^3*e*c
*b + 1/3*x^3*e*d*a + 1/3*x^3*f*c*a + x*e*c*a
```

Sympy [A] time = 0.062564, size = 63, normalized size = 1.12

$$acex+\frac{bdfx^{7}}{7}+x^{5}\left(\frac{adf}{5}+\frac{bcf}{5}+\frac{bde}{5}\right)+x^{3}\left(\frac{acf}{3}+\frac{ade}{3}+\frac{bce}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+c)*(f*x**2+e),x)
```

```
[Out] a*c*e*x + b*d*f*x**7/7 + x**5*(a*d*f/5 + b*c*f/5 + b*d*e/5) + x**3*(a*c*f/3
 + a*d*e/3 + b*c*e/3)
```

Giac [A] time = 1.14974, size = 89, normalized size = 1.59

$$\frac{1}{7}\,bdfx^{7}+\frac{1}{5}\,bcfx^{5}+\frac{1}{5}\,adfx^{5}+\frac{1}{5}\,bdx^{5}e+\frac{1}{3}\,acfx^{3}+\frac{1}{3}\,bcx^{3}e+\frac{1}{3}\,adx^{3}e+acxe$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)*(f*x^2+e),x, algorithm="giac")
```

```
[Out] 1/7*b*d*f*x^7 + 1/5*b*c*f*x^5 + 1/5*a*d*f*x^5 + 1/5*b*d*x^5*e + 1/3*a*c*f*x
^3 + 1/3*b*c*x^3*e + 1/3*a*d*x^3*e + a*c*x*e
```

3.5 $\int \frac{(a+bx^2)(c+dx^2)}{e+fx^2}\,dx$

Optimal. Leaf size=81

$$-\frac{x(-2adf-3bcf+3bde)}{3f^2}+\frac{(be-af)(de-cf)\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}f^{5/2}}+\frac{dx\left(a+bx^2\right)}{3f}$$

```
[Out] -((3*b*d*e - 3*b*c*f - 2*a*d*f)*x)/(3*f^2) + (d*x*(a + b*x^2))/(3*f) + ((b*
e - a*f)*(d*e - c*f)*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e]*f^(5/2))
```

Rubi [A] time = 0.0798565, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {528, 388, 205}

$$-\frac{x(-2adf-3bcf+3bde)}{3f^2}+\frac{(be-af)(de-cf)\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}f^{5/2}}+\frac{dx\left(a+bx^2\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(c + d*x^2))/(e + f*x^2),x]
```

```
[Out] -((3*b*d*e - 3*b*c*f - 2*a*d*f)*x)/(3*f^2) + (d*x*(a + b*x^2))/(3*f) + ((b*
e - a*f)*(d*e - c*f)*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e]*f^(5/2))
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(a+bx^2)(c+dx^2)}{e+fx^2}\,dx = \frac{dx(a+bx^2)}{3f} + \frac{\int \frac{-a(de-3cf)-(3bde-3bcf-2adf)x^2}{e+fx^2}\,dx}{3f}$$
$$= -\frac{(3bde-3bcf-2adf)x}{3f^2} + \frac{dx(a+bx^2)}{3f} + \frac{((be-af)(de-cf))\int \frac{1}{e+fx^2}\,dx}{f^2}$$
$$= -\frac{(3bde-3bcf-2adf)x}{3f^2} + \frac{dx(a+bx^2)}{3f} + \frac{(be-af)(de-cf)\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}f^{5/2}}$$

Mathematica [A] time = 0.0534778, size = 72, normalized size = 0.89

$$\frac{x(adf+bcf-bde)}{f^2} + \frac{(be-af)(de-cf)\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}f^{5/2}} + \frac{bdx^3}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(c + d*x^2))/(e + f*x^2),x]
```

```
[Out] ((-(b*d*e) + b*c*f + a*d*f)*x)/f^2 + (b*d*x^3)/(3*f) + ((b*e - a*f)*(d*e -
c*f)*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e]*f^(5/2))
```

Maple [A] time = 0.006, size = 119, normalized size = 1.5

$$\frac{x^3bd}{3f} + \frac{adx}{f} + \frac{bcx}{f} - \frac{bdex}{f^2} + ac\arctan\left(fx\frac{1}{\sqrt{ef}}\right)\frac{1}{\sqrt{ef}} - \frac{ade}{f}\arctan\left(fx\frac{1}{\sqrt{ef}}\right)\frac{1}{\sqrt{ef}} - \frac{bce}{f}\arctan\left(fx\frac{1}{\sqrt{ef}}\right)\frac{1}{\sqrt{ef}} + \frac{bde^2}{f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x^2+c)/(f*x^2+e),x)
```

```
[Out] 1/3/f*x^3*b*d+1/f*a*d*x+1/f*b*c*x-1/f^2*b*d*e*x+1/(e*f)^(1/2)*arctan(x*f/(e
*f)^(1/2))*a*c-1/f/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*a*d*e-1/f/(e*f)^(1/2
)*arctan(x*f/(e*f)^(1/2))*b*c*e+1/f^2/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*b
*d*e^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)/(f*x^2+e),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.48632, size = 423, normalized size = 5.22

$$\left[\frac{2bdef^2x^3 - 3\left(bde^2 + acf^2 - (bc+ad)ef\right)\sqrt{-ef}\log\left(\frac{fx^2-2\sqrt{-ef}x-e}{fx^2+e}\right) - 6\left(bde^2f - (bc+ad)ef^2\right)x}{6ef^3}, \frac{bdef^2x^3 + 3\left(bde^2 + ac\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)/(f*x^2+e),x, algorithm="fricas")
```

```
[Out] [1/6*(2*b*d*e*f^2*x^3 - 3*(b*d*e^2 + a*c*f^2 - (b*c + a*d)*e*f)*sqrt(-e*f)*
log((f*x^2 - 2*sqrt(-e*f)*x - e)/(f*x^2 + e)) - 6*(b*d*e^2*f - (b*c + a*d)*
e*f^2)*x)/(e*f^3), 1/3*(b*d*e*f^2*x^3 + 3*(b*d*e^2 + a*c*f^2 - (b*c + a*d)*
e*f)*sqrt(e*f)*arctan(sqrt(e*f)*x/e) - 3*(b*d*e^2*f - (b*c + a*d)*e*f^2)*x)
/(e*f^3)]
```

Sympy [B] time = 0.836548, size = 206, normalized size = 2.54

$$\frac{bdx^3}{3f} - \frac{\sqrt{-\frac{1}{ef^5}}\left(af-be\right)\left(cf-de\right)\log\left(-\frac{ef^2\sqrt{-\frac{1}{ef^5}}\left(af-be\right)\left(cf-de\right)}{acf^2-adef-bcef+bde^2}+x\right)}{2} + \frac{\sqrt{-\frac{1}{ef^5}}\left(af-be\right)\left(cf-de\right)\log\left(\frac{ef^2\sqrt{-\frac{1}{ef^5}}\left(af-be\right)(}{acf^2-adef-bcef+}\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+c)/(f*x**2+e),x)
```

```
[Out] b*d*x**3/(3*f) - sqrt(-1/(e*f**5))*(a*f - b*e)*(c*f - d*e)*log(-e*f**2*sqrt
(-1/(e*f**5))*(a*f - b*e)*(c*f - d*e)/(a*c*f**2 - a*d*e*f - b*c*e*f + b*d*e
**2) + x)/2 + sqrt(-1/(e*f**5))*(a*f - b*e)*(c*f - d*e)*log(e*f**2*sqrt(-1/
(e*f**5))*(a*f - b*e)*(c*f - d*e)/(a*c*f**2 - a*d*e*f - b*c*e*f + b*d*e**2)
 + x)/2 + x*(a*d*f + b*c*f - b*d*e)/f**2
```

Giac [A] time = 1.19611, size = 108, normalized size = 1.33

$$\frac{\left(acf^2 - bcfe - adfe + bde^2\right)\arctan\left(\sqrt{f}xe^{\left(-\frac{1}{2}\right)}\right)e^{\left(-\frac{1}{2}\right)}}{f^{\frac{5}{2}}} + \frac{bdf^2x^3 + 3\,bcf^2x + 3\,adf^2x - 3\,bdfxe}{3\,f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)/(f*x^2+e),x, algorithm="giac")
```

```
[Out] (a*c*f^2 - b*c*f*e - a*d*f*e + b*d*e^2)*arctan(sqrt(f)*x*e^(-1/2))*e^(-1/2)
/f^(5/2) + 1/3*(b*d*f^2*x^3 + 3*b*c*f^2*x + 3*a*d*f^2*x - 3*b*d*f*x*e)/f^3
```

3.6 $\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^2}\,dx$

Optimal. Leaf size=108

$$-\frac{\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(be(3de-cf)-af(cf+de))}{2e^{3/2}f^{5/2}}-\frac{x\left(a+bx^2\right)(de-cf)}{2ef\left(e+fx^2\right)}+\frac{bx(3de-cf)}{2ef^2}$$

```
[Out] (b*(3*d*e - c*f)*x)/(2*e*f^2) - ((d*e - c*f)*x*(a + b*x^2))/(2*e*f*(e + f*x
^2)) - ((b*e*(3*d*e - c*f) - a*f*(d*e + c*f))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/
(2*e^(3/2)*f^(5/2))
```

Rubi [A] time = 0.0849137, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {526, 388, 205}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(be(3de-cf)-af(cf+de))}{2e^{3/2}f^{5/2}}-\frac{x\left(a+bx^2\right)(de-cf)}{2ef\left(e+fx^2\right)}+\frac{bx(3de-cf)}{2ef^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(c + d*x^2))/(e + f*x^2)^2,x]
```

```
[Out] (b*(3*d*e - c*f)*x)/(2*e*f^2) - ((d*e - c*f)*x*(a + b*x^2))/(2*e*f*(e + f*x
^2)) - ((b*e*(3*d*e - c*f) - a*f*(d*e + c*f))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/
(2*e^(3/2)*f^(5/2))
```

Rule 526

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^q)/(a*b*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p +
 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^2}\,dx = -\frac{(de-cf)x(a+bx^2)}{2ef(e+fx^2)} - \frac{\int \frac{-a(de+cf)-b(3de-cf)x^2}{e+fx^2}\,dx}{2ef}$$

$$= \frac{b(3de-cf)x}{2ef^2} - \frac{(de-cf)x(a+bx^2)}{2ef(e+fx^2)} - \frac{(be(3de-cf)-af(de+cf))\int \frac{1}{e+fx^2}\,dx}{2ef^2}$$

$$= \frac{b(3de-cf)x}{2ef^2} - \frac{(de-cf)x(a+bx^2)}{2ef(e+fx^2)} - \frac{(be(3de-cf)-af(de+cf))\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{2e^{3/2}f^{5/2}}$$

Mathematica [A] time = 0.0666011, size = 95, normalized size = 0.88

$$-\frac{\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(be(3de-cf)-af(cf+de))}{2e^{3/2}f^{5/2}} + \frac{x(be-af)(de-cf)}{2ef^2(e+fx^2)} + \frac{bdx}{f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(c + d*x^2))/(e + f*x^2)^2,x]
```

```
[Out] (b*d*x)/f^2 + ((b*e - a*f)*(d*e - c*f)*x)/(2*e*f^2*(e + f*x^2)) - ((b*e*(3*
d*e - c*f) - a*f*(d*e + c*f))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(2*e^(3/2)*f^(5/
2))
```

Maple [A] time = 0.008, size = 163, normalized size = 1.5

$$\frac{bdx}{f^2} + \frac{axc}{2e(fx^2+e)} - \frac{axd}{2f(fx^2+e)} - \frac{bcx}{2f(fx^2+e)} + \frac{bxed}{2f^2(fx^2+e)} + \frac{ac}{2e}\arctan\left(fx\frac{1}{\sqrt{ef}}\right)\frac{1}{\sqrt{ef}} + \frac{ad}{2f}\arctan\left(fx-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x^2+c)/(f*x^2+e)^2,x)
```

```
[Out] b*d/f^2*x+1/2/e*x/(f*x^2+e)*a*c-1/2/f*x/(f*x^2+e)*a*d-1/2/f*x/(f*x^2+e)*b*c
+1/2/f^2*e*x/(f*x^2+e)*b*d+1/2/e/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*a*c+1/
2/f/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*a*d+1/2/f/(e*f)^(1/2)*arctan(x*f/(e
*f)^(1/2))*b*c-3/2/f^2*e/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*b*d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)/(f*x^2+e)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.46482, size = 660, normalized size = 6.11

$$\left[\frac{4\,bde^2f^2x^3+\left(3\,bde^3-acef^2-(bc+ad)e^2f+\left(3\,bde^2f-acf^3-(bc+ad)ef^2\right)x^2\right)\sqrt{-ef}\log\left(\frac{fx^2-2\sqrt{-ef}x-e}{fx^2+e}\right)+2\left(3\,bde^3f\right.}{4\left(e^2f^4x^2+e^3f^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)/(f*x^2+e)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*b*d*e^2*f^2*x^3 + (3*b*d*e^3 - a*c*e*f^2 - (b*c + a*d)*e^2*f + (3*b
*d*e^2*f - a*c*f^3 - (b*c + a*d)*e*f^2)*x^2)*sqrt(-e*f)*log((f*x^2 - 2*sqrt
(-e*f)*x - e)/(f*x^2 + e)) + 2*(3*b*d*e^3*f + a*c*e*f^3 - (b*c + a*d)*e^2*f
^2)*x)/(e^2*f^4*x^2 + e^3*f^3), 1/2*(2*b*d*e^2*f^2*x^3 - (3*b*d*e^3 - a*c*e
*f^2 - (b*c + a*d)*e^2*f + (3*b*d*e^2*f - a*c*f^3 - (b*c + a*d)*e*f^2)*x^2)
*sqrt(e*f)*arctan(sqrt(e*f)*x/e) + (3*b*d*e^3*f + a*c*e*f^3 - (b*c + a*d)*e
^2*f^2)*x)/(e^2*f^4*x^2 + e^3*f^3)]
```

Sympy [A] time = 1.58296, size = 190, normalized size = 1.76

$$\frac{bdx}{f^2}+\frac{x\left(acf^2-adef-bcef+bde^2\right)}{2e^2f^2+2ef^3x^2}-\frac{\sqrt{-\frac{1}{e^3f^5}}\left(acf^2+adef+bcef-3bde^2\right)\log\left(-e^2f^2\sqrt{-\frac{1}{e^3f^5}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{e^3f^5}}\left(acf^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+c)/(f*x**2+e)**2,x)
```

```
[Out] b*d*x/f**2 + x*(a*c*f**2 - a*d*e*f - b*c*e*f + b*d*e**2)/(2*e**2*f**2 + 2*e
*f**3*x**2) - sqrt(-1/(e**3*f**5))*(a*c*f**2 + a*d*e*f + b*c*e*f - 3*b*d*e*
*2)*log(-e**2*f**2*sqrt(-1/(e**3*f**5)) + x)/4 + sqrt(-1/(e**3*f**5))*(a*c*
f**2 + a*d*e*f + b*c*e*f - 3*b*d*e**2)*log(e**2*f**2*sqrt(-1/(e**3*f**5)) +
 x)/4
```

Giac [A] time = 1.14734, size = 128, normalized size = 1.19

$$\frac{bdx}{f^2}+\frac{\left(acf^2+bcfe+adfe-3\,bde^2\right)\arctan\left(\sqrt{f}xe^{\left(-\frac{1}{2}\right)}\right)e^{\left(-\frac{3}{2}\right)}}{2\,f^{\frac{5}{2}}}+\frac{\left(acf^2x-bcfxe-adfxe+bdxe^2\right)e^{(-1)}}{2\left(fx^2+e\right)f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)/(f*x^2+e)^2,x, algorithm="giac")
```

```
[Out] b*d*x/f^2 + 1/2*(a*c*f^2 + b*c*f*e + a*d*f*e - 3*b*d*e^2)*arctan(sqrt(f)*x*
e^(-1/2))*e^(-3/2)/f^(5/2) + 1/2*(a*c*f^2*x - b*c*f*x*e - a*d*f*x*e + b*d*x
*e^2)*e^(-1)/((f*x^2 + e)*f^2)
```

3.7 $\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^3} dx$

Optimal. Leaf size=130

$$-\frac{x(be(cf+3de)-af(3cf+de))}{8e^2f^2(e+fx^2)}+\frac{\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(af(3cf+de)+be(cf+3de))}{8e^{5/2}f^{5/2}}-\frac{x(a+bx^2)(de-cf)}{4ef(e+fx^2)^2}$$

```
[Out] -((d*e - c*f)*x*(a + b*x^2))/(4*e*f*(e + f*x^2)^2) - ((b*e*(3*d*e + c*f) -
a*f*(d*e + 3*c*f))*x)/(8*e^2*f^2*(e + f*x^2)) + ((b*e*(3*d*e + c*f) + a*f*(
d*e + 3*c*f))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(8*e^(5/2)*f^(5/2))
```

Rubi [A] time = 0.108164, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {526, 385, 205}

$$-\frac{x(be(cf+3de)-af(3cf+de))}{8e^2f^2(e+fx^2)}+\frac{\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(af(3cf+de)+be(cf+3de))}{8e^{5/2}f^{5/2}}-\frac{x(a+bx^2)(de-cf)}{4ef(e+fx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(c + d*x^2))/(e + f*x^2)^3,x]
```

```
[Out] -((d*e - c*f)*x*(a + b*x^2))/(4*e*f*(e + f*x^2)^2) - ((b*e*(3*d*e + c*f) -
a*f*(d*e + 3*c*f))*x)/(8*e^2*f^2*(e + f*x^2)) + ((b*e*(3*d*e + c*f) + a*f*(
d*e + 3*c*f))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(8*e^(5/2)*f^(5/2))
```

Rule 526

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^q)/(a*b*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p +
 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^3}\,dx = -\frac{(de-cf)x(a+bx^2)}{4ef(e+fx^2)^2} - \frac{\int \frac{-a(de+3cf)-b(3de+cf)x^2}{(e+fx^2)^2}\,dx}{4ef}$$

$$= -\frac{(de-cf)x(a+bx^2)}{4ef(e+fx^2)^2} - \frac{(be(3de+cf)-af(de+3cf))x}{8e^2f^2(e+fx^2)} + \frac{(be(3de+cf)+af(de+3cf))\int \frac{}{e}}{8e^2f^2}$$

$$= -\frac{(de-cf)x(a+bx^2)}{4ef(e+fx^2)^2} - \frac{(be(3de+cf)-af(de+3cf))x}{8e^2f^2(e+fx^2)} + \frac{(be(3de+cf)+af(de+3cf))\tan}{8e^{5/2}f^{5/2}}$$

Mathematica [A] time = 0.0804119, size = 130, normalized size = 1.

$$\frac{x(af(3cf+de)+be(cf-5de))}{8e^2f^2(e+fx^2)} + \frac{\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(af(3cf+de)+be(cf+3de))}{8e^{5/2}f^{5/2}} + \frac{x(be-af)(de-cf)}{4ef^2(e+fx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(c + d*x^2))/(e + f*x^2)^3,x]
```

```
[Out] ((b*e - a*f)*(d*e - c*f)*x)/(4*e*f^2*(e + f*x^2)^2) + ((b*e*(-5*d*e + c*f)
+ a*f*(d*e + 3*c*f))*x)/(8*e^2*f^2*(e + f*x^2)) + ((b*e*(3*d*e + c*f) + a*f
*(d*e + 3*c*f))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(8*e^(5/2)*f^(5/2))
```

Maple [A] time = 0.009, size = 175, normalized size = 1.4

$$\frac{1}{(fx^2+e)^2}\left(\frac{(3\,acf^2+adef+bcef-5\,bde^2)\,x^3}{8\,e^2f} + \frac{(5\,acf^2-adef-bcef-3\,bde^2)\,x}{8\,f^2e}\right) + \frac{3\,ac}{8\,e^2}\arctan\left(fx\frac{1}{\sqrt{ef}}\right)\frac{1}{\sqrt{ef}} + \frac{ac}{8\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x^2+c)/(f*x^2+e)^3,x)
```

```
[Out] (1/8*(3*a*c*f^2+a*d*e*f+b*c*e*f-5*b*d*e^2)/e^2/f*x^3+1/8*(5*a*c*f^2-a*d*e*f
-b*c*e*f-3*b*d*e^2)/f^2/e*x)/(f*x^2+e)^2+3/8/e^2/(e*f)^(1/2)*arctan(x*f/(e*
f)^(1/2))*a*c+1/8/e/f/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*a*d+1/8/e/f/(e*f)
^(1/2)*arctan(x*f/(e*f)^(1/2))*b*c+3/8/f^2/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/
2))*b*d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)/(f*x^2+e)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.50555, size = 1002, normalized size = 7.71

$$\left[-\frac{2\left(5\,bde^3f^2-3\,acef^4-(bc+ad)e^2f^3\right)x^3+\left(3\,bde^4+3\,ace^2f^2+(bc+ad)e^3f+\left(3\,bde^2f^2+3\,acf^4+(bc+ad)ef^3\right)x\right.}{16\left(e^3f^5x^4+2\,e^4\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)/(f*x^2+e)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(2*(5*b*d*e^3*f^2 - 3*a*c*e*f^4 - (b*c + a*d)*e^2*f^3)*x^3 + (3*b*d*
e^4 + 3*a*c*e^2*f^2 + (b*c + a*d)*e^3*f + (3*b*d*e^2*f^2 + 3*a*c*f^4 + (b*c
 + a*d)*e*f^3)*x^4 + 2*(3*b*d*e^3*f + 3*a*c*e*f^3 + (b*c + a*d)*e^2*f^2)*x^
2)*sqrt(-e*f)*log((f*x^2 - 2*sqrt(-e*f)*x - e)/(f*x^2 + e)) + 2*(3*b*d*e^4*
f - 5*a*c*e^2*f^3 + (b*c + a*d)*e^3*f^2)*x)/(e^3*f^5*x^4 + 2*e^4*f^4*x^2 +
e^5*f^3), -1/8*((5*b*d*e^3*f^2 - 3*a*c*e*f^4 - (b*c + a*d)*e^2*f^3)*x^3 - (
3*b*d*e^4 + 3*a*c*e^2*f^2 + (b*c + a*d)*e^3*f + (3*b*d*e^2*f^2 + 3*a*c*f^4
+ (b*c + a*d)*e*f^3)*x^4 + 2*(3*b*d*e^3*f + 3*a*c*e*f^3 + (b*c + a*d)*e^2*f
^2)*x^2)*sqrt(e*f)*arctan(sqrt(e*f)*x/e) + (3*b*d*e^4*f - 5*a*c*e^2*f^3 + (
b*c + a*d)*e^3*f^2)*x)/(e^3*f^5*x^4 + 2*e^4*f^4*x^2 + e^5*f^3)]
```

Sympy [B] time = 3.64234, size = 246, normalized size = 1.89

$$-\frac{\sqrt{-\frac{1}{e^5f^5}}\left(3acf^2+adef+bcef+3bde^2\right)\log\left(-e^3f^2\sqrt{-\frac{1}{e^5f^5}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{e^5f^5}}\left(3acf^2+adef+bcef+3bde^2\right)\log\left(e^3f^2\right.}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+c)/(f*x**2+e)**3,x)
```

```
[Out] -sqrt(-1/(e**5*f**5))*(3*a*c*f**2 + a*d*e*f + b*c*e*f + 3*b*d*e**2)*log(-e*
*3*f**2*sqrt(-1/(e**5*f**5)) + x)/16 + sqrt(-1/(e**5*f**5))*(3*a*c*f**2 + a
*d*e*f + b*c*e*f + 3*b*d*e**2)*log(e**3*f**2*sqrt(-1/(e**5*f**5)) + x)/16 +
 (x**3*(3*a*c*f**3 + a*d*e*f**2 + b*c*e*f**2 - 5*b*d*e**2*f) + x*(5*a*c*e*f
**2 - a*d*e**2*f - b*c*e**2*f - 3*b*d*e**3))/(8*e**4*f**2 + 16*e**3*f**3*x*
*2 + 8*e**2*f**4*x**4)
```

Giac [A] time = 1.15831, size = 182, normalized size = 1.4

$$\frac{\left(3\,acf^2+bcfe+adfe+3\,bde^2\right)\arctan\left(\sqrt{f}xe^{\left(-\frac{1}{2}\right)}\right)e^{\left(-\frac{5}{2}\right)}}{8\,f^{\frac{5}{2}}}+\frac{\left(3\,acf^3x^3+bcf^2x^3e+adf^2x^3e-5\,bdfx^3e^2+5\,acf^2xe-\right.}{8\left(fx^2+e\right)^2f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)/(f*x^2+e)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*a*c*f^2 + b*c*f*e + a*d*f*e + 3*b*d*e^2)*arctan(sqrt(f)*x*e^(-1/2))*
e^(-5/2)/f^(5/2) + 1/8*(3*a*c*f^3*x^3 + b*c*f^2*x^3*e + a*d*f^2*x^3*e - 5*b
*d*f*x^3*e^2 + 5*a*c*f^2*x*e - b*c*f*x*e^2 - a*d*f*x*e^2 - 3*b*d*x*e^3)*e^(
-2)/((f*x^2 + e)^2*f^2)
```

3.8 $\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^4} dx$

Optimal. Leaf size=171

$$\frac{x(af(5cf+de)+be(cf+de))}{16e^3f^2(e+fx^2)} - \frac{x(3be(cf+de)-af(5cf+de))}{24e^2f^2(e+fx^2)^2} + \frac{\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(af(5cf+de)+be(cf+de))}{16e^{7/2}f^{5/2}} - \frac{x(a+bx}{6ef(e}$$

```
[Out] -((d*e - c*f)*x*(a + b*x^2))/(6*e*f*(e + f*x^2)^3) - ((3*b*e*(d*e + c*f) -
a*f*(d*e + 5*c*f))*x)/(24*e^2*f^2*(e + f*x^2)^2) + ((b*e*(d*e + c*f) + a*f*
(d*e + 5*c*f))*x)/(16*e^3*f^2*(e + f*x^2)) + ((b*e*(d*e + c*f) + a*f*(d*e +
 5*c*f))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(16*e^(7/2)*f^(5/2))
```

Rubi [A] time = 0.163406, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {526, 385, 199, 205}

$$\frac{x(af(5cf+de)+be(cf+de))}{16e^3f^2(e+fx^2)} - \frac{x(3be(cf+de)-af(5cf+de))}{24e^2f^2(e+fx^2)^2} + \frac{\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(af(5cf+de)+be(cf+de))}{16e^{7/2}f^{5/2}} - \frac{x(a+bx}{6ef(e}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(c + d*x^2))/(e + f*x^2)^4,x]
```


```
[Out] -((d*e - c*f)*x*(a + b*x^2))/(6*e*f*(e + f*x^2)^3) - ((3*b*e*(d*e + c*f) -
a*f*(d*e + 5*c*f))*x)/(24*e^2*f^2*(e + f*x^2)^2) + ((b*e*(d*e + c*f) + a*f*
(d*e + 5*c*f))*x)/(16*e^3*f^2*(e + f*x^2)) + ((b*e*(d*e + c*f) + a*f*(d*e +
 5*c*f))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(16*e^(7/2)*f^(5/2))
```


Rule 526

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^q)/(a*b*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p +
 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)\left(c+dx^2\right)}{\left(e+fx^2\right)^4}\,dx = -\frac{(de-cf)x\left(a+bx^2\right)}{6ef\left(e+fx^2\right)^3} - \frac{\int \frac{-a(de+5cf)-3b(de+cf)x^2}{\left(e+fx^2\right)^3}\,dx}{6ef}$$

$$= -\frac{(de-cf)x\left(a+bx^2\right)}{6ef\left(e+fx^2\right)^3} - \frac{(3be(de+cf)-af(de+5cf))x}{24e^2f^2\left(e+fx^2\right)^2} + \frac{(be(de+cf)+af(de+5cf))\int}{8e^2f^2}$$

$$= -\frac{(de-cf)x\left(a+bx^2\right)}{6ef\left(e+fx^2\right)^3} - \frac{(3be(de+cf)-af(de+5cf))x}{24e^2f^2\left(e+fx^2\right)^2} + \frac{(be(de+cf)+af(de+5cf))x}{16e^3f^2\left(e+fx^2\right)}$$

$$= -\frac{(de-cf)x\left(a+bx^2\right)}{6ef\left(e+fx^2\right)^3} - \frac{(3be(de+cf)-af(de+5cf))x}{24e^2f^2\left(e+fx^2\right)^2} + \frac{(be(de+cf)+af(de+5cf))x}{16e^3f^2\left(e+fx^2\right)}$$

Mathematica [A] time = 0.101899, size = 171, normalized size = 1.

$$\frac{x(af(5cf+de)+be(cf+de))}{16e^3f^2\left(e+fx^2\right)} + \frac{x(af(5cf+de)+be(cf-7de))}{24e^2f^2\left(e+fx^2\right)^2} + \frac{\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(af(5cf+de)+be(cf+de))}{16e^{7/2}f^{5/2}} + \frac{x(be-}{6ef}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(c + d*x^2))/(e + f*x^2)^4,x]
```

```
[Out] ((b*e - a*f)*(d*e - c*f)*x)/(6*e*f^2*(e + f*x^2)^3) + ((b*e*(-7*d*e + c*f)
+ a*f*(d*e + 5*c*f))*x)/(24*e^2*f^2*(e + f*x^2)^2) + ((b*e*(d*e + c*f) + a*
f*(d*e + 5*c*f))*x)/(16*e^3*f^2*(e + f*x^2)) + ((b*e*(d*e + c*f) + a*f*(d*e
 + 5*c*f))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(16*e^(7/2)*f^(5/2))
```

Maple [A] time = 0.01, size = 210, normalized size = 1.2

$$\frac{1}{\left(fx^2+e\right)^3}\left(\frac{\left(5\,acf^2+adef+bcef+bde^2\right)x^5}{16\,e^3} + \frac{\left(5\,acf^2+adef+bcef-bde^2\right)x^3}{6\,e^2f} + \frac{\left(11\,acf^2-adef-bcef-bde^2\right)x}{16\,ef^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x^2+c)/(f*x^2+e)^4,x)
```

```
[Out] (1/16*(5*a*c*f^2+a*d*e*f+b*c*e*f+b*d*e^2)/e^3*x^5+1/6*(5*a*c*f^2+a*d*e*f+b*
c*e*f-b*d*e^2)/e^2/f*x^3+1/16*(11*a*c*f^2-a*d*e*f-b*c*e*f-b*d*e^2)/e/f^2*x)
/(f*x^2+e)^3+5/16/e^3/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*a*c+1/16/e^2/f/(e
*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*a*d+1/16/e^2/f/(e*f)^(1/2)*arctan(x*f/(e*
f)^(1/2))*b*c+1/16/e/f^2/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*b*d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)/(f*x^2+e)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.55886, size = 1355, normalized size = 7.92

$$\left[\frac{6\left(bde^3f^3 + 5\,acef^5 + (bc + ad)e^2f^4\right)x^5 - 16\left(bde^4f^2 - 5\,ace^2f^4 - (bc + ad)e^3f^3\right)x^3 - 3\left(bde^5 + 5\,ace^3f^2 + \left(bde^2f^3 + 5\,a\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)/(f*x^2+e)^4,x, algorithm="fricas")
```

```
[Out] [1/96*(6*(b*d*e^3*f^3 + 5*a*c*e*f^5 + (b*c + a*d)*e^2*f^4)*x^5 - 16*(b*d*e^
4*f^2 - 5*a*c*e^2*f^4 - (b*c + a*d)*e^3*f^3)*x^3 - 3*(b*d*e^5 + 5*a*c*e^3*f
^2 + (b*d*e^2*f^3 + 5*a*c*f^5 + (b*c + a*d)*e*f^4)*x^6 + (b*c + a*d)*e^4*f
+ 3*(b*d*e^3*f^2 + 5*a*c*e*f^4 + (b*c + a*d)*e^2*f^3)*x^4 + 3*(b*d*e^4*f +
5*a*c*e^2*f^3 + (b*c + a*d)*e^3*f^2)*x^2)*sqrt(-e*f)*log((f*x^2 - 2*sqrt(-e
*f)*x - e)/(f*x^2 + e)) - 6*(b*d*e^5*f - 11*a*c*e^3*f^3 + (b*c + a*d)*e^4*f
^2)*x)/(e^4*f^6*x^6 + 3*e^5*f^5*x^4 + 3*e^6*f^4*x^2 + e^7*f^3), 1/48*(3*(b*
d*e^3*f^3 + 5*a*c*e*f^5 + (b*c + a*d)*e^2*f^4)*x^5 - 8*(b*d*e^4*f^2 - 5*a*c
*e^2*f^4 - (b*c + a*d)*e^3*f^3)*x^3 + 3*(b*d*e^5 + 5*a*c*e^3*f^2 + (b*d*e^2
*f^3 + 5*a*c*f^5 + (b*c + a*d)*e*f^4)*x^6 + (b*c + a*d)*e^4*f + 3*(b*d*e^3*
f^2 + 5*a*c*e*f^4 + (b*c + a*d)*e^2*f^3)*x^4 + 3*(b*d*e^4*f + 5*a*c*e^2*f^3
 + (b*c + a*d)*e^3*f^2)*x^2)*sqrt(e*f)*arctan(sqrt(e*f)*x/e) - 3*(b*d*e^5*f
 - 11*a*c*e^3*f^3 + (b*c + a*d)*e^4*f^2)*x)/(e^4*f^6*x^6 + 3*e^5*f^5*x^4 +
3*e^6*f^4*x^2 + e^7*f^3)]
```

Sympy [A] time = 8.42358, size = 313, normalized size = 1.83

$$-\frac{\sqrt{-\frac{1}{e^7f^5}}\left(5acf^2 + adef + bcef + bde^2\right)\log\left(-e^4f^2\sqrt{-\frac{1}{e^7f^5}} + x\right)}{32} + \frac{\sqrt{-\frac{1}{e^7f^5}}\left(5acf^2 + adef + bcef + bde^2\right)\log\left(e^4f^2\sqrt{-\frac{1}{e}}\right.}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+c)/(f*x**2+e)**4,x)
```

```
[Out] -sqrt(-1/(e**7*f**5))*(5*a*c*f**2 + a*d*e*f + b*c*e*f + b*d*e**2)*log(-e**4
*f**2*sqrt(-1/(e**7*f**5)) + x)/32 + sqrt(-1/(e**7*f**5))*(5*a*c*f**2 + a*d
*e*f + b*c*e*f + b*d*e**2)*log(e**4*f**2*sqrt(-1/(e**7*f**5)) + x)/32 + (x*
*5*(15*a*c*f**4 + 3*a*d*e*f**3 + 3*b*c*e*f**3 + 3*b*d*e**2*f**2) + x**3*(40
*a*c*e*f**3 + 8*a*d*e**2*f**2 + 8*b*c*e**2*f**2 - 8*b*d*e**3*f) + x*(33*a*c
*e**2*f**2 - 3*a*d*e**3*f - 3*b*c*e**3*f - 3*b*d*e**4))/(48*e**6*f**2 + 144
*e**5*f**3*x**2 + 144*e**4*f**4*x**4 + 48*e**3*f**5*x**6)
```

Giac [A] time = 1.19956, size = 248, normalized size = 1.45

$$\frac{\left(5\,acf^2 + bcfe + adfe + bde^2\right)\arctan\left(\sqrt{f}xe^{\left(-\frac{1}{2}\right)}\right)e^{\left(-\frac{7}{2}\right)}}{16\,f^{\frac{5}{2}}} + \frac{\left(15\,acf^4x^5 + 3\,bcf^3x^5e + 3\,adf^3x^5e + 3\,bdf^2x^5e^2 + 40\,acf\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)/(f*x^2+e)^4,x, algorithm="giac")
```

```
[Out] 1/16*(5*a*c*f^2 + b*c*f*e + a*d*f*e + b*d*e^2)*arctan(sqrt(f)*x*e^(-1/2))*e
^(-7/2)/f^(5/2) + 1/48*(15*a*c*f^4*x^5 + 3*b*c*f^3*x^5*e + 3*a*d*f^3*x^5*e
+ 3*b*d*f^2*x^5*e^2 + 40*a*c*f^3*x^3*e + 8*b*c*f^2*x^3*e^2 + 8*a*d*f^2*x^3*
e^2 - 8*b*d*f*x^3*e^3 + 33*a*c*f^2*x*e^2 - 3*b*c*f*x*e^3 - 3*a*d*f*x*e^3 -
3*b*d*x*e^4)*e^(-3)/((f*x^2 + e)^3*f^2)
```

3.9 $\int \left(a+bx^2\right)\left(c+dx^2\right)^2\left(e+fx^2\right)^3 \, dx$

Optimal. Leaf size=226

$$\frac{1}{9}fx^9\left(adf(2cf+3de)+b\left(c^2f^2+6cdef+3d^2e^2\right)\right)+\frac{1}{7}x^7\left(af\left(c^2f^2+6cdef+3d^2e^2\right)+be\left(3c^2f^2+6cdef+d^2e^2\right)\right)+\frac{1}{5}e$$

```
[Out] a*c^2*e^3*x + (c*e^2*(b*c*e + 2*a*d*e + 3*a*c*f)*x^3)/3 + (e*(b*c*e*(2*d*e
+ 3*c*f) + a*(d^2*e^2 + 6*c*d*e*f + 3*c^2*f^2))*x^5)/5 + ((a*f*(3*d^2*e^2 +
 6*c*d*e*f + c^2*f^2) + b*e*(d^2*e^2 + 6*c*d*e*f + 3*c^2*f^2))*x^7)/7 + (f*
(a*d*f*(3*d*e + 2*c*f) + b*(3*d^2*e^2 + 6*c*d*e*f + c^2*f^2))*x^9)/9 + (d*f
^2*(3*b*d*e + 2*b*c*f + a*d*f)*x^11)/11 + (b*d^2*f^3*x^13)/13
```

Rubi [A] time = 0.214069, antiderivative size = 226, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {521}

$$\frac{1}{9}fx^9\left(adf(2cf+3de)+b\left(c^2f^2+6cdef+3d^2e^2\right)\right)+\frac{1}{7}x^7\left(af\left(c^2f^2+6cdef+3d^2e^2\right)+be\left(3c^2f^2+6cdef+d^2e^2\right)\right)+\frac{1}{5}e$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)*(c + d*x^2)^2*(e + f*x^2)^3,x]
```

```
[Out] a*c^2*e^3*x + (c*e^2*(b*c*e + 2*a*d*e + 3*a*c*f)*x^3)/3 + (e*(b*c*e*(2*d*e
+ 3*c*f) + a*(d^2*e^2 + 6*c*d*e*f + 3*c^2*f^2))*x^5)/5 + ((a*f*(3*d^2*e^2 +
 6*c*d*e*f + c^2*f^2) + b*e*(d^2*e^2 + 6*c*d*e*f + 3*c^2*f^2))*x^7)/7 + (f*
(a*d*f*(3*d*e + 2*c*f) + b*(3*d^2*e^2 + 6*c*d*e*f + c^2*f^2))*x^9)/9 + (d*f
^2*(3*b*d*e + 2*b*c*f + a*d*f)*x^11)/11 + (b*d^2*f^3*x^13)/13
```

Rule 521

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c +
d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[p
, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rubi steps

$$\int \left(a+bx^2\right)\left(c+dx^2\right)^2\left(e+fx^2\right)^3 \, dx = \int \left(ac^2e^3+ce^2(bce+2ade+3acf)x^2+e\left(bce(2de+3cf)+a\left(d^2e^2+6cdef+3c^2\right.\right.\right.$$

$$= ac^2e^3x+\frac{1}{3}ce^2(bce+2ade+3acf)x^3+\frac{1}{5}e\left(bce(2de+3cf)+a\left(d^2e^2+6cdef+3c\right.\right.$$

Mathematica [A] time = 0.081365, size = 226, normalized size = 1.

$$\frac{1}{9}fx^9\left(adf(2cf+3de)+b\left(c^2f^2+6cdef+3d^2e^2\right)\right)+\frac{1}{7}x^7\left(af\left(c^2f^2+6cdef+3d^2e^2\right)+be\left(3c^2f^2+6cdef+d^2e^2\right)\right)+\frac{1}{5}e$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)*(c + d*x^2)^2*(e + f*x^2)^3,x]
```

```
[Out] a*c^2*e^3*x + (c*e^2*(b*c*e + 2*a*d*e + 3*a*c*f)*x^3)/3 + (e*(b*c*e*(2*d*e
+ 3*c*f) + a*(d^2*e^2 + 6*c*d*e*f + 3*c^2*f^2))*x^5)/5 + ((a*f*(3*d^2*e^2 +
```

```
 6*c*d*e*f + c^2*f^2) + b*e*(d^2*e^2 + 6*c*d*e*f + 3*c^2*f^2))*x^7)/7 + (f*
(a*d*f*(3*d*e + 2*c*f) + b*(3*d^2*e^2 + 6*c*d*e*f + c^2*f^2))*x^9)/9 + (d*f
^2*(3*b*d*e + 2*b*c*f + a*d*f)*x^11)/11 + (b*d^2*f^3*x^13)/13
```

Maple [A] time = 0.001, size = 237, normalized size = 1.1

$$\frac{bd^2f^3x^{13}}{13} + \frac{\left(\left(ad^2+2\,bcd\right)f^3+3\,bd^2ef^2\right)x^{11}}{11} + \frac{\left(\left(2\,acd+bc^2\right)f^3+3\,\left(ad^2+2\,bcd\right)ef^2+3\,bd^2e^2f\right)x^9}{9} + \frac{\left(ac^2f^3+3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x^2+c)^2*(f*x^2+e)^3,x)
```

```
[Out] 1/13*b*d^2*f^3*x^13+1/11*((a*d^2+2*b*c*d)*f^3+3*b*d^2*e*f^2)*x^11+1/9*((2*a
*c*d+b*c^2)*f^3+3*(a*d^2+2*b*c*d)*e*f^2+3*b*d^2*e^2*f)*x^9+1/7*(a*c^2*f^3+3
*(2*a*c*d+b*c^2)*e*f^2+3*(a*d^2+2*b*c*d)*e^2*f+b*d^2*e^3)*x^7+1/5*(3*a*c^2*
e*f^2+3*(2*a*c*d+b*c^2)*e^2*f+(a*d^2+2*b*c*d)*e^3)*x^5+1/3*(3*a*c^2*e^2*f+(
2*a*c*d+b*c^2)*e^3)*x^3+a*c^2*e^3*x
```

Maxima [A] time = 1.03594, size = 319, normalized size = 1.41

$$\frac{1}{13}\,bd^2f^3x^{13} + \frac{1}{11}\left(3\,bd^2ef^2+\left(2\,bcd+ad^2\right)f^3\right)x^{11} + \frac{1}{9}\left(3\,bd^2e^2f+3\left(2\,bcd+ad^2\right)ef^2+\left(bc^2+2\,acd\right)f^3\right)x^9 + \frac{1}{7}\left(bd^2e\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^2*(f*x^2+e)^3,x, algorithm="maxima")
```

```
[Out] 1/13*b*d^2*f^3*x^13 + 1/11*(3*b*d^2*e*f^2 + (2*b*c*d + a*d^2)*f^3)*x^11 + 1
/9*(3*b*d^2*e^2*f + 3*(2*b*c*d + a*d^2)*e*f^2 + (b*c^2 + 2*a*c*d)*f^3)*x^9
+ 1/7*(b*d^2*e^3 + a*c^2*f^3 + 3*(2*b*c*d + a*d^2)*e^2*f + 3*(b*c^2 + 2*a*c
*d)*e*f^2)*x^7 + a*c^2*e^3*x + 1/5*(3*a*c^2*e*f^2 + (2*b*c*d + a*d^2)*e^3 +
 3*(b*c^2 + 2*a*c*d)*e^2*f)*x^5 + 1/3*(3*a*c^2*e^2*f + (b*c^2 + 2*a*c*d)*e^
3)*x^3
```

Fricas [A] time = 1.2578, size = 676, normalized size = 2.99

$$\frac{1}{13}x^{13}f^3d^2b + \frac{3}{11}x^{11}f^2ed^2b + \frac{2}{11}x^{11}f^3dcb + \frac{1}{11}x^{11}f^3d^2a + \frac{1}{3}x^9fe^2d^2b + \frac{2}{3}x^9f^2edcb + \frac{1}{9}x^9f^3c^2b + \frac{1}{3}x^9f^2ed^2a + \frac{2}{9}x^9f$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^2*(f*x^2+e)^3,x, algorithm="fricas")
```

```
[Out] 1/13*x^13*f^3*d^2*b + 3/11*x^11*f^2*e*d^2*b + 2/11*x^11*f^3*d*c*b + 1/11*x^
11*f^3*d^2*a + 1/3*x^9*f*e^2*d^2*b + 2/3*x^9*f^2*e*d*c*b + 1/9*x^9*f^3*c^2*
b + 1/3*x^9*f^2*e*d^2*a + 2/9*x^9*f^3*d*c*a + 1/7*x^7*e^3*d^2*b + 6/7*x^7*f
*e^2*d*c*b + 3/7*x^7*f^2*e*c^2*b + 3/7*x^7*f*e^2*d^2*a + 6/7*x^7*f^2*e*d*c*
a + 1/7*x^7*f^3*c^2*a + 2/5*x^5*e^3*d*c*b + 3/5*x^5*f*e^2*c^2*b + 1/5*x^5*e
^3*d^2*a + 6/5*x^5*f*e^2*d*c*a + 3/5*x^5*f^2*e*c^2*a + 1/3*x^3*e^3*c^2*b +
2/3*x^3*e^3*d*c*a + x^3*f*e^2*c^2*a + x*e^3*c^2*a
```

Sympy [A] time = 0.098201, size = 304, normalized size = 1.35

$$ac^2e^3x + \frac{bd^2f^3x^{13}}{13} + x^{11}\left(\frac{ad^2f^3}{11} + \frac{2bcdf^3}{11} + \frac{3bd^2ef^2}{11}\right) + x^9\left(\frac{2acdf^3}{9} + \frac{ad^2ef^2}{3} + \frac{bc^2f^3}{9} + \frac{2bcdef^2}{3} + \frac{bd^2e^2f}{3}\right) + x^7\left(\frac{ac^2}{7}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+c)**2*(f*x**2+e)**3,x)
```

```
[Out] a*c**2*e**3*x + b*d**2*f**3*x**13/13 + x**11*(a*d**2*f**3/11 + 2*b*c*d*f**3
/11 + 3*b*d**2*e*f**2/11) + x**9*(2*a*c*d*f**3/9 + a*d**2*e*f**2/3 + b*c**2
*f**3/9 + 2*b*c*d*e*f**2/3 + b*d**2*e**2*f/3) + x**7*(a*c**2*f**3/7 + 6*a*c
*d*e*f**2/7 + 3*a*d**2*e**2*f/7 + 3*b*c**2*e*f**2/7 + 6*b*c*d*e**2*f/7 + b*
d**2*e**3/7) + x**5*(3*a*c**2*e*f**2/5 + 6*a*c*d*e**2*f/5 + a*d**2*e**3/5 +
 3*b*c**2*e**2*f/5 + 2*b*c*d*e**3/5) + x**3*(a*c**2*e**2*f + 2*a*c*d*e**3/3
 + b*c**2*e**3/3)
```

Giac [A] time = 1.17603, size = 382, normalized size = 1.69

$$\frac{1}{13}bd^2f^3x^{13} + \frac{2}{11}bcdf^3x^{11} + \frac{1}{11}ad^2f^3x^{11} + \frac{3}{11}bd^2f^2x^{11}e + \frac{1}{9}bc^2f^3x^9 + \frac{2}{9}acdf^3x^9 + \frac{2}{3}bcdf^2x^9e + \frac{1}{3}ad^2f^2x^9e + \frac{1}{3}bd^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^2*(f*x^2+e)^3,x, algorithm="giac")
```

```
[Out] 1/13*b*d^2*f^3*x^13 + 2/11*b*c*d*f^3*x^11 + 1/11*a*d^2*f^3*x^11 + 3/11*b*d^
2*f^2*x^11*e + 1/9*b*c^2*f^3*x^9 + 2/9*a*c*d*f^3*x^9 + 2/3*b*c*d*f^2*x^9*e
+ 1/3*a*d^2*f^2*x^9*e + 1/3*b*d^2*f*x^9*e^2 + 1/7*a*c^2*f^3*x^7 + 3/7*b*c^2
*f^2*x^7*e + 6/7*a*c*d*f^2*x^7*e + 6/7*b*c*d*f*x^7*e^2 + 3/7*a*d^2*f*x^7*e^
2 + 1/7*b*d^2*x^7*e^3 + 3/5*a*c^2*f^2*x^5*e + 3/5*b*c^2*f*x^5*e^2 + 6/5*a*c
*d*f*x^5*e^2 + 2/5*b*c*d*x^5*e^3 + 1/5*a*d^2*x^5*e^3 + a*c^2*f*x^3*e^2 + 1/
3*b*c^2*x^3*e^3 + 2/3*a*c*d*x^3*e^3 + a*c^2*x*e^3
```

3.10 $\int \left(a+bx^2\right)\left(c+dx^2\right)^2\left(e+fx^2\right)^2\,dx$

Optimal. Leaf size=158

$$\frac{1}{7}x^7\left(2adf(cf+de)+b\left(c^2f^2+4cdef+d^2e^2\right)\right)+\frac{1}{5}x^5\left(a\left(c^2f^2+4cdef+d^2e^2\right)+2bce(cf+de)\right)+\frac{1}{9}dfx^9(adf+2b(c$$

```
[Out] a*c^2*e^2*x + (c*e*(b*c*e + 2*a*(d*e + c*f))*x^3)/3 + ((2*b*c*e*(d*e + c*f)
 + a*(d^2*e^2 + 4*c*d*e*f + c^2*f^2))*x^5)/5 + ((2*a*d*f*(d*e + c*f) + b*(d
^2*e^2 + 4*c*d*e*f + c^2*f^2))*x^7)/7 + (d*f*(a*d*f + 2*b*(d*e + c*f))*x^9)
/9 + (b*d^2*f^2*x^11)/11
```

Rubi [A] time = 0.167084, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {521}

$$\frac{1}{7}x^7\left(2adf(cf+de)+b\left(c^2f^2+4cdef+d^2e^2\right)\right)+\frac{1}{5}x^5\left(a\left(c^2f^2+4cdef+d^2e^2\right)+2bce(cf+de)\right)+\frac{1}{9}dfx^9(adf+2b(c$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)*(c + d*x^2)^2*(e + f*x^2)^2,x]
```

```
[Out] a*c^2*e^2*x + (c*e*(b*c*e + 2*a*(d*e + c*f))*x^3)/3 + ((2*b*c*e*(d*e + c*f)
 + a*(d^2*e^2 + 4*c*d*e*f + c^2*f^2))*x^5)/5 + ((2*a*d*f*(d*e + c*f) + b*(d
^2*e^2 + 4*c*d*e*f + c^2*f^2))*x^7)/7 + (d*f*(a*d*f + 2*b*(d*e + c*f))*x^9)
/9 + (b*d^2*f^2*x^11)/11
```

Rule 521

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c +
d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[p
, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rubi steps

$$\int \left(a+bx^2\right)\left(c+dx^2\right)^2\left(e+fx^2\right)^2\,dx=\int \left(ac^2e^2+ce(bce+2a(de+cf))x^2+\left(2bce(de+cf)+a\left(d^2e^2+4cdef+c^2f\right.\right.\right.$$

$$=ac^2e^2x+\frac{1}{3}ce(bce+2a(de+cf))x^3+\frac{1}{5}\left(2bce(de+cf)+a\left(d^2e^2+4cdef+c^2\right.\right.$$

Mathematica [A] time = 0.0595616, size = 158, normalized size = 1.

$$\frac{1}{7}x^7\left(2adf(cf+de)+b\left(c^2f^2+4cdef+d^2e^2\right)\right)+\frac{1}{5}x^5\left(a\left(c^2f^2+4cdef+d^2e^2\right)+2bce(cf+de)\right)+\frac{1}{9}dfx^9(adf+2b(c$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)*(c + d*x^2)^2*(e + f*x^2)^2,x]
```

```
[Out] a*c^2*e^2*x + (c*e*(b*c*e + 2*a*(d*e + c*f))*x^3)/3 + ((2*b*c*e*(d*e + c*f)
 + a*(d^2*e^2 + 4*c*d*e*f + c^2*f^2))*x^5)/5 + ((2*a*d*f*(d*e + c*f) + b*(d
^2*e^2 + 4*c*d*e*f + c^2*f^2))*x^7)/7 + (d*f*(a*d*f + 2*b*(d*e + c*f))*x^9)
```

```
/9 + (b*d^2*f^2*x^11)/11
```

Maple [A] time = 0., size = 169, normalized size = 1.1

$$\frac{bd^2f^2x^{11}}{11}+\frac{\left(\left(ad^2+2\,bcd\right)f^2+2\,bd^2ef\right)x^9}{9}+\frac{\left(\left(2\,acd+bc^2\right)f^2+2\,\left(ad^2+2\,bcd\right)ef+bd^2e^2\right)x^7}{7}+\frac{\left(ac^2f^2+2\,\left(2\,acd+b\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x^2+c)^2*(f*x^2+e)^2,x)
```

```
[Out] 1/11*b*d^2*f^2*x^11+1/9*((a*d^2+2*b*c*d)*f^2+2*b*d^2*e*f)*x^9+1/7*((2*a*c*d
+b*c^2)*f^2+2*(a*d^2+2*b*c*d)*e*f+b*d^2*e^2)*x^7+1/5*(a*c^2*f^2+2*(2*a*c*d+
b*c^2)*e*f+(a*d^2+2*b*c*d)*e^2)*x^5+1/3*(2*a*c^2*e*f+(2*a*c*d+b*c^2)*e^2)*x
^3+a*c^2*e^2*x
```

Maxima [A] time = 1.01531, size = 227, normalized size = 1.44

$$\frac{1}{11}\,bd^2f^2x^{11}+\frac{1}{9}\left(2\,bd^2ef+\left(2\,bcd+ad^2\right)f^2\right)x^9+\frac{1}{7}\left(bd^2e^2+2\left(2\,bcd+ad^2\right)ef+\left(bc^2+2\,acd\right)f^2\right)x^7+ac^2e^2x+\frac{1}{5}\left(ac^2f^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^2*(f*x^2+e)^2,x, algorithm="maxima")
```

```
[Out] 1/11*b*d^2*f^2*x^11 + 1/9*(2*b*d^2*e*f + (2*b*c*d + a*d^2)*f^2)*x^9 + 1/7*(
b*d^2*e^2 + 2*(2*b*c*d + a*d^2)*e*f + (b*c^2 + 2*a*c*d)*f^2)*x^7 + a*c^2*e^
2*x + 1/5*(a*c^2*f^2 + (2*b*c*d + a*d^2)*e^2 + 2*(b*c^2 + 2*a*c*d)*e*f)*x^5
 + 1/3*(2*a*c^2*e*f + (b*c^2 + 2*a*c*d)*e^2)*x^3
```

Fricas [A] time = 1.27589, size = 479, normalized size = 3.03

$$\frac{1}{11}x^{11}f^2d^2b+\frac{2}{9}x^9fed^2b+\frac{2}{9}x^9f^2dcb+\frac{1}{9}x^9f^2d^2a+\frac{1}{7}x^7e^2d^2b+\frac{4}{7}x^7fedcb+\frac{1}{7}x^7f^2c^2b+\frac{2}{7}x^7fed^2a+\frac{2}{7}x^7f^2dca+\frac{2}{5}x^5e^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^2*(f*x^2+e)^2,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*f^2*d^2*b + 2/9*x^9*f*e*d^2*b + 2/9*x^9*f^2*d*c*b + 1/9*x^9*f^2*d
^2*a + 1/7*x^7*e^2*d^2*b + 4/7*x^7*f*e*d*c*b + 1/7*x^7*f^2*c^2*b + 2/7*x^7*
f*e*d^2*a + 2/7*x^7*f^2*d*c*a + 2/5*x^5*e^2*d*c*b + 2/5*x^5*f*e*c^2*b + 1/5
*x^5*e^2*d^2*a + 4/5*x^5*f*e*d*c*a + 1/5*x^5*f^2*c^2*a + 1/3*x^3*e^2*c^2*b
+ 2/3*x^3*e^2*d*c*a + 2/3*x^3*f*e*c^2*a + x*e^2*c^2*a
```

Sympy [A] time = 0.086888, size = 216, normalized size = 1.37

$$ac^2e^2x+\frac{bd^2f^2x^{11}}{11}+x^9\left(\frac{ad^2f^2}{9}+\frac{2bcdf^2}{9}+\frac{2bd^2ef}{9}\right)+x^7\left(\frac{2acdf^2}{7}+\frac{2ad^2ef}{7}+\frac{bc^2f^2}{7}+\frac{4bcdef}{7}+\frac{bd^2e^2}{7}\right)+x^5\left(\frac{ac^2f^2}{5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+c)**2*(f*x**2+e)**2,x)
```

```
[Out] a*c**2*e**2*x + b*d**2*f**2*x**11/11 + x**9*(a*d**2*f**2/9 + 2*b*c*d*f**2/9
 + 2*b*d**2*e*f/9) + x**7*(2*a*c*d*f**2/7 + 2*a*d**2*e*f/7 + b*c**2*f**2/7
+ 4*b*c*d*e*f/7 + b*d**2*e**2/7) + x**5*(a*c**2*f**2/5 + 4*a*c*d*e*f/5 + a*
d**2*e**2/5 + 2*b*c**2*e*f/5 + 2*b*c*d*e**2/5) + x**3*(2*a*c**2*e*f/3 + 2*a
*c*d*e**2/3 + b*c**2*e**2/3)
```

<u>Giac</u> [A] time = 1.14455, size = 273, normalized size = 1.73

$$\frac{1}{11}\,bd^2f^2x^{11} + \frac{2}{9}\,bcdf^2x^9 + \frac{1}{9}\,ad^2f^2x^9 + \frac{2}{9}\,bd^2fx^9e + \frac{1}{7}\,bc^2f^2x^7 + \frac{2}{7}\,acdf^2x^7 + \frac{4}{7}\,bcdfx^7e + \frac{2}{7}\,ad^2fx^7e + \frac{1}{7}\,bd^2x^7e^2 +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^2*(f*x^2+e)^2,x, algorithm="giac")
```

```
[Out] 1/11*b*d^2*f^2*x^11 + 2/9*b*c*d*f^2*x^9 + 1/9*a*d^2*f^2*x^9 + 2/9*b*d^2*f*x
^9*e + 1/7*b*c^2*f^2*x^7 + 2/7*a*c*d*f^2*x^7 + 4/7*b*c*d*f*x^7*e + 2/7*a*d^
2*f*x^7*e + 1/7*b*d^2*x^7*e^2 + 1/5*a*c^2*f^2*x^5 + 2/5*b*c^2*f*x^5*e + 4/5
*a*c*d*f*x^5*e + 2/5*b*c*d*x^5*e^2 + 1/5*a*d^2*x^5*e^2 + 2/3*a*c^2*f*x^3*e
+ 1/3*b*c^2*x^3*e^2 + 2/3*a*c*d*x^3*e^2 + a*c^2*x*e^2
```

3.11 $\int (a+bx^2)(c+dx^2)^2(e+fx^2)\,dx$

Optimal. Leaf size=94

$$\frac{1}{7}dx^7(adf+2bcf+bde)+\frac{1}{5}x^5(ad(2cf+de)+bc(cf+2de))+\frac{1}{3}cx^3(acf+2ade+bce)+ac^2ex+\frac{1}{9}bd^2fx^9$$

```
[Out] a*c^2*e*x + (c*(b*c*e + 2*a*d*e + a*c*f)*x^3)/3 + ((b*c*(2*d*e + c*f) + a*d
*(d*e + 2*c*f))*x^5)/5 + (d*(b*d*e + 2*b*c*f + a*d*f)*x^7)/7 + (b*d^2*f*x^9
)/9
```

Rubi [A] time = 0.0812315, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {521}

$$\frac{1}{7}dx^7(adf+2bcf+bde)+\frac{1}{5}x^5(ad(2cf+de)+bc(cf+2de))+\frac{1}{3}cx^3(acf+2ade+bce)+ac^2ex+\frac{1}{9}bd^2fx^9$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)*(c + d*x^2)^2*(e + f*x^2),x]
```

```
[Out] a*c^2*e*x + (c*(b*c*e + 2*a*d*e + a*c*f)*x^3)/3 + ((b*c*(2*d*e + c*f) + a*d
*(d*e + 2*c*f))*x^5)/5 + (d*(b*d*e + 2*b*c*f + a*d*f)*x^7)/7 + (b*d^2*f*x^9
)/9
```

Rule 521

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c +
d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[p
, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rubi steps

$$\begin{aligned}\int (a+bx^2)(c+dx^2)^2(e+fx^2)\,dx &= \int \left(ac^2e+c(bce+2ade+acf)x^2+(bc(2de+cf)+ad(de+2cf))x^4+d(bde+2bc\right.\\ &= ac^2ex+\frac{1}{3}c(bce+2ade+acf)x^3+\frac{1}{5}(bc(2de+cf)+ad(de+2cf))x^5+\frac{1}{7}d(bde+\end{aligned}$$

Mathematica [A] time = 0.0288298, size = 96, normalized size = 1.02

$$\frac{1}{5}x^5\left(2acdf+ad^2e+bc^2f+2bcde\right)+\frac{1}{7}dx^7(adf+2bcf+bde)+\frac{1}{3}cx^3(acf+2ade+bce)+ac^2ex+\frac{1}{9}bd^2fx^9$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)*(c + d*x^2)^2*(e + f*x^2),x]
```

```
[Out] a*c^2*e*x + (c*(b*c*e + 2*a*d*e + a*c*f)*x^3)/3 + ((2*b*c*d*e + a*d^2*e + b
*c^2*f + 2*a*c*d*f)*x^5)/5 + (d*(b*d*e + 2*b*c*f + a*d*f)*x^7)/7 + (b*d^2*f
*x^9)/9
```

Maple [A] time = 0.002, size = 101, normalized size = 1.1

$$\frac{bd^2fx^9}{9}+\frac{\left(\left(ad^2+2\,bcd\right)f+bd^2e\right)x^7}{7}+\frac{\left(\left(2\,acd+bc^2\right)f+\left(ad^2+2\,bcd\right)e\right)x^5}{5}+\frac{\left(ac^2f+\left(2\,acd+bc^2\right)e\right)x^3}{3}+ac^2ex$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x^2+c)^2*(f*x^2+e),x)
```

```
[Out] 1/9*b*d^2*f*x^9+1/7*((a*d^2+2*b*c*d)*f+b*d^2*e)*x^7+1/5*((2*a*c*d+b*c^2)*f+
(a*d^2+2*b*c*d)*e)*x^5+1/3*(a*c^2*f+(2*a*c*d+b*c^2)*e)*x^3+a*c^2*e*x
```

Maxima [A] time = 1.01608, size = 135, normalized size = 1.44

$$\frac{1}{9}\,bd^2fx^9+\frac{1}{7}\left(bd^2e+\left(2\,bcd+ad^2\right)f\right)x^7+\frac{1}{5}\left(\left(2\,bcd+ad^2\right)e+\left(bc^2+2\,acd\right)f\right)x^5+ac^2ex+\frac{1}{3}\left(ac^2f+\left(bc^2+2\,acd\right)e\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^2*(f*x^2+e),x, algorithm="maxima")
```

```
[Out] 1/9*b*d^2*f*x^9 + 1/7*(b*d^2*e + (2*b*c*d + a*d^2)*f)*x^7 + 1/5*((2*b*c*d +
 a*d^2)*e + (b*c^2 + 2*a*c*d)*f)*x^5 + a*c^2*e*x + 1/3*(a*c^2*f + (b*c^2 +
2*a*c*d)*e)*x^3
```

Fricas [A] time = 1.26207, size = 282, normalized size = 3.

$$\frac{1}{9}x^9fd^2b+\frac{1}{7}x^7ed^2b+\frac{2}{7}x^7fdcb+\frac{1}{7}x^7fd^2a+\frac{2}{5}x^5edcb+\frac{1}{5}x^5fc^2b+\frac{1}{5}x^5ed^2a+\frac{2}{5}x^5fdca+\frac{1}{3}x^3ec^2b+\frac{2}{3}x^3edca+\frac{1}{3}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^2*(f*x^2+e),x, algorithm="fricas")
```

```
[Out] 1/9*x^9*f*d^2*b + 1/7*x^7*e*d^2*b + 2/7*x^7*f*d*c*b + 1/7*x^7*f*d^2*a + 2/5
*x^5*e*d*c*b + 1/5*x^5*f*c^2*b + 1/5*x^5*e*d^2*a + 2/5*x^5*f*d*c*a + 1/3*x^
3*e*c^2*b + 2/3*x^3*e*d*c*a + 1/3*x^3*f*c^2*a + x*e*c^2*a
```

Sympy [A] time = 0.073632, size = 121, normalized size = 1.29

$$ac^2ex+\frac{bd^2fx^9}{9}+x^7\left(\frac{ad^2f}{7}+\frac{2bcdf}{7}+\frac{bd^2e}{7}\right)+x^5\left(\frac{2acdf}{5}+\frac{ad^2e}{5}+\frac{bc^2f}{5}+\frac{2bcde}{5}\right)+x^3\left(\frac{ac^2f}{3}+\frac{2acde}{3}+\frac{bc^2e}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+c)**2*(f*x**2+e),x)
```

```
[Out] a*c**2*e*x + b*d**2*f*x**9/9 + x**7*(a*d**2*f/7 + 2*b*c*d*f/7 + b*d**2*e/7)
 + x**5*(2*a*c*d*f/5 + a*d**2*e/5 + b*c**2*f/5 + 2*b*c*d*e/5) + x**3*(a*c**
2*f/3 + 2*a*c*d*e/3 + b*c**2*e/3)
```

Giac [A] time = 1.21616, size = 162, normalized size = 1.72

$$\frac{1}{9}bd^2fx^9 + \frac{2}{7}bcdfx^7 + \frac{1}{7}ad^2fx^7 + \frac{1}{7}bd^2x^7e + \frac{1}{5}bc^2fx^5 + \frac{2}{5}acdfx^5 + \frac{2}{5}bcdx^5e + \frac{1}{5}ad^2x^5e + \frac{1}{3}ac^2fx^3 + \frac{1}{3}bc^2x^3e + \frac{2}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^2*(f*x^2+e),x, algorithm="giac")
```

```
[Out] 1/9*b*d^2*f*x^9 + 2/7*b*c*d*f*x^7 + 1/7*a*d^2*f*x^7 + 1/7*b*d^2*x^7*e + 1/5
*b*c^2*f*x^5 + 2/5*a*c*d*f*x^5 + 2/5*b*c*d*x^5*e + 1/5*a*d^2*x^5*e + 1/3*a*
c^2*f*x^3 + 1/3*b*c^2*x^3*e + 2/3*a*c*d*x^3*e + a*c^2*x*e
```

3.12 $\int \frac{(a+bx^2)(c+dx^2)^2}{e+fx^2}\,dx$

Optimal. Leaf size=142

$$-\frac{x\left(5adf(3de-5cf)-b\left(8c^2f^2-25cdef+15d^2e^2\right)\right)}{15f^3}-\frac{x\left(c+dx^2\right)(-5adf-4bcf+5bde)}{15f^2}-\frac{(be-af)(de-cf)^2\tan}{\sqrt{e}f^{7/2}}$$

```
[Out] -((5*a*d*f*(3*d*e - 5*c*f) - b*(15*d^2*e^2 - 25*c*d*e*f + 8*c^2*f^2))*x)/(1
5*f^3) - ((5*b*d*e - 4*b*c*f - 5*a*d*f)*x*(c + d*x^2))/(15*f^2) + (b*x*(c +
 d*x^2)^2)/(5*f) - ((b*e - a*f)*(d*e - c*f)^2*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/
(Sqrt[e]*f^(7/2))
```

Rubi [A] time = 0.208257, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {528, 388, 205}

$$-\frac{x\left(5adf(3de-5cf)-b\left(8c^2f^2-25cdef+15d^2e^2\right)\right)}{15f^3}-\frac{x\left(c+dx^2\right)(-5adf-4bcf+5bde)}{15f^2}-\frac{(be-af)(de-cf)^2\tan}{\sqrt{e}f^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(c + d*x^2)^2)/(e + f*x^2),x]
```

```
[Out] -((5*a*d*f*(3*d*e - 5*c*f) - b*(15*d^2*e^2 - 25*c*d*e*f + 8*c^2*f^2))*x)/(1
5*f^3) - ((5*b*d*e - 4*b*c*f - 5*a*d*f)*x*(c + d*x^2))/(15*f^2) + (b*x*(c +
 d*x^2)^2)/(5*f) - ((b*e - a*f)*(d*e - c*f)^2*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/
(Sqrt[e]*f^(7/2))
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(a+bx^2)(c+dx^2)^2}{e+fx^2}\,dx = \frac{bx(c+dx^2)^2}{5f} + \frac{\int \frac{(c+dx^2)(-c(be-5af)+(-5bde+4bcf+5adf)x^2)}{e+fx^2}\,dx}{5f}$$
$$= -\frac{(5bde-4bcf-5adf)x(c+dx^2)}{15f^2} + \frac{bx(c+dx^2)^2}{5f} + \frac{\int \frac{c(be(5de-7cf)-5af(de-3cf))-(5adf(3de-5cf)\cdot}{e+fx^2}}{15f^2}$$
$$= -\frac{(5adf(3de-5cf)-b(15d^2e^2-25cdef+8c^2f^2))x}{15f^3} - \frac{(5bde-4bcf-5adf)x(c+dx^2)}{15f^2} +$$
$$= -\frac{(5adf(3de-5cf)-b(15d^2e^2-25cdef+8c^2f^2))x}{15f^3} - \frac{(5bde-4bcf-5adf)x(c+dx^2)}{15f^2} +$$

Mathematica [A] time = 0.0636626, size = 115, normalized size = 0.81

$$\frac{dx^3(adf+2bcf-bde)}{3f^2} + \frac{x(adf(2cf-de)+b(de-cf)^2)}{f^3} - \frac{(be-af)(de-cf)^2\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}f^{7/2}} + \frac{bd^2x^5}{5f}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(c + d*x^2)^2)/(e + f*x^2),x]
```

```
[Out] ((b*(d*e - c*f)^2 + a*d*f*(-(d*e) + 2*c*f))*x)/f^3 + (d*(-(b*d*e) + 2*b*c*f
 + a*d*f)*x^3)/(3*f^2) + (b*d^2*x^5)/(5*f) - ((b*e - a*f)*(d*e - c*f)^2*Arc
Tan[(Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e]*f^(7/2))
```

Maple [A] time = 0.004, size = 243, normalized size = 1.7

$$\frac{bd^2x^5}{5f} + \frac{x^3ad^2}{3f} + \frac{2x^3bcd}{3f} - \frac{x^3bd^2e}{3f^2} + 2\frac{acdx}{f} - \frac{ad^2ex}{f^2} + \frac{bc^2x}{f} - 2\frac{bcdex}{f^2} + \frac{bd^2e^2x}{f^3} + ac^2\arctan\left(fx\frac{1}{\sqrt{ef}}\right)\frac{1}{\sqrt{ef}} - 2\frac{acde}{f\sqrt{ef}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x^2+c)^2/(f*x^2+e),x)
```

```
[Out] 1/5/f*b*d^2*x^5+1/3/f*x^3*a*d^2+2/3/f*x^3*b*c*d-1/3/f^2*x^3*b*d^2*e+2/f*a*c
*d*x-1/f^2*a*d^2*e*x+1/f*b*c^2*x-2/f^2*b*c*d*e*x+1/f^3*b*d^2*e^2*x+1/(e*f)^
(1/2)*arctan(x*f/(e*f)^(1/2))*a*c^2-2/f/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))
*a*c*d*e+1/f^2/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*a*d^2*e^2-1/f/(e*f)^(1/2
)*arctan(x*f/(e*f)^(1/2))*b*c^2*e+2/f^2/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))
*b*c*d*e^2-1/f^3/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*b*d^2*e^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^2/(f*x^2+e),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.48085, size = 775, normalized size = 5.46

$$\left[\frac{6\,bd^2ef^3x^5 - 10\left(bd^2e^2f^2 - \left(2\,bcd + ad^2\right)ef^3\right)x^3 + 15\left(bd^2e^3 - ac^2f^3 - \left(2\,bcd + ad^2\right)e^2f + \left(bc^2 + 2\,acd\right)ef^2\right)\sqrt{-ef}\log}{30\,ef^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^2/(f*x^2+e),x, algorithm="fricas")
```

```
[Out] [1/30*(6*b*d^2*e*f^3*x^5 - 10*(b*d^2*e^2*f^2 - (2*b*c*d + a*d^2)*e*f^3)*x^3
 + 15*(b*d^2*e^3 - a*c^2*f^3 - (2*b*c*d + a*d^2)*e^2*f + (b*c^2 + 2*a*c*d)*
e*f^2)*sqrt(-e*f)*log((f*x^2 - 2*sqrt(-e*f)*x - e)/(f*x^2 + e)) + 30*(b*d^2
*e^3*f - (2*b*c*d + a*d^2)*e^2*f^2 + (b*c^2 + 2*a*c*d)*e*f^3)*x)/(e*f^4), 1
/15*(3*b*d^2*e*f^3*x^5 - 5*(b*d^2*e^2*f^2 - (2*b*c*d + a*d^2)*e*f^3)*x^3 -
15*(b*d^2*e^3 - a*c^2*f^3 - (2*b*c*d + a*d^2)*e^2*f + (b*c^2 + 2*a*c*d)*e*f
^2)*sqrt(e*f)*arctan(sqrt(e*f)*x/e) + 15*(b*d^2*e^3*f - (2*b*c*d + a*d^2)*e
^2*f^2 + (b*c^2 + 2*a*c*d)*e*f^3)*x)/(e*f^4)]
```

Sympy [B] time = 1.33411, size = 343, normalized size = 2.42

$$\frac{bd^2x^5}{5f} - \frac{\sqrt{-\frac{1}{ef^7}}\left(af - be\right)\left(cf - de\right)^2\log\left(-\frac{ef^3\sqrt{-\frac{1}{ef^7}}\left(af-be\right)\left(cf-de\right)^2}{ac^2f^3-2acdef^2+ad^2e^2f-bc^2ef^2+2bcde^2f-bd^2e^3} + x\right)}{2} + \frac{\sqrt{-\frac{1}{ef^7}}\left(af - be\right)\left(cf - de\right)^2\,\mathrm{l}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+c)**2/(f*x**2+e),x)
```

```
[Out] b*d**2*x**5/(5*f) - sqrt(-1/(e*f**7))*(a*f - b*e)*(c*f - d*e)**2*log(-e*f**
3*sqrt(-1/(e*f**7))*(a*f - b*e)*(c*f - d*e)**2/(a*c**2*f**3 - 2*a*c*d*e*f**
2 + a*d**2*e**2*f - b*c**2*e*f**2 + 2*b*c*d*e**2*f - b*d**2*e**3) + x)/2 +
sqrt(-1/(e*f**7))*(a*f - b*e)*(c*f - d*e)**2*log(e*f**3*sqrt(-1/(e*f**7))*(
a*f - b*e)*(c*f - d*e)**2/(a*c**2*f**3 - 2*a*c*d*e*f**2 + a*d**2*e**2*f - b
*c**2*e*f**2 + 2*b*c*d*e**2*f - b*d**2*e**3) + x)/2 + x**3*(a*d**2*f + 2*b*
c*d*f - b*d**2*e)/(3*f**2) + x*(2*a*c*d*f**2 - a*d**2*e*f + b*c**2*f**2 - 2
*b*c*d*e*f + b*d**2*e**2)/f**3
```

Giac [A] time = 1.17317, size = 240, normalized size = 1.69

$$\frac{\left(ac^2f^3 - bc^2f^2e - 2\,acdf^2e + 2\,bcdfe^2 + ad^2fe^2 - bd^2e^3\right)\arctan\left(\sqrt{f}xe^{\left(-\frac{1}{2}\right)}\right)e^{\left(-\frac{1}{2}\right)}}{f^{\frac{7}{2}}} + \frac{3\,bd^2f^4x^5 + 10\,bcdf^4x^3 + 5\,ad^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^2/(f*x^2+e),x, algorithm="giac")
```

```
[Out] (a*c^2*f^3 - b*c^2*f^2*e - 2*a*c*d*f^2*e + 2*b*c*d*f*e^2 + a*d^2*f*e^2 - b*
d^2*e^3)*arctan(sqrt(f)*x*e^(-1/2))*e^(-1/2)/f^(7/2) + 1/15*(3*b*d^2*f^4*x^
5 + 10*b*c*d*f^4*x^3 + 5*a*d^2*f^4*x^3 - 5*b*d^2*f^3*x^3*e + 15*b*c^2*f^4*x
 + 30*a*c*d*f^4*x - 30*b*c*d*f^3*x*e - 15*a*d^2*f^3*x*e + 15*b*d^2*f^2*x*e^
2)/f^5
```

3.13 $\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^2}\,dx$

Optimal. Leaf size=164

$$\frac{(de-cf)\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(be(5de-cf)-af(cf+3de))}{2e^{3/2}f^{7/2}}+\frac{dx\left(c+dx^2\right)(5be-3af)}{6ef^2}-\frac{dx(be(15de-13cf)-3af(3de-cf}{6ef^3}$$

```
[Out] -(d*(b*e*(15*d*e - 13*c*f) - 3*a*f*(3*d*e - c*f))*x)/(6*e*f^3) + (d*(5*b*e
- 3*a*f)*x*(c + d*x^2))/(6*e*f^2) - ((b*e - a*f)*x*(c + d*x^2)^2)/(2*e*f*(e
 + f*x^2)) + ((d*e - c*f)*(b*e*(5*d*e - c*f) - a*f*(3*d*e + c*f))*ArcTan[(S
qrt[f]*x)/Sqrt[e]])/(2*e^(3/2)*f^(7/2))
```

Rubi [A] time = 0.232371, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {526, 528, 388, 205}

$$\frac{(de-cf)\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(be(5de-cf)-af(cf+3de))}{2e^{3/2}f^{7/2}}+\frac{dx\left(c+dx^2\right)(5be-3af)}{6ef^2}-\frac{dx(be(15de-13cf)-3af(3de-cf}{6ef^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(c + d*x^2)^2)/(e + f*x^2)^2,x]
```

```
[Out] -(d*(b*e*(15*d*e - 13*c*f) - 3*a*f*(3*d*e - c*f))*x)/(6*e*f^3) + (d*(5*b*e
- 3*a*f)*x*(c + d*x^2))/(6*e*f^2) - ((b*e - a*f)*x*(c + d*x^2)^2)/(2*e*f*(e
 + f*x^2)) + ((d*e - c*f)*(b*e*(5*d*e - c*f) - a*f*(3*d*e + c*f))*ArcTan[(S
qrt[f]*x)/Sqrt[e]])/(2*e^(3/2)*f^(7/2))
```

Rule 526

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^q)/(a*b*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p +
 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^2}\,dx = -\frac{(be-af)x(c+dx^2)^2}{2ef(e+fx^2)} - \frac{\int \frac{(c+dx^2)(-c(be+af)-d(5be-3af)x^2)}{e+fx^2}\,dx}{2ef}$$

$$= \frac{d(5be-3af)x(c+dx^2)}{6ef^2} - \frac{(be-af)x(c+dx^2)^2}{2ef(e+fx^2)} - \frac{\int \frac{c(be(5de-3cf)-3af(de+cf))+d(be(15de-13cf)-3a}{e+fx^2}}{6ef^2}$$

$$= -\frac{d(be(15de-13cf)-3af(3de-cf))x}{6ef^3} + \frac{d(5be-3af)x(c+dx^2)}{6ef^2} - \frac{(be-af)x(c+dx^2)^2}{2ef(e+fx^2)} +$$

$$= -\frac{d(be(15de-13cf)-3af(3de-cf))x}{6ef^3} + \frac{d(5be-3af)x(c+dx^2)}{6ef^2} - \frac{(be-af)x(c+dx^2)^2}{2ef(e+fx^2)} +$$

Mathematica [A] time = 0.0925017, size = 134, normalized size = 0.82

$$\frac{(de-cf)\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(be(5de-cf)-af(cf+3de))}{2e^{3/2}f^{7/2}} - \frac{x(be-af)(de-cf)^2}{2ef^3(e+fx^2)} + \frac{dx(adf+2bcf-2bde)}{f^3} + \frac{bd^2x^3}{3f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(c + d*x^2)^2)/(e + f*x^2)^2,x]
```

```
[Out] (d*(-2*b*d*e + 2*b*c*f + a*d*f)*x)/f^3 + (b*d^2*x^3)/(3*f^2) - ((b*e - a*f)
*(d*e - c*f)^2*x)/(2*e*f^3*(e + f*x^2)) + ((d*e - c*f)*(b*e*(5*d*e - c*f) -
 a*f*(3*d*e + c*f))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(2*e^(3/2)*f^(7/2))
```

Maple [B] time = 0.009, size = 299, normalized size = 1.8

$$\frac{d^2x^3b}{3f^2} + \frac{ad^2x}{f^2} + 2\frac{bcdx}{f^2} - 2\frac{bd^2ex}{f^3} + \frac{axc^2}{2e(fx^2+e)} - \frac{axcd}{f(fx^2+e)} + \frac{exad^2}{2f^2(fx^2+e)} - \frac{bxc^2}{2f(fx^2+e)} + \frac{bxecd}{f^2(fx^2+e)} - \frac{e}{2f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x^2+c)^2/(f*x^2+e)^2,x)
```

```
[Out] 1/3*d^2/f^2*x^3*b+d^2/f^2*a*x+2*d/f^2*b*c*x-2*d^2/f^3*b*e*x+1/2/e*x/(f*x^2+
e)*a*c^2-1/f*x/(f*x^2+e)*a*c*d+1/2/f^2*e*x/(f*x^2+e)*a*d^2-1/2/f*x/(f*x^2+e
)*b*c^2+1/f^2*e*x/(f*x^2+e)*b*c*d-1/2/f^3*e^2*x/(f*x^2+e)*b*d^2+1/2/e/(e*f)
^(1/2)*arctan(x*f/(e*f)^(1/2))*a*c^2+1/f/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2)
)*a*c*d-3/2/f^2*e/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*a*d^2+1/2/f/(e*f)^(1/
2)*arctan(x*f/(e*f)^(1/2))*b*c^2-3/f^2*e/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2)
)*b*c*d+5/2/f^3*e^2/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*b*d^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^2/(f*x^2+e)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.59117, size = 1150, normalized size = 7.01

$$\left[\frac{4\,bd^2e^2f^3x^5 - 4\left(5\,bd^2e^3f^2 - 3\left(2\,bcd + ad^2\right)e^2f^3\right)x^3 - 3\left(5\,bd^2e^4 + ac^2ef^3 - 3\left(2\,bcd + ad^2\right)e^3f + \left(bc^2 + 2\,acd\right)e^2f^2 + \cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^2/(f*x^2+e)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(4*b*d^2*e^2*f^3*x^5 - 4*(5*b*d^2*e^3*f^2 - 3*(2*b*c*d + a*d^2)*e^2*f
^3)*x^3 - 3*(5*b*d^2*e^4 + a*c^2*e*f^3 - 3*(2*b*c*d + a*d^2)*e^3*f + (b*c^2
 + 2*a*c*d)*e^2*f^2 + (5*b*d^2*e^3*f + a*c^2*f^4 - 3*(2*b*c*d + a*d^2)*e^2*
f^2 + (b*c^2 + 2*a*c*d)*e*f^3)*x^2)*sqrt(-e*f)*log((f*x^2 - 2*sqrt(-e*f)*x
- e)/(f*x^2 + e)) - 6*(5*b*d^2*e^4*f - a*c^2*e*f^4 - 3*(2*b*c*d + a*d^2)*e^
3*f^2 + (b*c^2 + 2*a*c*d)*e^2*f^3)*x)/(e^2*f^5*x^2 + e^3*f^4), 1/6*(2*b*d^2
*e^2*f^3*x^5 - 2*(5*b*d^2*e^3*f^2 - 3*(2*b*c*d + a*d^2)*e^2*f^3)*x^3 + 3*(5
*b*d^2*e^4 + a*c^2*e*f^3 - 3*(2*b*c*d + a*d^2)*e^3*f + (b*c^2 + 2*a*c*d)*e^
2*f^2 + (5*b*d^2*e^3*f + a*c^2*f^4 - 3*(2*b*c*d + a*d^2)*e^2*f^2 + (b*c^2 +
 2*a*c*d)*e*f^3)*x^2)*sqrt(e*f)*arctan(sqrt(e*f)*x/e) - 3*(5*b*d^2*e^4*f -
a*c^2*e*f^4 - 3*(2*b*c*d + a*d^2)*e^3*f^2 + (b*c^2 + 2*a*c*d)*e^2*f^3)*x)/(
e^2*f^5*x^2 + e^3*f^4)]
```

Sympy [B] time = 3.64659, size = 479, normalized size = 2.92

$$\frac{bd^2x^3}{3f^2} + \frac{x\left(ac^2f^3 - 2acdef^2 + ad^2e^2f - bc^2ef^2 + 2bcde^2f - bd^2e^3\right)}{2e^2f^3 + 2ef^4x^2} - \frac{\sqrt{-\frac{1}{e^3f^7}}\left(cf - de\right)\left(acf^2 + 3adef + bcef - 5bde^{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+c)**2/(f*x**2+e)**2,x)
```

```
[Out] b*d**2*x**3/(3*f**2) + x*(a*c**2*f**3 - 2*a*c*d*e*f**2 + a*d**2*e**2*f - b*
c**2*e*f**2 + 2*b*c*d*e**2*f - b*d**2*e**3)/(2*e**2*f**3 + 2*e*f**4*x**2) -
 sqrt(-1/(e**3*f**7))*(c*f - d*e)*(a*c*f**2 + 3*a*d*e*f + b*c*e*f - 5*b*d*e
**2)*log(-e**2*f**3*sqrt(-1/(e**3*f**7))*(c*f - d*e)*(a*c*f**2 + 3*a*d*e*f
+ b*c*e*f - 5*b*d*e**2)/(a*c**2*f**3 + 2*a*c*d*e*f**2 - 3*a*d**2*e**2*f + b
*c**2*e*f**2 - 6*b*c*d*e**2*f + 5*b*d**2*e**3) + x)/4 + sqrt(-1/(e**3*f**7)
)*(c*f - d*e)*(a*c*f**2 + 3*a*d*e*f + b*c*e*f - 5*b*d*e**2)*log(e**2*f**3*s
qrt(-1/(e**3*f**7))*(c*f - d*e)*(a*c*f**2 + 3*a*d*e*f + b*c*e*f - 5*b*d*e**
2)/(a*c**2*f**3 + 2*a*c*d*e*f**2 - 3*a*d**2*e**2*f + b*c**2*e*f**2 - 6*b*c*
```

```
d*e**2*f + 5*b*d**2*e**3) + x)/4 + x*(a*d**2*f + 2*b*c*d*f - 2*b*d**2*e)/f*
*3
```

Giac [A] time = 1.16065, size = 263, normalized size = 1.6

$$\frac{\left(ac^2f^3 + bc^2f^2e + 2\,acdf^2e - 6\,bcdfe^2 - 3\,ad^2fe^2 + 5\,bd^2e^3\right)\arctan\left(\sqrt{f}xe^{\left(-\frac{1}{2}\right)}\right)e^{\left(-\frac{3}{2}\right)}}{2\,f^{\frac{7}{2}}} + \frac{\left(ac^2f^3x - bc^2f^2xe - 2\,acdf^2xe\right.}{2\left(f\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^2/(f*x^2+e)^2,x, algorithm="giac")
```

```
[Out] 1/2*(a*c^2*f^3 + b*c^2*f^2*e + 2*a*c*d*f^2*e - 6*b*c*d*f*e^2 - 3*a*d^2*f*e^
2 + 5*b*d^2*e^3)*arctan(sqrt(f)*x*e^(-1/2))*e^(-3/2)/f^(7/2) + 1/2*(a*c^2*f
^3*x - b*c^2*f^2*x*e - 2*a*c*d*f^2*x*e + 2*b*c*d*f*x*e^2 + a*d^2*f*x*e^2 -
b*d^2*x*e^3)*e^(-1)/((f*x^2 + e)*f^3) + 1/3*(b*d^2*f^4*x^3 + 6*b*c*d*f^4*x
+ 3*a*d^2*f^4*x - 6*b*d^2*f^3*x*e)/f^6
```

3.14 $\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^3} dx$

Optimal. Leaf size=207

$$-\frac{\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\left(be\left(-c^2f^2-6cdef+15d^2e^2\right)-af\left(3c^2f^2+2cdef+3d^2e^2\right)\right)}{8e^{5/2}f^{7/2}}-\frac{x\left(c+dx^2\right)(be(5de-cf)-af(3cf+de}{8e^2f^2\left(e+fx^2\right)}$$

```
[Out] (d*(b*e*(15*d*e - c*f) - 3*a*f*(d*e + c*f))*x)/(8*e^2*f^3) - ((b*e - a*f)*x
*(c + d*x^2)^2)/(4*e*f*(e + f*x^2)^2) - ((b*e*(5*d*e - c*f) - a*f*(d*e + 3*
c*f))*x*(c + d*x^2))/(8*e^2*f^2*(e + f*x^2)) - ((b*e*(15*d^2*e^2 - 6*c*d*e*
f - c^2*f^2) - a*f*(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2))*ArcTan[(Sqrt[f]*x)/
Sqrt[e]])/(8*e^(5/2)*f^(7/2))
```

Rubi [A] time = 0.238958, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {526, 388, 205}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\left(be\left(-c^2f^2-6cdef+15d^2e^2\right)-af\left(3c^2f^2+2cdef+3d^2e^2\right)\right)}{8e^{5/2}f^{7/2}}-\frac{x\left(c+dx^2\right)(be(5de-cf)-af(3cf+de}{8e^2f^2\left(e+fx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(c + d*x^2)^2)/(e + f*x^2)^3,x]
```



```
[Out] (d*(b*e*(15*d*e - c*f) - 3*a*f*(d*e + c*f))*x)/(8*e^2*f^3) - ((b*e - a*f)*x
*(c + d*x^2)^2)/(4*e*f*(e + f*x^2)^2) - ((b*e*(5*d*e - c*f) - a*f*(d*e + 3*
c*f))*x*(c + d*x^2))/(8*e^2*f^2*(e + f*x^2)) - ((b*e*(15*d^2*e^2 - 6*c*d*e*
f - c^2*f^2) - a*f*(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2))*ArcTan[(Sqrt[f]*x)/
Sqrt[e]])/(8*e^(5/2)*f^(7/2))
```


Rule 526

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^q)/(a*b*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p +
 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^3}dx = -\frac{(be-af)x(c+dx^2)^2}{4ef(e+fx^2)^2} - \frac{\int \frac{(c+dx^2)(-c(be+3af)-d(5be-af)x^2)}{(e+fx^2)^2}dx}{4ef}$$

$$= -\frac{(be-af)x(c+dx^2)^2}{4ef(e+fx^2)^2} - \frac{(be(5de-cf)-af(de+3cf))x(c+dx^2)}{8e^2f^2(e+fx^2)} + \frac{\int \frac{-c(af(de-3cf)-be(5de+c}{}}{}$$

$$= \frac{d(be(15de-cf)-3af(de+cf))x}{8e^2f^3} - \frac{(be-af)x(c+dx^2)^2}{4ef(e+fx^2)^2} - \frac{(be(5de-cf)-af(de+3cf))x}{8e^2f^2(e+fx^2)}$$

$$= \frac{d(be(15de-cf)-3af(de+cf))x}{8e^2f^3} - \frac{(be-af)x(c+dx^2)^2}{4ef(e+fx^2)^2} - \frac{(be(5de-cf)-af(de+3cf))x}{8e^2f^2(e+fx^2)}$$

Mathematica [A] time = 0.127148, size = 183, normalized size = 0.88

$$-\frac{\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\left(be\left(-c^2f^2-6cdef+15d^2e^2\right)-af\left(3c^2f^2+2cdef+3d^2e^2\right)\right)}{8e^{5/2}f^{7/2}} + \frac{x(de-cf)(be(9de-cf)-af(3cf+5de))}{8e^2f^3(e+fx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(c + d*x^2)^2)/(e + f*x^2)^3,x]
```

```
[Out] (b*d^2*x)/f^3 - ((b*e - a*f)*(d*e - c*f)^2*x)/(4*e*f^3*(e + f*x^2)^2) + ((d
*e - c*f)*(b*e*(9*d*e - c*f) - a*f*(5*d*e + 3*c*f))*x)/(8*e^2*f^3*(e + f*x^
2)) - ((b*e*(15*d^2*e^2 - 6*c*d*e*f - c^2*f^2) - a*f*(3*d^2*e^2 + 2*c*d*e*f
 + 3*c^2*f^2))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(8*e^(5/2)*f^(7/2))
```

Maple [B] time = 0.011, size = 397, normalized size = 1.9

$$\frac{bd^2x}{f^3} + \frac{3fx^3ac^2}{8(fx^2+e)^2e^2} + \frac{x^3acd}{4(fx^2+e)^2e} - \frac{5x^3ad^2}{8f(fx^2+e)^2} + \frac{x^3bc^2}{8(fx^2+e)^2e} - \frac{5x^3bcd}{4f(fx^2+e)^2} + \frac{9x^3bd^2e}{8f^2(fx^2+e)^2} + \frac{5ax}{8(fx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x^2+c)^2/(f*x^2+e)^3,x)
```

```
[Out] b*d^2/f^3*x+3/8*f/(f*x^2+e)^2/e^2*x^3*a*c^2+1/4/(f*x^2+e)^2/e*x^3*a*c*d-5/8
/f/(f*x^2+e)^2*x^3*a*d^2+1/8/(f*x^2+e)^2/e*x^3*b*c^2-5/4/f/(f*x^2+e)^2*x^3*
b*c*d+9/8/f^2/(f*x^2+e)^2*x^3*b*d^2*e+5/8/(f*x^2+e)^2/e*x*a*c^2-1/4/f/(f*x^
2+e)^2*a*c*d*x-3/8/f^2/(f*x^2+e)^2*a*d^2*e*x-1/8/f/(f*x^2+e)^2*b*c^2*x-3/4/
f^2/(f*x^2+e)^2*b*c*d*e*x+7/8/f^3/(f*x^2+e)^2*b*d^2*e^2*x+3/8/e^2/(e*f)^(1/
2)*arctan(x*f/(e*f)^(1/2))*a*c^2+1/4/f/e/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2)
)*a*c*d+3/8/f^2/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*a*d^2+1/8/f/e/(e*f)^(1/
2)*arctan(x*f/(e*f)^(1/2))*b*c^2+3/4/f^2/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2)
)*b*c*d-15/8/f^3*e/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*b*d^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^2/(f*x^2+e)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.5857, size = 1613, normalized size = 7.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^2/(f*x^2+e)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(16*b*d^2*e^3*f^3*x^5 + 2*(25*b*d^2*e^4*f^2 + 3*a*c^2*e*f^5 - 5*(2*b*
c*d + a*d^2)*e^3*f^3 + (b*c^2 + 2*a*c*d)*e^2*f^4)*x^3 + (15*b*d^2*e^5 - 3*a
*c^2*e^2*f^3 - 3*(2*b*c*d + a*d^2)*e^4*f - (b*c^2 + 2*a*c*d)*e^3*f^2 + (15*
b*d^2*e^3*f^2 - 3*a*c^2*f^5 - 3*(2*b*c*d + a*d^2)*e^2*f^3 - (b*c^2 + 2*a*c*
d)*e*f^4)*x^4 + 2*(15*b*d^2*e^4*f - 3*a*c^2*e*f^4 - 3*(2*b*c*d + a*d^2)*e^3
*f^2 - (b*c^2 + 2*a*c*d)*e^2*f^3)*x^2)*sqrt(-e*f)*log((f*x^2 - 2*sqrt(-e*f)
*x - e)/(f*x^2 + e)) + 2*(15*b*d^2*e^5*f + 5*a*c^2*e^2*f^4 - 3*(2*b*c*d + a
*d^2)*e^4*f^2 - (b*c^2 + 2*a*c*d)*e^3*f^3)*x)/(e^3*f^6*x^4 + 2*e^4*f^5*x^2
+ e^5*f^4), 1/8*(8*b*d^2*e^3*f^3*x^5 + (25*b*d^2*e^4*f^2 + 3*a*c^2*e*f^5 -
5*(2*b*c*d + a*d^2)*e^3*f^3 + (b*c^2 + 2*a*c*d)*e^2*f^4)*x^3 - (15*b*d^2*e^
5 - 3*a*c^2*e^2*f^3 - 3*(2*b*c*d + a*d^2)*e^4*f - (b*c^2 + 2*a*c*d)*e^3*f^2
 + (15*b*d^2*e^3*f^2 - 3*a*c^2*f^5 - 3*(2*b*c*d + a*d^2)*e^2*f^3 - (b*c^2 +
 2*a*c*d)*e*f^4)*x^4 + 2*(15*b*d^2*e^4*f - 3*a*c^2*e*f^4 - 3*(2*b*c*d + a*d
^2)*e^3*f^2 - (b*c^2 + 2*a*c*d)*e^2*f^3)*x^2)*sqrt(e*f)*arctan(sqrt(e*f)*x/
e) + (15*b*d^2*e^5*f + 5*a*c^2*e^2*f^4 - 3*(2*b*c*d + a*d^2)*e^4*f^2 - (b*c
^2 + 2*a*c*d)*e^3*f^3)*x)/(e^3*f^6*x^4 + 2*e^4*f^5*x^2 + e^5*f^4)]
```

Sympy [B] time = 18.9233, size = 400, normalized size = 1.93

$$\frac{bd^2x}{f^3} - \frac{\sqrt{-\frac{1}{e^5f^7}}\left(3ac^2f^3 + 2acdef^2 + 3ad^2e^2f + bc^2ef^2 + 6bcde^2f - 15bd^2e^3\right)\log\left(-e^3f^3\sqrt{-\frac{1}{e^5f^7}} + x\right)}{16} + \frac{\sqrt{-\frac{1}{e^5f^7}}\left(3ac\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+c)**2/(f*x**2+e)**3,x)
```

```
[Out] b*d**2*x/f**3 - sqrt(-1/(e**5*f**7))*(3*a*c**2*f**3 + 2*a*c*d*e*f**2 + 3*a*
d**2*e**2*f + b*c**2*e*f**2 + 6*b*c*d*e**2*f - 15*b*d**2*e**3)*log(-e**3*f*
*3*sqrt(-1/(e**5*f**7)) + x)/16 + sqrt(-1/(e**5*f**7))*(3*a*c**2*f**3 + 2*a
*c*d*e*f**2 + 3*a*d**2*e**2*f + b*c**2*e*f**2 + 6*b*c*d*e**2*f - 15*b*d**2*
e**3)*log(e**3*f**3*sqrt(-1/(e**5*f**7)) + x)/16 + (x**3*(3*a*c**2*f**4 + 2
*a*c*d*e*f**3 - 5*a*d**2*e**2*f**2 + b*c**2*e*f**3 - 10*b*c*d*e**2*f**2 + 9
*b*d**2*e**3*f) + x*(5*a*c**2*e*f**3 - 2*a*c*d*e**2*f**2 - 3*a*d**2*e**3*f
- b*c**2*e**2*f**2 - 6*b*c*d*e**3*f + 7*b*d**2*e**4))/(8*e**4*f**3 + 16*e**
3*f**4*x**2 + 8*e**2*f**5*x**4)
```

Giac [A] time = 1.19987, size = 321, normalized size = 1.55

$$\frac{bd^2x}{f^3} + \frac{\left(3\,ac^2f^3 + bc^2f^2e + 2\,acdf^2e + 6\,bcdfe^2 + 3\,ad^2fe^2 - 15\,bd^2e^3\right)\arctan\left(\sqrt{f}xe^{\left(-\frac{1}{2}\right)}\right)e^{\left(-\frac{5}{2}\right)}}{8\,f^{\frac{7}{2}}} + \frac{\left(3\,ac^2f^4x^3 + bc^2f^3x^3 \cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^2/(f*x^2+e)^3,x, algorithm="giac")
```

```
[Out] b*d^2*x/f^3 + 1/8*(3*a*c^2*f^3 + b*c^2*f^2*e + 2*a*c*d*f^2*e + 6*b*c*d*f*e^
2 + 3*a*d^2*f*e^2 - 15*b*d^2*e^3)*arctan(sqrt(f)*x*e^(-1/2))*e^(-5/2)/f^(7/
2) + 1/8*(3*a*c^2*f^4*x^3 + b*c^2*f^3*x^3*e + 2*a*c*d*f^3*x^3*e - 10*b*c*d*
f^2*x^3*e^2 - 5*a*d^2*f^2*x^3*e^2 + 9*b*d^2*f*x^3*e^3 + 5*a*c^2*f^3*x*e - b
*c^2*f^2*x*e^2 - 2*a*c*d*f^2*x*e^2 - 6*b*c*d*f*x*e^3 - 3*a*d^2*f*x*e^3 + 7*
b*d^2*x*e^4)*e^(-2)/((f*x^2 + e)^2*f^3)
```

3.15 $\int \frac{\left(a+bx^2\right)\left(c+dx^2\right)^2}{\left(e+fx^2\right)^4}\,dx$

Optimal. Leaf size=240

$$-\frac{x\left(af\left(-15c^2f^2+4cdef+3d^2e^2\right)+be\left(-3c^2f^2-4cdef+15d^2e^2\right)\right)}{48e^3f^3\left(e+fx^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\left(af\left(5c^2f^2+2cdef+d^2e^2\right)+be\,(\right.}{16e^{7/2}f^{7/2}}$$

```
[Out] -((b*e - a*f)*x*(c + d*x^2)^2)/(6*e*f*(e + f*x^2)^3) - ((d*e*(5*b*e + a*f)
- c*f*(b*e + 5*a*f))*x*(c + d*x^2))/(24*e^2*f^2*(e + f*x^2)^2) - ((a*f*(3*d
^2*e^2 + 4*c*d*e*f - 15*c^2*f^2) + b*e*(15*d^2*e^2 - 4*c*d*e*f - 3*c^2*f^2)
)*x)/(48*e^3*f^3*(e + f*x^2)) + ((b*e*(5*d^2*e^2 + 2*c*d*e*f + c^2*f^2) + a
*f*(d^2*e^2 + 2*c*d*e*f + 5*c^2*f^2))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(16*e^(7
/2)*f^(7/2))
```

Rubi [A] time = 0.280437, antiderivative size = 240, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {526, 385, 205}

$$-\frac{x\left(af\left(-15c^2f^2+4cdef+3d^2e^2\right)+be\left(-3c^2f^2-4cdef+15d^2e^2\right)\right)}{48e^3f^3\left(e+fx^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\left(af\left(5c^2f^2+2cdef+d^2e^2\right)+be\,(\right.}{16e^{7/2}f^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(c + d*x^2)^2)/(e + f*x^2)^4,x]
```

```
[Out] -((b*e - a*f)*x*(c + d*x^2)^2)/(6*e*f*(e + f*x^2)^3) - ((d*e*(5*b*e + a*f)
- c*f*(b*e + 5*a*f))*x*(c + d*x^2))/(24*e^2*f^2*(e + f*x^2)^2) - ((a*f*(3*d
^2*e^2 + 4*c*d*e*f - 15*c^2*f^2) + b*e*(15*d^2*e^2 - 4*c*d*e*f - 3*c^2*f^2)
)*x)/(48*e^3*f^3*(e + f*x^2)) + ((b*e*(5*d^2*e^2 + 2*c*d*e*f + c^2*f^2) + a
*f*(d^2*e^2 + 2*c*d*e*f + 5*c^2*f^2))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(16*e^(7
/2)*f^(7/2))
```

Rule 526

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^q)/(a*b*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p +
 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^4}\,dx = -\frac{(be-af)x(c+dx^2)^2}{6ef(e+fx^2)^3} - \frac{\int \frac{(c+dx^2)(-c(be+5af)-d(5be+af)x^2)}{(e+fx^2)^3}\,dx}{6ef}$$

$$= -\frac{(be-af)x(c+dx^2)^2}{6ef(e+fx^2)^3} - \frac{(de(5be+af)-cf(be+5af))x(c+dx^2)}{24e^2f^2(e+fx^2)^2} + \frac{\int \frac{c(de(5be+af)+3cf(be+5a}{}}{}$$

$$= -\frac{(be-af)x(c+dx^2)^2}{6ef(e+fx^2)^3} - \frac{(de(5be+af)-cf(be+5af))x(c+dx^2)}{24e^2f^2(e+fx^2)^2} - \frac{(af(3d^2e^2+4cdef-}{}$$

$$= -\frac{(be-af)x(c+dx^2)^2}{6ef(e+fx^2)^3} - \frac{(de(5be+af)-cf(be+5af))x(c+dx^2)}{24e^2f^2(e+fx^2)^2} - \frac{(af(3d^2e^2+4cdef-}{}$$

Mathematica [A] time = 0.160394, size = 242, normalized size = 1.01

$$\frac{x(af(5c^2f^2+2cdef+d^2e^2)+be(c^2f^2+2cdef-11d^2e^2))}{16e^3f^3(e+fx^2)} + \frac{\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(af(5c^2f^2+2cdef+d^2e^2)+be(c^2f^2+2cde}{16e^{7/2}f^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(c + d*x^2)^2)/(e + f*x^2)^4,x]
```

```
[Out] -((b*e - a*f)*(d*e - c*f)^2*x)/(6*e*f^3*(e + f*x^2)^3) + ((d*e - c*f)*(b*e*
(13*d*e - c*f) - a*f*(7*d*e + 5*c*f))*x)/(24*e^2*f^3*(e + f*x^2)^2) + ((b*e
*(-11*d^2*e^2 + 2*c*d*e*f + c^2*f^2) + a*f*(d^2*e^2 + 2*c*d*e*f + 5*c^2*f^2
))*x)/(16*e^3*f^3*(e + f*x^2)) + ((b*e*(5*d^2*e^2 + 2*c*d*e*f + c^2*f^2) +
a*f*(d^2*e^2 + 2*c*d*e*f + 5*c^2*f^2))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(16*e^(
7/2)*f^(7/2))
```

Maple [A] time = 0.01, size = 360, normalized size = 1.5

$$\frac{1}{(fx^2+e)^3}\left(\frac{(5ac^2f^3+2acdef^2+ad^2e^2f+bc^2ef^2+2bcde^2f-11bd^2e^3)x^5}{16e^3f} + \frac{(5ac^2f^3+2acdef^2-ad^2e^2f+bc^2ef^2-}{6e^2f^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x^2+c)^2/(f*x^2+e)^4,x)
```

```
[Out] (1/16*(5*a*c^2*f^3+2*a*c*d*e*f^2+a*d^2*e^2*f+b*c^2*e*f^2+2*b*c*d*e^2*f-11*b
*d^2*e^3)/e^3/f*x^5+1/6*(5*a*c^2*f^3+2*a*c*d*e*f^2-a*d^2*e^2*f+b*c^2*e*f^2-
2*b*c*d*e^2*f-5*b*d^2*e^3)/e^2/f^2*x^3+1/16*(11*a*c^2*f^3-2*a*c*d*e*f^2-a*d
^2*e^2*f-b*c^2*e*f^2-2*b*c*d*e^2*f-5*b*d^2*e^3)/f^3/e*x)/(f*x^2+e)^3+5/16/e
^3/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*a*c^2+1/8/e^2/f/(e*f)^(1/2)*arctan(x
*f/(e*f)^(1/2))*a*c*d+1/16/e/f^2/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*a*d^2+
```

```
1/16/e^2/f/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*b*c^2+1/8/e/f^2/(e*f)^(1/2)*
arctan(x*f/(e*f)^(1/2))*b*c*d+5/16/f^3/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*
b*d^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^2/(f*x^2+e)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.60621, size = 2117, normalized size = 8.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^2/(f*x^2+e)^4,x, algorithm="fricas")
```

```
[Out] [-1/96*(6*(11*b*d^2*e^4*f^3 - 5*a*c^2*e*f^6 - (2*b*c*d + a*d^2)*e^3*f^4 - (
b*c^2 + 2*a*c*d)*e^2*f^5)*x^5 + 16*(5*b*d^2*e^5*f^2 - 5*a*c^2*e^2*f^5 + (2*
b*c*d + a*d^2)*e^4*f^3 - (b*c^2 + 2*a*c*d)*e^3*f^4)*x^3 + 3*(5*b*d^2*e^6 +
5*a*c^2*e^3*f^3 + (2*b*c*d + a*d^2)*e^5*f + (b*c^2 + 2*a*c*d)*e^4*f^2 + (5*
b*d^2*e^3*f^3 + 5*a*c^2*f^6 + (2*b*c*d + a*d^2)*e^2*f^4 + (b*c^2 + 2*a*c*d)
*e*f^5)*x^6 + 3*(5*b*d^2*e^4*f^2 + 5*a*c^2*e*f^5 + (2*b*c*d + a*d^2)*e^3*f^
3 + (b*c^2 + 2*a*c*d)*e^2*f^4)*x^4 + 3*(5*b*d^2*e^5*f + 5*a*c^2*e^2*f^4 + (
2*b*c*d + a*d^2)*e^4*f^2 + (b*c^2 + 2*a*c*d)*e^3*f^3)*x^2)*sqrt(-e*f)*log((
f*x^2 - 2*sqrt(-e*f)*x - e)/(f*x^2 + e)) + 6*(5*b*d^2*e^6*f - 11*a*c^2*e^3*
f^4 + (2*b*c*d + a*d^2)*e^5*f^2 + (b*c^2 + 2*a*c*d)*e^4*f^3)*x)/(e^4*f^7*x^
6 + 3*e^5*f^6*x^4 + 3*e^6*f^5*x^2 + e^7*f^4), -1/48*(3*(11*b*d^2*e^4*f^3 -
5*a*c^2*e*f^6 - (2*b*c*d + a*d^2)*e^3*f^4 - (b*c^2 + 2*a*c*d)*e^2*f^5)*x^5
+ 8*(5*b*d^2*e^5*f^2 - 5*a*c^2*e^2*f^5 + (2*b*c*d + a*d^2)*e^4*f^3 - (b*c^2
 + 2*a*c*d)*e^3*f^4)*x^3 - 3*(5*b*d^2*e^6 + 5*a*c^2*e^3*f^3 + (2*b*c*d + a*
d^2)*e^5*f + (b*c^2 + 2*a*c*d)*e^4*f^2 + (5*b*d^2*e^3*f^3 + 5*a*c^2*f^6 + (
2*b*c*d + a*d^2)*e^2*f^4 + (b*c^2 + 2*a*c*d)*e*f^5)*x^6 + 3*(5*b*d^2*e^4*f^
2 + 5*a*c^2*e*f^5 + (2*b*c*d + a*d^2)*e^3*f^3 + (b*c^2 + 2*a*c*d)*e^2*f^4)*
x^4 + 3*(5*b*d^2*e^5*f + 5*a*c^2*e^2*f^4 + (2*b*c*d + a*d^2)*e^4*f^2 + (b*c
^2 + 2*a*c*d)*e^3*f^3)*x^2)*sqrt(e*f)*arctan(sqrt(e*f)*x/e) + 3*(5*b*d^2*e^
6*f - 11*a*c^2*e^3*f^4 + (2*b*c*d + a*d^2)*e^5*f^2 + (b*c^2 + 2*a*c*d)*e^4*
f^3)*x)/(e^4*f^7*x^6 + 3*e^5*f^6*x^4 + 3*e^6*f^5*x^2 + e^7*f^4)]
```

Sympy [B] time = 96.292, size = 486, normalized size = 2.02

$$-\frac{\sqrt{-\frac{1}{e^7 f^7}}\left(5ac^2f^3 + 2acdef^2 + ad^2e^2f + bc^2ef^2 + 2bcde^2f + 5bd^2e^3\right)\log\left(-e^4 f^3\sqrt{-\frac{1}{e^7 f^7}} + x\right)}{32} + \frac{\sqrt{-\frac{1}{e^7 f^7}}\left(5ac^2f^3 + 2a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+c)**2/(f*x**2+e)**4,x)
```

```
[Out] -sqrt(-1/(e**7*f**7))*(5*a*c**2*f**3 + 2*a*c*d*e*f**2 + a*d**2*e**2*f + b*c
**2*e*f**2 + 2*b*c*d*e**2*f + 5*b*d**2*e**3)*log(-e**4*f**3*sqrt(-1/(e**7*f
**7)) + x)/32 + sqrt(-1/(e**7*f**7))*(5*a*c**2*f**3 + 2*a*c*d*e*f**2 + a*d*
*2*e**2*f + b*c**2*e*f**2 + 2*b*c*d*e**2*f + 5*b*d**2*e**3)*log(e**4*f**3*s
qrt(-1/(e**7*f**7)) + x)/32 + (x**5*(15*a*c**2*f**5 + 6*a*c*d*e*f**4 + 3*a*
d**2*e**2*f**3 + 3*b*c**2*e*f**4 + 6*b*c*d*e**2*f**3 - 33*b*d**2*e**3*f**2)
 + x**3*(40*a*c**2*e*f**4 + 16*a*c*d*e**2*f**3 - 8*a*d**2*e**3*f**2 + 8*b*c
**2*e**2*f**3 - 16*b*c*d*e**3*f**2 - 40*b*d**2*e**4*f) + x*(33*a*c**2*e**2*
f**3 - 6*a*c*d*e**3*f**2 - 3*a*d**2*e**4*f - 3*b*c**2*e**3*f**2 - 6*b*c*d*e
**4*f - 15*b*d**2*e**5))/(48*e**6*f**3 + 144*e**5*f**4*x**2 + 144*e**4*f**5
*x**4 + 48*e**3*f**6*x**6)
```

Giac [A] time = 1.15767, size = 420, normalized size = 1.75

$$\frac{\left(5\,ac^2f^3 + bc^2f^2e + 2\,acdf^2e + 2\,bcdfe^2 + ad^2fe^2 + 5\,bd^2e^3\right)\arctan\left(\sqrt{f}xe^{\left(-\frac{1}{2}\right)}\right)e^{\left(-\frac{7}{2}\right)}}{16\,f^{\frac{7}{2}}} + \frac{\left(15\,ac^2f^5x^5 + 3\,bc^2f^4x^5e + 6\,ac\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^2/(f*x^2+e)^4,x, algorithm="giac")
```

```
[Out] 1/16*(5*a*c^2*f^3 + b*c^2*f^2*e + 2*a*c*d*f^2*e + 2*b*c*d*f*e^2 + a*d^2*f*e
^2 + 5*b*d^2*e^3)*arctan(sqrt(f)*x*e^(-1/2))*e^(-7/2)/f^(7/2) + 1/48*(15*a*
c^2*f^5*x^5 + 3*b*c^2*f^4*x^5*e + 6*a*c*d*f^4*x^5*e + 6*b*c*d*f^3*x^5*e^2 +
 3*a*d^2*f^3*x^5*e^2 - 33*b*d^2*f^2*x^5*e^3 + 40*a*c^2*f^4*x^3*e + 8*b*c^2*
f^3*x^3*e^2 + 16*a*c*d*f^3*x^3*e^2 - 16*b*c*d*f^2*x^3*e^3 - 8*a*d^2*f^2*x^3
*e^3 - 40*b*d^2*f*x^3*e^4 + 33*a*c^2*f^3*x*e^2 - 3*b*c^2*f^2*x*e^3 - 6*a*c*
d*f^2*x*e^3 - 6*b*c*d*f*x*e^4 - 3*a*d^2*f*x*e^4 - 15*b*d^2*x*e^5)*e^(-3)/((
f*x^2 + e)^3*f^3)
```

3.16 $\int \left(a+bx^2\right)\left(c+dx^2\right)^3\left(e+fx^2\right)^3\,dx$

Optimal. Leaf size=310

$$\frac{3}{11}dfx^{11}\left(adf(cf+de)+b\left(c^2f^2+3cdef+d^2e^2\right)\right)+\frac{1}{9}x^9\left(3adf\left(c^2f^2+3cdef+d^2e^2\right)+b\left(9c^2def^2+c^3f^3+9cd^2e^2f\right.\right.$$

```
[Out] a*c^3*e^3*x + (c^2*e^2*(b*c*e + 3*a*(d*e + c*f))*x^3)/3 + (3*c*e*(b*c*e*(d*
e + c*f) + a*(d^2*e^2 + 3*c*d*e*f + c^2*f^2))*x^5)/5 + ((3*b*c*e*(d^2*e^2 +
 3*c*d*e*f + c^2*f^2) + a*(d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 + c^3*f^
3))*x^7)/7 + ((3*a*d*f*(d^2*e^2 + 3*c*d*e*f + c^2*f^2) + b*(d^3*e^3 + 9*c*d
^2*e^2*f + 9*c^2*d*e*f^2 + c^3*f^3))*x^9)/9 + (3*d*f*(a*d*f*(d*e + c*f) + b
*(d^2*e^2 + 3*c*d*e*f + c^2*f^2))*x^11)/11 + (d^2*f^2*(a*d*f + 3*b*(d*e + c
*f))*x^13)/13 + (b*d^3*f^3*x^15)/15
```

Rubi [A] time = 0.386571, antiderivative size = 310, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {521}

$$\frac{3}{11}dfx^{11}\left(adf(cf+de)+b\left(c^2f^2+3cdef+d^2e^2\right)\right)+\frac{1}{9}x^9\left(3adf\left(c^2f^2+3cdef+d^2e^2\right)+b\left(9c^2def^2+c^3f^3+9cd^2e^2f\right.\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)*(c + d*x^2)^3*(e + f*x^2)^3,x]
```

```
[Out] a*c^3*e^3*x + (c^2*e^2*(b*c*e + 3*a*(d*e + c*f))*x^3)/3 + (3*c*e*(b*c*e*(d*
e + c*f) + a*(d^2*e^2 + 3*c*d*e*f + c^2*f^2))*x^5)/5 + ((3*b*c*e*(d^2*e^2 +
 3*c*d*e*f + c^2*f^2) + a*(d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 + c^3*f^
3))*x^7)/7 + ((3*a*d*f*(d^2*e^2 + 3*c*d*e*f + c^2*f^2) + b*(d^3*e^3 + 9*c*d
^2*e^2*f + 9*c^2*d*e*f^2 + c^3*f^3))*x^9)/9 + (3*d*f*(a*d*f*(d*e + c*f) + b
*(d^2*e^2 + 3*c*d*e*f + c^2*f^2))*x^11)/11 + (d^2*f^2*(a*d*f + 3*b*(d*e + c
*f))*x^13)/13 + (b*d^3*f^3*x^15)/15
```

Rule 521

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c +
d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[p
, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^2\right)\left(c+dx^2\right)^3\left(e+fx^2\right)^3\,dx &= \int \left(ac^3e^3+c^2e^2(bce+3a(de+cf))x^2+3ce\left(bce(de+cf)+a\left(d^2e^2+3cdef+\right.\right.\right.\\ &= ac^3e^3x+\frac{1}{3}c^2e^2(bce+3a(de+cf))x^3+\frac{3}{5}ce\left(bce(de+cf)+a\left(d^2e^2+3cdef+\right.\right.\end{aligned}$$

Mathematica [A] time = 0.114383, size = 310, normalized size = 1.

$$\frac{3}{11}dfx^{11}\left(adf(cf+de)+b\left(c^2f^2+3cdef+d^2e^2\right)\right)+\frac{1}{9}x^9\left(3adf\left(c^2f^2+3cdef+d^2e^2\right)+b\left(9c^2def^2+c^3f^3+9cd^2e^2f\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)*(c + d*x^2)^3*(e + f*x^2)^3,x]
```

```
[Out] a*c^3*e^3*x + (c^2*e^2*(b*c*e + 3*a*(d*e + c*f))*x^3)/3 + (3*c*e*(b*c*e*(d*
e + c*f) + a*(d^2*e^2 + 3*c*d*e*f + c^2*f^2))*x^5)/5 + ((3*b*c*e*(d^2*e^2 +
 3*c*d*e*f + c^2*f^2) + a*(d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 + c^3*f^
3))*x^7)/7 + ((3*a*d*f*(d^2*e^2 + 3*c*d*e*f + c^2*f^2) + b*(d^3*e^3 + 9*c*d
^2*e^2*f + 9*c^2*d*e*f^2 + c^3*f^3))*x^9)/9 + (3*d*f*(a*d*f*(d*e + c*f) + b
*(d^2*e^2 + 3*c*d*e*f + c^2*f^2))*x^11)/11 + (d^2*f^2*(a*d*f + 3*b*(d*e + c
*f))*x^13)/13 + (b*d^3*f^3*x^15)/15
```

Maple [A] time = 0., size = 339, normalized size = 1.1

$$\frac{bd^3f^3x^{15}}{15}+\frac{\left(\left(ad^3+3\,bcd^2\right)f^3+3\,bd^3ef^2\right)x^{13}}{13}+\frac{\left(\left(3\,acd^2+3\,bc^2d\right)f^3+3\,\left(ad^3+3\,bcd^2\right)ef^2+3\,bd^3e^2f\right)x^{11}}{11}+\frac{\left(\left(3\,ac^2d\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x^2+c)^3*(f*x^2+e)^3,x)
```

```
[Out] 1/15*b*d^3*f^3*x^15+1/13*((a*d^3+3*b*c*d^2)*f^3+3*b*d^3*e*f^2)*x^13+1/11*((
3*a*c*d^2+3*b*c^2*d)*f^3+3*(a*d^3+3*b*c*d^2)*e*f^2+3*b*d^3*e^2*f)*x^11+1/9*
((3*a*c^2*d+b*c^3)*f^3+3*(3*a*c*d^2+3*b*c^2*d)*e*f^2+3*(a*d^3+3*b*c*d^2)*e^
2*f+b*d^3*e^3)*x^9+1/7*(a*c^3*f^3+3*(3*a*c^2*d+b*c^3)*e*f^2+3*(3*a*c*d^2+3*
b*c^2*d)*e^2*f+(a*d^3+3*b*c*d^2)*e^3)*x^7+1/5*(3*a*c^3*e*f^2+3*(3*a*c^2*d+b
*c^3)*e^2*f+(3*a*c*d^2+3*b*c^2*d)*e^3)*x^5+1/3*(3*a*c^3*e^2*f+(3*a*c^2*d+b*
c^3)*e^3)*x^3+a*c^3*e^3*x
```

Maxima [A] time = 0.985705, size = 440, normalized size = 1.42

$$\frac{1}{15}\,bd^3f^3x^{15}+\frac{1}{13}\left(3\,bd^3ef^2+\left(3\,bcd^2+ad^3\right)f^3\right)x^{13}+\frac{3}{11}\left(bd^3e^2f+\left(3\,bcd^2+ad^3\right)ef^2+\left(bc^2d+acd^2\right)f^3\right)x^{11}+\frac{1}{9}\left(bd^3e^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^3*(f*x^2+e)^3,x, algorithm="maxima")
```

```
[Out] 1/15*b*d^3*f^3*x^15 + 1/13*(3*b*d^3*e*f^2 + (3*b*c*d^2 + a*d^3)*f^3)*x^13 +
 3/11*(b*d^3*e^2*f + (3*b*c*d^2 + a*d^3)*e*f^2 + (b*c^2*d + a*c*d^2)*f^3)*x
^11 + 1/9*(b*d^3*e^3 + 3*(3*b*c*d^2 + a*d^3)*e^2*f + 9*(b*c^2*d + a*c*d^2)*
e*f^2 + (b*c^3 + 3*a*c^2*d)*f^3)*x^9 + a*c^3*e^3*x + 1/7*(a*c^3*f^3 + (3*b*
c*d^2 + a*d^3)*e^3 + 9*(b*c^2*d + a*c*d^2)*e^2*f + 3*(b*c^3 + 3*a*c^2*d)*e*
f^2)*x^7 + 3/5*(a*c^3*e*f^2 + (b*c^2*d + a*c*d^2)*e^3 + (b*c^3 + 3*a*c^2*d)
*e^2*f)*x^5 + 1/3*(3*a*c^3*e^2*f + (b*c^3 + 3*a*c^2*d)*e^3)*x^3
```

Fricas [A] time = 1.27603, size = 938, normalized size = 3.03

$$\frac{1}{15}x^{15}f^3d^3b+\frac{3}{13}x^{13}f^2ed^3b+\frac{3}{13}x^{13}f^3d^2cb+\frac{1}{13}x^{13}f^3d^3a+\frac{3}{11}x^{11}fe^2d^3b+\frac{9}{11}x^{11}f^2ed^2cb+\frac{3}{11}x^{11}f^3dc^2b+\frac{3}{11}x^{11}f^2ed^3a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^3*(f*x^2+e)^3,x, algorithm="fricas")
```

```
[Out] 1/15*x^15*f^3*d^3*b + 3/13*x^13*f^2*e*d^3*b + 3/13*x^13*f^3*d^2*c*b + 1/13*
x^13*f^3*d^3*a + 3/11*x^11*f*e^2*d^3*b + 9/11*x^11*f^2*e*d^2*c*b + 3/11*x^1
1*f^3*d*c^2*b + 3/11*x^11*f^2*e*d^3*a + 3/11*x^11*f^3*d^2*c*a + 1/9*x^9*e^3
*d^3*b + x^9*f*e^2*d^2*c*b + x^9*f^2*e*d*c^2*b + 1/9*x^9*f^3*c^3*b + 1/3*x^
9*f*e^2*d^3*a + x^9*f^2*e*d^2*c*a + 1/3*x^9*f^3*d*c^2*a + 3/7*x^7*e^3*d^2*c
*b + 9/7*x^7*f*e^2*d*c^2*b + 3/7*x^7*f^2*e*c^3*b + 1/7*x^7*e^3*d^3*a + 9/7*
x^7*f*e^2*d^2*c*a + 9/7*x^7*f^2*e*d*c^2*a + 1/7*x^7*f^3*c^3*a + 3/5*x^5*e^3
*d*c^2*b + 3/5*x^5*f*e^2*c^3*b + 3/5*x^5*e^3*d^2*c*a + 9/5*x^5*f*e^2*d*c^2*
a + 3/5*x^5*f^2*e*c^3*a + 1/3*x^3*e^3*c^3*b + x^3*e^3*d*c^2*a + x^3*f*e^2*c
^3*a + x*e^3*c^3*a
```

Sympy [A] time = 0.113477, size = 423, normalized size = 1.36

$$ac^3e^3x + \frac{bd^3f^3x^{15}}{15} + x^{13}\left(\frac{ad^3f^3}{13} + \frac{3bcd^2f^3}{13} + \frac{3bd^3ef^2}{13}\right) + x^{11}\left(\frac{3acd^2f^3}{11} + \frac{3ad^3ef^2}{11} + \frac{3bc^2df^3}{11} + \frac{9bcd^2ef^2}{11} + \frac{3bd^3e^2}{11}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+c)**3*(f*x**2+e)**3,x)
```

```
[Out] a*c**3*e**3*x + b*d**3*f**3*x**15/15 + x**13*(a*d**3*f**3/13 + 3*b*c*d**2*f
**3/13 + 3*b*d**3*e*f**2/13) + x**11*(3*a*c*d**2*f**3/11 + 3*a*d**3*e*f**2/
11 + 3*b*c**2*d*f**3/11 + 9*b*c*d**2*e*f**2/11 + 3*b*d**3*e**2*f/11) + x**9
*(a*c**2*d*f**3/3 + a*c*d**2*e*f**2 + a*d**3*e**2*f/3 + b*c**3*f**3/9 + b*c
**2*d*e*f**2 + b*c*d**2*e**2*f + b*d**3*e**3/9) + x**7*(a*c**3*f**3/7 + 9*a
*c**2*d*e*f**2/7 + 9*a*c*d**2*e**2*f/7 + a*d**3*e**3/7 + 3*b*c**3*e*f**2/7
+ 9*b*c**2*d*e**2*f/7 + 3*b*c*d**2*e**3/7) + x**5*(3*a*c**3*e*f**2/5 + 9*a*
c**2*d*e**2*f/5 + 3*a*c*d**2*e**3/5 + 3*b*c**3*e**2*f/5 + 3*b*c**2*d*e**3/5
) + x**3*(a*c**3*e**2*f + a*c**2*d*e**3 + b*c**3*e**3/3)
```

Giac [A] time = 1.14176, size = 541, normalized size = 1.75

$$\frac{1}{15}bd^3f^3x^{15} + \frac{3}{13}bcd^2f^3x^{13} + \frac{1}{13}ad^3f^3x^{13} + \frac{3}{13}bd^3f^2x^{13}e + \frac{3}{11}bc^2df^3x^{11} + \frac{3}{11}acd^2f^3x^{11} + \frac{9}{11}bcd^2f^2x^{11}e + \frac{3}{11}ad^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^3*(f*x^2+e)^3,x, algorithm="giac")
```

```
[Out] 1/15*b*d^3*f^3*x^15 + 3/13*b*c*d^2*f^3*x^13 + 1/13*a*d^3*f^3*x^13 + 3/13*b*
d^3*f^2*x^13*e + 3/11*b*c^2*d*f^3*x^11 + 3/11*a*c*d^2*f^3*x^11 + 9/11*b*c*d
^2*f^2*x^11*e + 3/11*a*d^3*f^2*x^11*e + 3/11*b*d^3*f*x^11*e^2 + 1/9*b*c^3*f
^3*x^9 + 1/3*a*c^2*d*f^3*x^9 + b*c^2*d*f^2*x^9*e + a*c*d^2*f^2*x^9*e + b*c*
d^2*f*x^9*e^2 + 1/3*a*d^3*f*x^9*e^2 + 1/7*a*c^3*f^3*x^7 + 1/9*b*d^3*x^9*e^3
 + 3/7*b*c^3*f^2*x^7*e + 9/7*a*c^2*d*f^2*x^7*e + 9/7*b*c^2*d*f*x^7*e^2 + 9/
7*a*c*d^2*f*x^7*e^2 + 3/7*b*c*d^2*x^7*e^3 + 1/7*a*d^3*x^7*e^3 + 3/5*a*c^3*f
^2*x^5*e + 3/5*b*c^3*f*x^5*e^2 + 9/5*a*c^2*d*f*x^5*e^2 + 3/5*b*c^2*d*x^5*e^
3 + 3/5*a*c*d^2*x^5*e^3 + a*c^3*f*x^3*e^2 + 1/3*b*c^3*x^3*e^3 + a*c^2*d*x^3
*e^3 + a*c^3*x*e^3
```

3.17 $\int \left(a+bx^2\right)\left(c+dx^2\right)^3\left(e+fx^2\right)^2\,dx$

Optimal. Leaf size=226

$$\frac{1}{9}dx^9\left(adf(3cf+2de)+b\left(3c^2f^2+6cdef+d^2e^2\right)\right)+\frac{1}{7}x^7\left(ad\left(3c^2f^2+6cdef+d^2e^2\right)+bc\left(c^2f^2+6cdef+3d^2e^2\right)\right)+\frac{1}{5}c$$

```
[Out] a*c^3*e^2*x + (c^2*e*(b*c*e + 3*a*d*e + 2*a*c*f)*x^3)/3 + (c*(b*c*e*(3*d*e
+ 2*c*f) + a*(3*d^2*e^2 + 6*c*d*e*f + c^2*f^2))*x^5)/5 + ((b*c*(3*d^2*e^2 +
 6*c*d*e*f + c^2*f^2) + a*d*(d^2*e^2 + 6*c*d*e*f + 3*c^2*f^2))*x^7)/7 + (d*
(a*d*f*(2*d*e + 3*c*f) + b*(d^2*e^2 + 6*c*d*e*f + 3*c^2*f^2))*x^9)/9 + (d^2
*f*(2*b*d*e + 3*b*c*f + a*d*f)*x^11)/11 + (b*d^3*f^2*x^13)/13
```

Rubi [A] time = 0.216233, antiderivative size = 226, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {521}

$$\frac{1}{9}dx^9\left(adf(3cf+2de)+b\left(3c^2f^2+6cdef+d^2e^2\right)\right)+\frac{1}{7}x^7\left(ad\left(3c^2f^2+6cdef+d^2e^2\right)+bc\left(c^2f^2+6cdef+3d^2e^2\right)\right)+\frac{1}{5}c$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)*(c + d*x^2)^3*(e + f*x^2)^2,x]
```

```
[Out] a*c^3*e^2*x + (c^2*e*(b*c*e + 3*a*d*e + 2*a*c*f)*x^3)/3 + (c*(b*c*e*(3*d*e
+ 2*c*f) + a*(3*d^2*e^2 + 6*c*d*e*f + c^2*f^2))*x^5)/5 + ((b*c*(3*d^2*e^2 +
 6*c*d*e*f + c^2*f^2) + a*d*(d^2*e^2 + 6*c*d*e*f + 3*c^2*f^2))*x^7)/7 + (d*
(a*d*f*(2*d*e + 3*c*f) + b*(d^2*e^2 + 6*c*d*e*f + 3*c^2*f^2))*x^9)/9 + (d^2
*f*(2*b*d*e + 3*b*c*f + a*d*f)*x^11)/11 + (b*d^3*f^2*x^13)/13
```

Rule 521

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c +
d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[p
, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rubi steps

$$\int \left(a+bx^2\right)\left(c+dx^2\right)^3\left(e+fx^2\right)^2\,dx=\int \left(ac^3e^2+c^2e(bce+3ade+2acf)x^2+c\left(bce(3de+2cf)+a\left(3d^2e^2+6cdef+c^2\right.\right.\right.$$
$$=ac^3e^2x+\frac{1}{3}c^2e(bce+3ade+2acf)x^3+\frac{1}{5}c\left(bce(3de+2cf)+a\left(3d^2e^2+6cdef+c\right.\right.$$

Mathematica [A] time = 0.0829819, size = 226, normalized size = 1.

$$\frac{1}{9}dx^9\left(adf(3cf+2de)+b\left(3c^2f^2+6cdef+d^2e^2\right)\right)+\frac{1}{7}x^7\left(ad\left(3c^2f^2+6cdef+d^2e^2\right)+bc\left(c^2f^2+6cdef+3d^2e^2\right)\right)+\frac{1}{5}c$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)*(c + d*x^2)^3*(e + f*x^2)^2,x]
```

```
[Out] a*c^3*e^2*x + (c^2*e*(b*c*e + 3*a*d*e + 2*a*c*f)*x^3)/3 + (c*(b*c*e*(3*d*e
+ 2*c*f) + a*(3*d^2*e^2 + 6*c*d*e*f + c^2*f^2))*x^5)/5 + ((b*c*(3*d^2*e^2 +
```

```
 6*c*d*e*f + c^2*f^2) + a*d*(d^2*e^2 + 6*c*d*e*f + 3*c^2*f^2))*x^7)/7 + (d*
(a*d*f*(2*d*e + 3*c*f) + b*(d^2*e^2 + 6*c*d*e*f + 3*c^2*f^2))*x^9)/9 + (d^2
*f*(2*b*d*e + 3*b*c*f + a*d*f)*x^11)/11 + (b*d^3*f^2*x^13)/13
```

Maple [A] time = 0., size = 244, normalized size = 1.1

$$\frac{bd^3f^2x^{13}}{13} + \frac{\left(\left(ad^3+3\,bcd^2\right)f^2+2\,bd^3ef\right)x^{11}}{11} + \frac{\left(\left(3\,acd^2+3\,bc^2d\right)f^2+2\,\left(ad^3+3\,bcd^2\right)ef+bd^3e^2\right)x^9}{9} + \frac{\left(\left(3\,ac^2d+\cdots\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x^2+c)^3*(f*x^2+e)^2,x)
```

```
[Out] 1/13*b*d^3*f^2*x^13+1/11*((a*d^3+3*b*c*d^2)*f^2+2*b*d^3*e*f)*x^11+1/9*((3*a
*c*d^2+3*b*c^2*d)*f^2+2*(a*d^3+3*b*c*d^2)*e*f+b*d^3*e^2)*x^9+1/7*((3*a*c^2*
d+b*c^3)*f^2+2*(3*a*c*d^2+3*b*c^2*d)*e*f+(a*d^3+3*b*c*d^2)*e^2)*x^7+1/5*(a*
c^3*f^2+2*(3*a*c^2*d+b*c^3)*e*f+(3*a*c*d^2+3*b*c^2*d)*e^2)*x^5+1/3*(2*a*c^3
*e*f+(3*a*c^2*d+b*c^3)*e^2)*x^3+a*c^3*e^2*x
```

Maxima [A] time = 1.01063, size = 323, normalized size = 1.43

$$\frac{1}{13}\,bd^3f^2x^{13} + \frac{1}{11}\left(2\,bd^3ef + \left(3\,bcd^2 + ad^3\right)f^2\right)x^{11} + \frac{1}{9}\left(bd^3e^2 + 2\left(3\,bcd^2 + ad^3\right)ef + 3\left(bc^2d + acd^2\right)f^2\right)x^9 + \frac{1}{7}\left(\left(3\,bc\cdots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^3*(f*x^2+e)^2,x, algorithm="maxima")
```

```
[Out] 1/13*b*d^3*f^2*x^13 + 1/11*(2*b*d^3*e*f + (3*b*c*d^2 + a*d^3)*f^2)*x^11 + 1
/9*(b*d^3*e^2 + 2*(3*b*c*d^2 + a*d^3)*e*f + 3*(b*c^2*d + a*c*d^2)*f^2)*x^9
+ 1/7*((3*b*c*d^2 + a*d^3)*e^2 + 6*(b*c^2*d + a*c*d^2)*e*f + (b*c^3 + 3*a*c
^2*d)*f^2)*x^7 + a*c^3*e^2*x + 1/5*(a*c^3*f^2 + 3*(b*c^2*d + a*c*d^2)*e^2 +
 2*(b*c^3 + 3*a*c^2*d)*e*f)*x^5 + 1/3*(2*a*c^3*e*f + (b*c^3 + 3*a*c^2*d)*e^
2)*x^3
```

Fricas [A] time = 1.24677, size = 676, normalized size = 2.99

$$\frac{1}{13}x^{13}f^2d^3b + \frac{2}{11}x^{11}fed^3b + \frac{3}{11}x^{11}f^2d^2cb + \frac{1}{11}x^{11}f^2d^3a + \frac{1}{9}x^9e^2d^3b + \frac{2}{3}x^9fed^2cb + \frac{1}{3}x^9f^2dc^2b + \frac{2}{9}x^9fed^3a + \frac{1}{3}x^9f^2\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^3*(f*x^2+e)^2,x, algorithm="fricas")
```

```
[Out] 1/13*x^13*f^2*d^3*b + 2/11*x^11*f*e*d^3*b + 3/11*x^11*f^2*d^2*c*b + 1/11*x^
11*f^2*d^3*a + 1/9*x^9*e^2*d^3*b + 2/3*x^9*f*e*d^2*c*b + 1/3*x^9*f^2*d*c^2*
b + 2/9*x^9*f*e*d^3*a + 1/3*x^9*f^2*d^2*c*a + 3/7*x^7*e^2*d^2*c*b + 6/7*x^7
*f*e*d*c^2*b + 1/7*x^7*f^2*c^3*b + 1/7*x^7*e^2*d^3*a + 6/7*x^7*f*e*d^2*c*a
+ 3/7*x^7*f^2*d*c^2*a + 3/5*x^5*e^2*d*c^2*b + 2/5*x^5*f*e*c^3*b + 3/5*x^5*e
^2*d^2*c*a + 6/5*x^5*f*e*d*c^2*a + 1/5*x^5*f^2*c^3*a + 1/3*x^3*e^2*c^3*b +
x^3*e^2*d*c^2*a + 2/3*x^3*f*e*c^3*a + x*e^2*c^3*a
```

Sympy [A] time = 0.098346, size = 304, normalized size = 1.35

$$ac^3e^2x + \frac{bd^3f^2x^{13}}{13} + x^{11}\left(\frac{ad^3f^2}{11} + \frac{3bcd^2f^2}{11} + \frac{2bd^3ef}{11}\right) + x^9\left(\frac{acd^2f^2}{3} + \frac{2ad^3ef}{9} + \frac{bc^2df^2}{3} + \frac{2bcd^2ef}{3} + \frac{bd^3e^2}{9}\right) + x^7\left(\frac{3ac}{\ }\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+c)**3*(f*x**2+e)**2,x)
```

```
[Out] a*c**3*e**2*x + b*d**3*f**2*x**13/13 + x**11*(a*d**3*f**2/11 + 3*b*c*d**2*f
**2/11 + 2*b*d**3*e*f/11) + x**9*(a*c*d**2*f**2/3 + 2*a*d**3*e*f/9 + b*c**2
*d*f**2/3 + 2*b*c*d**2*e*f/3 + b*d**3*e**2/9) + x**7*(3*a*c**2*d*f**2/7 + 6
*a*c*d**2*e*f/7 + a*d**3*e**2/7 + b*c**3*f**2/7 + 6*b*c**2*d*e*f/7 + 3*b*c*
d**2*e**2/7) + x**5*(a*c**3*f**2/5 + 6*a*c**2*d*e*f/5 + 3*a*c*d**2*e**2/5 +
 2*b*c**3*e*f/5 + 3*b*c**2*d*e**2/5) + x**3*(2*a*c**3*e*f/3 + a*c**2*d*e**2
 + b*c**3*e**2/3)
```

Giac [A] time = 1.15961, size = 390, normalized size = 1.73

$$\frac{1}{13}bd^3f^2x^{13} + \frac{3}{11}bcd^2f^2x^{11} + \frac{1}{11}ad^3f^2x^{11} + \frac{2}{11}bd^3fx^{11}e + \frac{1}{3}bc^2df^2x^9 + \frac{1}{3}acd^2f^2x^9 + \frac{2}{3}bcd^2fx^9e + \frac{2}{9}ad^3fx^9e + \frac{1}{9}bd$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^3*(f*x^2+e)^2,x, algorithm="giac")
```

```
[Out] 1/13*b*d^3*f^2*x^13 + 3/11*b*c*d^2*f^2*x^11 + 1/11*a*d^3*f^2*x^11 + 2/11*b*
d^3*f*x^11*e + 1/3*b*c^2*d*f^2*x^9 + 1/3*a*c*d^2*f^2*x^9 + 2/3*b*c*d^2*f*x^
9*e + 2/9*a*d^3*f*x^9*e + 1/9*b*d^3*x^9*e^2 + 1/7*b*c^3*f^2*x^7 + 3/7*a*c^2
*d*f^2*x^7 + 6/7*b*c^2*d*f*x^7*e + 6/7*a*c*d^2*f*x^7*e + 3/7*b*c*d^2*x^7*e^
2 + 1/7*a*d^3*x^7*e^2 + 1/5*a*c^3*f^2*x^5 + 2/5*b*c^3*f*x^5*e + 6/5*a*c^2*d
*f*x^5*e + 3/5*b*c^2*d*x^5*e^2 + 3/5*a*c*d^2*x^5*e^2 + 2/3*a*c^3*f*x^3*e +
1/3*b*c^3*x^3*e^2 + a*c^2*d*x^3*e^2 + a*c^3*x*e^2
```

3.18 $\int \left(a+bx^2\right)\left(c+dx^2\right)^3\left(e+fx^2\right)\,dx$

Optimal. Leaf size=130

$$\frac{1}{3}c^2x^3(acf+3ade+bce)+\frac{1}{9}d^2x^9(adf+3bcf+bde)+\frac{1}{7}dx^7(ad(3cf+de)+3bc(cf+de))+\frac{1}{5}cx^5(3ad(cf+de)+bc($$

```
[Out] a*c^3*e*x + (c^2*(b*c*e + 3*a*d*e + a*c*f)*x^3)/3 + (c*(3*a*d*(d*e + c*f) +
 b*c*(3*d*e + c*f))*x^5)/5 + (d*(3*b*c*(d*e + c*f) + a*d*(d*e + 3*c*f))*x^7
)/7 + (d^2*(b*d*e + 3*b*c*f + a*d*f)*x^9)/9 + (b*d^3*f*x^11)/11
```

Rubi [A] time = 0.13479, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {521}

$$\frac{1}{3}c^2x^3(acf+3ade+bce)+\frac{1}{9}d^2x^9(adf+3bcf+bde)+\frac{1}{7}dx^7(ad(3cf+de)+3bc(cf+de))+\frac{1}{5}cx^5(3ad(cf+de)+bc($$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)*(c + d*x^2)^3*(e + f*x^2),x]
```

```
[Out] a*c^3*e*x + (c^2*(b*c*e + 3*a*d*e + a*c*f)*x^3)/3 + (c*(3*a*d*(d*e + c*f) +
 b*c*(3*d*e + c*f))*x^5)/5 + (d*(3*b*c*(d*e + c*f) + a*d*(d*e + 3*c*f))*x^7
)/7 + (d^2*(b*d*e + 3*b*c*f + a*d*f)*x^9)/9 + (b*d^3*f*x^11)/11
```

Rule 521

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c +
d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[p
, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^2\right)\left(c+dx^2\right)^3\left(e+fx^2\right)\,dx &= \int \left(ac^3e+c^2(bce+3ade+acf)x^2+c(3ad(de+cf)+bc(3de+cf))x^4+d(3bc(\right.\\ &= ac^3ex+\frac{1}{3}c^2(bce+3ade+acf)x^3+\frac{1}{5}c(3ad(de+cf)+bc(3de+cf))x^5+\frac{1}{7}d(3\end{aligned}$$

Mathematica [A] time = 0.0497375, size = 130, normalized size = 1.

$$\frac{1}{3}c^2x^3(acf+3ade+bce)+\frac{1}{9}d^2x^9(adf+3bcf+bde)+\frac{1}{7}dx^7(ad(3cf+de)+3bc(cf+de))+\frac{1}{5}cx^5(3ad(cf+de)+bc($$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)*(c + d*x^2)^3*(e + f*x^2),x]
```

```
[Out] a*c^3*e*x + (c^2*(b*c*e + 3*a*d*e + a*c*f)*x^3)/3 + (c*(3*a*d*(d*e + c*f) +
 b*c*(3*d*e + c*f))*x^5)/5 + (d*(3*b*c*(d*e + c*f) + a*d*(d*e + 3*c*f))*x^7
)/7 + (d^2*(b*d*e + 3*b*c*f + a*d*f)*x^9)/9 + (b*d^3*f*x^11)/11
```

Maple [A] time = 0.002, size = 149, normalized size = 1.2

$$\frac{bd^3fx^{11}}{11}+\frac{\left(\left(ad^3+3\,bcd^2\right)f+bd^3e\right)x^9}{9}+\frac{\left(\left(3\,acd^2+3\,bc^2d\right)f+\left(ad^3+3\,bcd^2\right)e\right)x^7}{7}+\frac{\left(\left(3\,ac^2d+bc^3\right)f+\left(3\,acd^2+3\,b\right.\right.}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x^2+c)^3*(f*x^2+e),x)
```

```
[Out] 1/11*b*d^3*f*x^11+1/9*((a*d^3+3*b*c*d^2)*f+b*d^3*e)*x^9+1/7*((3*a*c*d^2+3*b
*c^2*d)*f+(a*d^3+3*b*c*d^2)*e)*x^7+1/5*((3*a*c^2*d+b*c^3)*f+(3*a*c*d^2+3*b*
c^2*d)*e)*x^5+1/3*(a*c^3*f+(3*a*c^2*d+b*c^3)*e)*x^3+a*c^3*e*x
```

Maxima [A] time = 0.995879, size = 197, normalized size = 1.52

$$\frac{1}{11}\,bd^3fx^{11}+\frac{1}{9}\left(bd^3e+\left(3\,bcd^2+ad^3\right)f\right)x^9+\frac{1}{7}\left(\left(3\,bcd^2+ad^3\right)e+3\left(bc^2d+acd^2\right)f\right)x^7+ac^3ex+\frac{1}{5}\left(3\left(bc^2d+acd^2\right)e+\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^3*(f*x^2+e),x, algorithm="maxima")
```

```
[Out] 1/11*b*d^3*f*x^11 + 1/9*(b*d^3*e + (3*b*c*d^2 + a*d^3)*f)*x^9 + 1/7*((3*b*c
*d^2 + a*d^3)*e + 3*(b*c^2*d + a*c*d^2)*f)*x^7 + a*c^3*e*x + 1/5*(3*(b*c^2*
d + a*c*d^2)*e + (b*c^3 + 3*a*c^2*d)*f)*x^5 + 1/3*(a*c^3*f + (b*c^3 + 3*a*c
^2*d)*e)*x^3
```

Fricas [A] time = 1.23814, size = 398, normalized size = 3.06

$$\frac{1}{11}x^{11}fd^3b+\frac{1}{9}x^9ed^3b+\frac{1}{3}x^9fd^2cb+\frac{1}{9}x^9fd^3a+\frac{3}{7}x^7ed^2cb+\frac{3}{7}x^7fdc^2b+\frac{1}{7}x^7ed^3a+\frac{3}{7}x^7fd^2ca+\frac{3}{5}x^5edc^2b+\frac{1}{5}x^5fc^3b+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^3*(f*x^2+e),x, algorithm="fricas")
```

```
[Out] 1/11*x^11*f*d^3*b + 1/9*x^9*e*d^3*b + 1/3*x^9*f*d^2*c*b + 1/9*x^9*f*d^3*a +
 3/7*x^7*e*d^2*c*b + 3/7*x^7*f*d*c^2*b + 1/7*x^7*e*d^3*a + 3/7*x^7*f*d^2*c*
a + 3/5*x^5*e*d*c^2*b + 1/5*x^5*f*c^3*b + 3/5*x^5*e*d^2*c*a + 3/5*x^5*f*d*c
^2*a + 1/3*x^3*e*c^3*b + x^3*e*d*c^2*a + 1/3*x^3*f*c^3*a + x*e*c^3*a
```

Sympy [A] time = 0.085075, size = 173, normalized size = 1.33

$$ac^3ex+\frac{bd^3fx^{11}}{11}+x^9\left(\frac{ad^3f}{9}+\frac{bcd^2f}{3}+\frac{bd^3e}{9}\right)+x^7\left(\frac{3acd^2f}{7}+\frac{ad^3e}{7}+\frac{3bc^2df}{7}+\frac{3bcd^2e}{7}\right)+x^5\left(\frac{3ac^2df}{5}+\frac{3acd^2e}{5}+\frac{bc^3}{5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+c)**3*(f*x**2+e),x)
```

```
[Out] a*c**3*e*x + b*d**3*f*x**11/11 + x**9*(a*d**3*f/9 + b*c*d**2*f/3 + b*d**3*e
/9) + x**7*(3*a*c*d**2*f/7 + a*d**3*e/7 + 3*b*c**2*d*f/7 + 3*b*c*d**2*e/7)
```

```
+ x**5*(3*a*c**2*d*f/5 + 3*a*c*d**2*e/5 + b*c**3*f/5 + 3*b*c**2*d*e/5) + x*
*3*(a*c**3*f/3 + a*c**2*d*e + b*c**3*e/3)
```

Giac [A] time = 1.13893, size = 234, normalized size = 1.8

$$\frac{1}{11} b d^3 f x^{11} + \frac{1}{3} b c d^2 f x^9 + \frac{1}{9} a d^3 f x^9 + \frac{1}{9} b d^3 x^9 e + \frac{3}{7} b c^2 d f x^7 + \frac{3}{7} a c d^2 f x^7 + \frac{3}{7} b c d^2 x^7 e + \frac{1}{7} a d^3 x^7 e + \frac{1}{5} b c^3 f x^5 + \frac{3}{5} a c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^3*(f*x^2+e),x, algorithm="giac")
```

```
[Out] 1/11*b*d^3*f*x^11 + 1/3*b*c*d^2*f*x^9 + 1/9*a*d^3*f*x^9 + 1/9*b*d^3*x^9*e +
 3/7*b*c^2*d*f*x^7 + 3/7*a*c*d^2*f*x^7 + 3/7*b*c*d^2*x^7*e + 1/7*a*d^3*x^7*
e + 1/5*b*c^3*f*x^5 + 3/5*a*c^2*d*f*x^5 + 3/5*b*c^2*d*x^5*e + 3/5*a*c*d^2*x
^5*e + 1/3*a*c^3*f*x^3 + 1/3*b*c^3*x^3*e + a*c^2*d*x^3*e + a*c^3*x*e
```

3.19 $\int \frac{(a+bx^2)(c+dx^2)^3}{e+fx^2} \, dx$

Optimal. Leaf size=227

$$-\frac{x\left(c+dx^2\right)\left(7adf(5de-9cf)-b\left(24c^2f^2-63cdef+35d^2e^2\right)\right)}{105f^3}+\frac{x\left(7adf\left(33c^2f^2-40cdef+15d^2e^2\right)-b\left(231c^2def^2-\cdots\right.\right.}{105f^4}$$

```
[Out] ((7*a*d*f*(15*d^2*e^2 - 40*c*d*e*f + 33*c^2*f^2) - b*(105*d^3*e^3 - 280*c*d
^2*e^2*f + 231*c^2*d*e*f^2 - 48*c^3*f^3))*x)/(105*f^4) - ((7*a*d*f*(5*d*e -
 9*c*f) - b*(35*d^2*e^2 - 63*c*d*e*f + 24*c^2*f^2))*x*(c + d*x^2))/(105*f^3
) - ((7*b*d*e - 6*b*c*f - 7*a*d*f)*x*(c + d*x^2)^2)/(35*f^2) + (b*x*(c + d*
x^2)^3)/(7*f) + ((b*e - a*f)*(d*e - c*f)^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(Sq
rt[e]*f^(9/2))
```

Rubi [A] time = 0.371285, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {528, 388, 205}

$$-\frac{x\left(c+dx^2\right)\left(7adf(5de-9cf)-b\left(24c^2f^2-63cdef+35d^2e^2\right)\right)}{105f^3}+\frac{x\left(7adf\left(33c^2f^2-40cdef+15d^2e^2\right)-b\left(231c^2def^2-\cdots\right.\right.}{105f^4}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(c + d*x^2)^3)/(e + f*x^2),x]
```

```
[Out] ((7*a*d*f*(15*d^2*e^2 - 40*c*d*e*f + 33*c^2*f^2) - b*(105*d^3*e^3 - 280*c*d
^2*e^2*f + 231*c^2*d*e*f^2 - 48*c^3*f^3))*x)/(105*f^4) - ((7*a*d*f*(5*d*e -
 9*c*f) - b*(35*d^2*e^2 - 63*c*d*e*f + 24*c^2*f^2))*x*(c + d*x^2))/(105*f^3
) - ((7*b*d*e - 6*b*c*f - 7*a*d*f)*x*(c + d*x^2)^2)/(35*f^2) + (b*x*(c + d*
x^2)^3)/(7*f) + ((b*e - a*f)*(d*e - c*f)^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(Sq
rt[e]*f^(9/2))
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(a+bx^2)(c+dx^2)^3}{e+fx^2}\,dx = \frac{bx(c+dx^2)^3}{7f} + \frac{\int \frac{(c+dx^2)^2(-c(be-7af)+(-7bde+6bcf+7adf)x^2)}{e+fx^2}\,dx}{7f}$$

$$= -\frac{(7bde-6bcf-7adf)x(c+dx^2)^2}{35f^2} + \frac{bx(c+dx^2)^3}{7f} + \frac{\int \frac{(c+dx^2)(c(be(7de-11cf)-7af(de-5cf))+(}{}}{}$$

$$= -\frac{\left(7adf(5de-9cf)-b\left(35d^2e^2-63cdef+24c^2f^2\right)\right)x(c+dx^2)}{105f^3} - \frac{(7bde-6bcf-7adf}{35f^2}$$

$$= \frac{\left(7adf\left(15d^2e^2-40cdef+33c^2f^2\right)-b\left(105d^3e^3-280cd^2e^2f+231c^2def^2-48c^3f^3\right)\right)x}{105f^4}$$

$$= \frac{\left(7adf\left(15d^2e^2-40cdef+33c^2f^2\right)-b\left(105d^3e^3-280cd^2e^2f+231c^2def^2-48c^3f^3\right)\right)x}{105f^4}$$

Mathematica [A] time = 0.0932308, size = 179, normalized size = 0.79

$$\frac{dx^3\left(adf(3cf-de)+b\left(3c^2f^2-3cdef+d^2e^2\right)\right)}{3f^3} + \frac{x\left(adf\left(3c^2f^2-3cdef+d^2e^2\right)-b(de-cf)^3\right)}{f^4} + \frac{d^2x^5(adf+3bcf-}{5f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(c + d*x^2)^3)/(e + f*x^2),x]
```

```
[Out] ((-(b*(d*e - c*f)^3) + a*d*f*(d^2*e^2 - 3*c*d*e*f + 3*c^2*f^2))*x)/f^4 + (d
*(a*d*f*(-(d*e) + 3*c*f) + b*(d^2*e^2 - 3*c*d*e*f + 3*c^2*f^2))*x^3)/(3*f^3
) + (d^2*(-(b*d*e) + 3*b*c*f + a*d*f)*x^5)/(5*f^2) + (b*d^3*x^7)/(7*f) + ((
b*e - a*f)*(d*e - c*f)^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e]*f^(9/2))
```

Maple [A] time = 0.005, size = 401, normalized size = 1.8

$$\frac{bd^3x^7}{7f} + \frac{x^5ad^3}{5f} + \frac{3x^5bcd^2}{5f} - \frac{x^5bd^3e}{5f^2} + \frac{x^3acd^2}{f} - \frac{x^3ad^3e}{3f^2} + \frac{x^3bc^2d}{f} - \frac{x^3bcd^2e}{f^2} + \frac{x^3bd^3e^2}{3f^3} + 3\frac{ac^2dx}{f} - 3\frac{acd^2ex}{f^2} + \frac{ad^3e}{f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x^2+c)^3/(f*x^2+e),x)
```

```
[Out] 1/7/f*b*d^3*x^7+1/5/f*x^5*a*d^3+3/5/f*x^5*b*c*d^2-1/5/f^2*x^5*b*d^3*e+1/f*x
^3*a*c*d^2-1/3/f^2*x^3*a*d^3*e+1/f*x^3*b*c^2*d-1/f^2*x^3*b*c*d^2*e+1/3/f^3*
x^3*b*d^3*e^2+3/f*a*c^2*d*x-3/f^2*a*c*d^2*e*x+1/f^3*a*d^3*e^2*x+1/f*b*c^3*x
-3/f^2*b*c^2*d*e*x+3/f^3*b*c*d^2*e^2*x-1/f^4*b*d^3*e^3*x+1/(e*f)^(1/2)*arct
an(x*f/(e*f)^(1/2))*a*c^3-3/f/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*a*c^2*d*e
+3/f^2/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*a*c*d^2*e^2-1/f^3/(e*f)^(1/2)*ar
ctan(x*f/(e*f)^(1/2))*a*d^3*e^3-1/f/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*b*c
^3*e+3/f^2/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*b*c^2*d*e^2-3/f^3/(e*f)^(1/2
)*arctan(x*f/(e*f)^(1/2))*b*c*d^2*e^3+1/f^4/(e*f)^(1/2)*arctan(x*f/(e*f)^(1
/2))*b*d^3*e^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^3/(f*x^2+e),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.55086, size = 1216, normalized size = 5.36

$$\left[\frac{30\,bd^3ef^4x^7 - 42\left(bd^3e^2f^3 - \left(3\,bcd^2 + ad^3\right)ef^4\right)x^5 + 70\left(bd^3e^3f^2 - \left(3\,bcd^2 + ad^3\right)e^2f^3 + 3\left(bc^2d + acd^2\right)ef^4\right)x^3 - 105\left(b\cdots\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^3/(f*x^2+e),x, algorithm="fricas")
```

```
[Out] [1/210*(30*b*d^3*e*f^4*x^7 - 42*(b*d^3*e^2*f^3 - (3*b*c*d^2 + a*d^3)*e*f^4)
*x^5 + 70*(b*d^3*e^3*f^2 - (3*b*c*d^2 + a*d^3)*e^2*f^3 + 3*(b*c^2*d + a*c*d
^2)*e*f^4)*x^3 - 105*(b*d^3*e^4 + a*c^3*f^4 - (3*b*c*d^2 + a*d^3)*e^3*f + 3
*(b*c^2*d + a*c*d^2)*e^2*f^2 - (b*c^3 + 3*a*c^2*d)*e*f^3)*sqrt(-e*f)*log((f
*x^2 - 2*sqrt(-e*f)*x - e)/(f*x^2 + e)) - 210*(b*d^3*e^4*f - (3*b*c*d^2 + a
*d^3)*e^3*f^2 + 3*(b*c^2*d + a*c*d^2)*e^2*f^3 - (b*c^3 + 3*a*c^2*d)*e*f^4)*
x)/(e*f^5), 1/105*(15*b*d^3*e*f^4*x^7 - 21*(b*d^3*e^2*f^3 - (3*b*c*d^2 + a*
d^3)*e*f^4)*x^5 + 35*(b*d^3*e^3*f^2 - (3*b*c*d^2 + a*d^3)*e^2*f^3 + 3*(b*c^
2*d + a*c*d^2)*e*f^4)*x^3 + 105*(b*d^3*e^4 + a*c^3*f^4 - (3*b*c*d^2 + a*d^3
)*e^3*f + 3*(b*c^2*d + a*c*d^2)*e^2*f^2 - (b*c^3 + 3*a*c^2*d)*e*f^3)*sqrt(e
*f)*arctan(sqrt(e*f)*x/e) - 105*(b*d^3*e^4*f - (3*b*c*d^2 + a*d^3)*e^3*f^2
+ 3*(b*c^2*d + a*c*d^2)*e^2*f^3 - (b*c^3 + 3*a*c^2*d)*e*f^4)*x)/(e*f^5)]
```

Sympy [B] time = 2.00094, size = 508, normalized size = 2.24

$$\frac{bd^3x^7}{7f} - \frac{\sqrt{-\frac{1}{ef^9}}\left(af - be\right)\left(cf - de\right)^3\log\left(-\frac{ef^4\sqrt{-\frac{1}{ef^9}}\left(af-be\right)\left(cf-de\right)^3}{ac^3f^4-3ac^2def^3+3acd^2e^2f^2-ad^3e^3f-bc^3ef^3+3bc^2de^2f^2-3bcd^2e^3f+bd^3e^4} + x\right)}{2} + \frac{\sqrt{-\frac{1}{ef^9}}\left(af - \cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+c)**3/(f*x**2+e),x)
```

```
[Out] b*d**3*x**7/(7*f) - sqrt(-1/(e*f**9))*(a*f - b*e)*(c*f - d*e)**3*log(-e*f**
4*sqrt(-1/(e*f**9))*(a*f - b*e)*(c*f - d*e)**3/(a*c**3*f**4 - 3*a*c**2*d*e*
f**3 + 3*a*c*d**2*e**2*f**2 - a*d**3*e**3*f - b*c**3*e*f**3 + 3*b*c**2*d*e*
*2*f**2 - 3*b*c*d**2*e**3*f + b*d**3*e**4) + x)/2 + sqrt(-1/(e*f**9))*(a*f
- b*e)*(c*f - d*e)**3*log(e*f**4*sqrt(-1/(e*f**9))*(a*f - b*e)*(c*f - d*e)*
*3/(a*c**3*f**4 - 3*a*c**2*d*e*f**3 + 3*a*c*d**2*e**2*f**2 - a*d**3*e**3*f
- b*c**3*e*f**3 + 3*b*c**2*d*e**2*f**2 - 3*b*c*d**2*e**3*f + b*d**3*e**4) +
 x)/2 + x**5*(a*d**3*f + 3*b*c*d**2*f - b*d**3*e)/(5*f**2) + x**3*(3*a*c*d*
*2*f**2 - a*d**3*e*f + 3*b*c**2*d*f**2 - 3*b*c*d**2*e*f + b*d**3*e**2)/(3*f
```

```
**3) + x*(3*a*c**2*d*f**3 - 3*a*c*d**2*e*f**2 + a*d**3*e**2*f + b*c**3*f**3
 - 3*b*c**2*d*e*f**2 + 3*b*c*d**2*e**2*f - b*d**3*e**3)/f**4
```

Giac [A] time = 1.14847, size = 414, normalized size = 1.82

$$\frac{\left(ac^3f^4 - bc^3f^3e - 3\,ac^2df^3e + 3\,bc^2df^2e^2 + 3\,acd^2f^2e^2 - 3\,bcd^2fe^3 - ad^3fe^3 + bd^3e^4\right)\arctan\left(\sqrt{f}xe^{\left(-\frac{1}{2}\right)}\right)e^{\left(-\frac{1}{2}\right)}}{f^{\frac{9}{2}}} + \frac{15\,b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^3/(f*x^2+e),x, algorithm="giac")
```

```
[Out] (a*c^3*f^4 - b*c^3*f^3*e - 3*a*c^2*d*f^3*e + 3*b*c^2*d*f^2*e^2 + 3*a*c*d^2*
f^2*e^2 - 3*b*c*d^2*f*e^3 - a*d^3*f*e^3 + b*d^3*e^4)*arctan(sqrt(f)*x*e^(-1
/2))*e^(-1/2)/f^(9/2) + 1/105*(15*b*d^3*f^6*x^7 + 63*b*c*d^2*f^6*x^5 + 21*a
*d^3*f^6*x^5 - 21*b*d^3*f^5*x^5*e + 105*b*c^2*d*f^6*x^3 + 105*a*c*d^2*f^6*x
^3 - 105*b*c*d^2*f^5*x^3*e - 35*a*d^3*f^5*x^3*e + 35*b*d^3*f^4*x^3*e^2 + 10
5*b*c^3*f^6*x + 315*a*c^2*d*f^6*x - 315*b*c^2*d*f^5*x*e - 315*a*c*d^2*f^5*x
*e + 315*b*c*d^2*f^4*x*e^2 + 105*a*d^3*f^4*x*e^2 - 105*b*d^3*f^3*x*e^3)/f^7
```

3.20 $\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^2} dx$

Optimal. Leaf size=242

$$-\frac{dx\left(5af\left(3c^2f^2-22cdef+15d^2e^2\right)-be\left(81c^2f^2-190cdef+105d^2e^2\right)\right)}{30ef^4}-\frac{(de-cf)^2\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(be(7de-cf)-af(cf}{2e^{3/2}f^{9/2}}$$

```
[Out] -(d*(5*a*f*(15*d^2*e^2 - 22*c*d*e*f + 3*c^2*f^2) - b*e*(105*d^2*e^2 - 190*c
*d*e*f + 81*c^2*f^2))*x)/(30*e*f^4) - (d*(b*e*(35*d*e - 33*c*f) - 5*a*f*(5*
d*e - 3*c*f))*x*(c + d*x^2))/(30*e*f^3) + (d*(7*b*e - 5*a*f)*x*(c + d*x^2)^
2)/(10*e*f^2) - ((b*e - a*f)*x*(c + d*x^2)^3)/(2*e*f*(e + f*x^2)) - ((d*e -
 c*f)^2*(b*e*(7*d*e - c*f) - a*f*(5*d*e + c*f))*ArcTan[(Sqrt[f]*x)/Sqrt[e]]
)/(2*e^(3/2)*f^(9/2))
```

Rubi [A] time = 0.398615, antiderivative size = 242, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {526, 528, 388, 205}

$$-\frac{dx\left(5af\left(3c^2f^2-22cdef+15d^2e^2\right)-be\left(81c^2f^2-190cdef+105d^2e^2\right)\right)}{30ef^4}-\frac{(de-cf)^2\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(be(7de-cf)-af(cf}{2e^{3/2}f^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(c + d*x^2)^3)/(e + f*x^2)^2,x]
```


```
[Out] -(d*(5*a*f*(15*d^2*e^2 - 22*c*d*e*f + 3*c^2*f^2) - b*e*(105*d^2*e^2 - 190*c
*d*e*f + 81*c^2*f^2))*x)/(30*e*f^4) - (d*(b*e*(35*d*e - 33*c*f) - 5*a*f*(5*
d*e - 3*c*f))*x*(c + d*x^2))/(30*e*f^3) + (d*(7*b*e - 5*a*f)*x*(c + d*x^2)^
2)/(10*e*f^2) - ((b*e - a*f)*x*(c + d*x^2)^3)/(2*e*f*(e + f*x^2)) - ((d*e -
 c*f)^2*(b*e*(7*d*e - c*f) - a*f*(5*d*e + c*f))*ArcTan[(Sqrt[f]*x)/Sqrt[e]]
)/(2*e^(3/2)*f^(9/2))
```


Rule 526

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^q)/(a*b*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p +
 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)\left(c+dx^2\right)^3}{\left(e+fx^2\right)^2}\,dx = -\frac{(be-af)x\left(c+dx^2\right)^3}{2ef\left(e+fx^2\right)} - \frac{\int \frac{\left(c+dx^2\right)^2\left(-c(be+af)-d(7be-5af)x^2\right)}{e+fx^2}\,dx}{2ef}$$

$$= \frac{d(7be-5af)x\left(c+dx^2\right)^2}{10ef^2} - \frac{(be-af)x\left(c+dx^2\right)^3}{2ef\left(e+fx^2\right)} - \frac{\int \frac{\left(c+dx^2\right)\left(c(be(7de-5cf)-5af(de+cf))+d(be(3\ldots}{e+fx^2}}{10ef^2}$$

$$= -\frac{d(be(35de-33cf)-5af(5de-3cf))x\left(c+dx^2\right)}{30ef^3} + \frac{d(7be-5af)x\left(c+dx^2\right)^2}{10ef^2} - \frac{(be-a\ldots}{2ef\ldots}$$

$$= -\frac{d\left(5af\left(15d^2e^2-22cdef+3c^2f^2\right)-be\left(105d^2e^2-190cdef+81c^2f^2\right)\right)x}{30ef^4} - \frac{d(be(35de-\ldots}{\ldots}$$

$$= -\frac{d\left(5af\left(15d^2e^2-22cdef+3c^2f^2\right)-be\left(105d^2e^2-190cdef+81c^2f^2\right)\right)x}{30ef^4} - \frac{d(be(35de-\ldots}{\ldots}$$

Mathematica [A] time = 0.128194, size = 176, normalized size = 0.73

$$\frac{d^2x^3(adf+3bcf-2bde)}{3f^3} - \frac{(de-cf)^2\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(be(7de-cf)-af(cf+5de))}{2e^{3/2}f^{9/2}} + \frac{x(be-af)(de-cf)^3}{2ef^4\left(e+fx^2\right)} + \frac{dx\left(adf(3\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(c + d*x^2)^3)/(e + f*x^2)^2,x]
```

```
[Out] (d*(3*b*(d*e - c*f)^2 + a*d*f*(-2*d*e + 3*c*f))*x)/f^4 + (d^2*(-2*b*d*e + 3
*b*c*f + a*d*f)*x^3)/(3*f^3) + (b*d^3*x^5)/(5*f^2) + ((b*e - a*f)*(d*e - c*
f)^3*x)/(2*e*f^4*(e + f*x^2)) - ((d*e - c*f)^2*(b*e*(7*d*e - c*f) - a*f*(5*
d*e + c*f))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(2*e^(3/2)*f^(9/2))
```

Maple [B] time = 0.013, size = 475, normalized size = 2.

$$\frac{15\,bcd^2e^2}{2\,f^3}\arctan\left(fx\frac{1}{\sqrt{ef}}\right)\frac{1}{\sqrt{ef}} - \frac{3\,e^2xbcd^2}{2\,f^3\left(fx^2+e\right)} - \frac{9\,bc^2de}{2\,f^2}\arctan\left(fx\frac{1}{\sqrt{ef}}\right)\frac{1}{\sqrt{ef}} - \frac{e^2xad^3}{2\,f^3\left(fx^2+e\right)} + \frac{e^3xbd^3}{2\,f^4\left(fx^2+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x^2+c)^3/(f*x^2+e)^2,x)
```

```
[Out] 15/2/f^3*e^2/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*b*c*d^2-3/2/f^3*e^2*x/(f*x
^2+e)*b*c*d^2-9/2/f^2*e/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*b*c^2*d-1/2/f^3
*e^2*x/(f*x^2+e)*a*d^3+1/2/f^4*e^3*x/(f*x^2+e)*b*d^3+3/2/f/(e*f)^(1/2)*arct
an(x*f/(e*f)^(1/2))*a*c^2*d+5/2/f^3*e^2/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))
*a*d^3-9/2/f^2*e/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*a*c*d^2+3/2/f^2*e*x/(f
*x^2+e)*a*c*d^2+3/2/f^2*e*x/(f*x^2+e)*b*c^2*d-1/2/f*x/(f*x^2+e)*b*c^3-2/3*d
^3/f^3*x^3*b*e+d^2/f^2*x^3*b*c-3/2/f*x/(f*x^2+e)*a*c^2*d+1/2/e/(e*f)^(1/2)*
arctan(x*f/(e*f)^(1/2))*a*c^3+1/2/f/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*b*c
^3+1/2/e*x/(f*x^2+e)*a*c^3+1/5*d^3/f^2*b*x^5+1/3*d^3/f^2*x^3*a+3*d^3/f^4*b*
e^2*x+3*d/f^2*b*c^2*x+3*d^2/f^2*a*c*x-2*d^3/f^3*a*e*x-6*d^2/f^3*b*c*e*x-7/2
/f^4*e^3/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*b*d^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^3/(f*x^2+e)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.55181, size = 1701, normalized size = 7.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^3/(f*x^2+e)^2,x, algorithm="fricas")
```

```
[Out] [1/60*(12*b*d^3*e^2*f^4*x^7 - 4*(7*b*d^3*e^3*f^3 - 5*(3*b*c*d^2 + a*d^3)*e^
2*f^4)*x^5 + 20*(7*b*d^3*e^4*f^2 - 5*(3*b*c*d^2 + a*d^3)*e^3*f^3 + 9*(b*c^2
*d + a*c*d^2)*e^2*f^4)*x^3 + 15*(7*b*d^3*e^5 - a*c^3*e*f^4 - 5*(3*b*c*d^2 +
 a*d^3)*e^4*f + 9*(b*c^2*d + a*c*d^2)*e^3*f^2 - (b*c^3 + 3*a*c^2*d)*e^2*f^3
 + (7*b*d^3*e^4*f - a*c^3*f^5 - 5*(3*b*c*d^2 + a*d^3)*e^3*f^2 + 9*(b*c^2*d
+ a*c*d^2)*e^2*f^3 - (b*c^3 + 3*a*c^2*d)*e*f^4)*x^2)*sqrt(-e*f)*log((f*x^2
- 2*sqrt(-e*f)*x - e)/(f*x^2 + e)) + 30*(7*b*d^3*e^5*f + a*c^3*e*f^5 - 5*(3
*b*c*d^2 + a*d^3)*e^4*f^2 + 9*(b*c^2*d + a*c*d^2)*e^3*f^3 - (b*c^3 + 3*a*c^
2*d)*e^2*f^4)*x)/(e^2*f^6*x^2 + e^3*f^5), 1/30*(6*b*d^3*e^2*f^4*x^7 - 2*(7*
b*d^3*e^3*f^3 - 5*(3*b*c*d^2 + a*d^3)*e^2*f^4)*x^5 + 10*(7*b*d^3*e^4*f^2 -
5*(3*b*c*d^2 + a*d^3)*e^3*f^3 + 9*(b*c^2*d + a*c*d^2)*e^2*f^4)*x^3 - 15*(7*
b*d^3*e^5 - a*c^3*e*f^4 - 5*(3*b*c*d^2 + a*d^3)*e^4*f + 9*(b*c^2*d + a*c*d^
2)*e^3*f^2 - (b*c^3 + 3*a*c^2*d)*e^2*f^3 + (7*b*d^3*e^4*f - a*c^3*f^5 - 5*(
3*b*c*d^2 + a*d^3)*e^3*f^2 + 9*(b*c^2*d + a*c*d^2)*e^2*f^3 - (b*c^3 + 3*a*c
^2*d)*e*f^4)*x^2)*sqrt(e*f)*arctan(sqrt(e*f)*x/e) + 15*(7*b*d^3*e^5*f + a*c
^3*e*f^5 - 5*(3*b*c*d^2 + a*d^3)*e^4*f^2 + 9*(b*c^2*d + a*c*d^2)*e^3*f^3 -
(b*c^3 + 3*a*c^2*d)*e^2*f^4)*x)/(e^2*f^6*x^2 + e^3*f^5)]
```

Sympy [B] time = 7.2812, size = 654, normalized size = 2.7

$$\frac{bd^3x^5}{5f^2} + \frac{x\left(ac^3f^4 - 3ac^2def^3 + 3acd^2e^2f^2 - ad^3e^3f - bc^3ef^3 + 3bc^2de^2f^2 - 3bcd^2e^3f + bd^3e^4\right)}{2e^2f^4 + 2ef^5x^2} - \frac{\sqrt{-\frac{1}{e^3f^9}}\left(cf - de\right)^2\left(acf\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+c)**3/(f*x**2+e)**2,x)
```

```
[Out] b*d**3*x**5/(5*f**2) + x*(a*c**3*f**4 - 3*a*c**2*d*e*f**3 + 3*a*c*d**2*e**2
*f**2 - a*d**3*e**3*f - b*c**3*e*f**3 + 3*b*c**2*d*e**2*f**2 - 3*b*c*d**2*e
**3*f + b*d**3*e**4)/(2*e**2*f**4 + 2*e*f**5*x**2) - sqrt(-1/(e**3*f**9))*(
c*f - d*e)**2*(a*c*f**2 + 5*a*d*e*f + b*c*e*f - 7*b*d*e**2)*log(-e**2*f**4*
sqrt(-1/(e**3*f**9))*(c*f - d*e)**2*(a*c*f**2 + 5*a*d*e*f + b*c*e*f - 7*b*d
*e**2)/(a*c**3*f**4 + 3*a*c**2*d*e*f**3 - 9*a*c*d**2*e**2*f**2 + 5*a*d**3*e
**3*f + b*c**3*e*f**3 - 9*b*c**2*d*e**2*f**2 + 15*b*c*d**2*e**3*f - 7*b*d**
3*e**4) + x)/4 + sqrt(-1/(e**3*f**9))*(c*f - d*e)**2*(a*c*f**2 + 5*a*d*e*f
+ b*c*e*f - 7*b*d*e**2)*log(e**2*f**4*sqrt(-1/(e**3*f**9))*(c*f - d*e)**2*(
a*c*f**2 + 5*a*d*e*f + b*c*e*f - 7*b*d*e**2)/(a*c**3*f**4 + 3*a*c**2*d*e*f*
*3 - 9*a*c*d**2*e**2*f**2 + 5*a*d**3*e**3*f + b*c**3*e*f**3 - 9*b*c**2*d*e*
*2*f**2 + 15*b*c*d**2*e**3*f - 7*b*d**3*e**4) + x)/4 + x**3*(a*d**3*f + 3*b
*c*d**2*f - 2*b*d**3*e)/(3*f**3) + x*(3*a*c*d**2*f**2 - 2*a*d**3*e*f + 3*b*
c**2*d*f**2 - 6*b*c*d**2*e*f + 3*b*d**3*e**2)/f**4
```

<u>Giac</u> [A] time = 1.21962, size = 433, normalized size = 1.79

$$\frac{\left(ac^3f^4 + bc^3f^3e + 3\,ac^2df^3e - 9\,bc^2df^2e^2 - 9\,acd^2f^2e^2 + 15\,bcd^2fe^3 + 5\,ad^3fe^3 - 7\,bd^3e^4\right)\arctan\left(\sqrt{f}xe^{\left(-\frac{1}{2}\right)}\right)e^{\left(-\frac{3}{2}\right)}}{2\,f^{\frac{9}{2}}} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^3/(f*x^2+e)^2,x, algorithm="giac")
```

```
[Out] 1/2*(a*c^3*f^4 + b*c^3*f^3*e + 3*a*c^2*d*f^3*e - 9*b*c^2*d*f^2*e^2 - 9*a*c*
d^2*f^2*e^2 + 15*b*c*d^2*f*e^3 + 5*a*d^3*f*e^3 - 7*b*d^3*e^4)*arctan(sqrt(f
)*x*e^(-1/2))*e^(-3/2)/f^(9/2) + 1/2*(a*c^3*f^4*x - b*c^3*f^3*x*e - 3*a*c^2
*d*f^3*x*e + 3*b*c^2*d*f^2*x*e^2 + 3*a*c*d^2*f^2*x*e^2 - 3*b*c*d^2*f*x*e^3
- a*d^3*f*x*e^3 + b*d^3*x*e^4)*e^(-1)/((f*x^2 + e)*f^4) + 1/15*(3*b*d^3*f^8
*x^5 + 15*b*c*d^2*f^8*x^3 + 5*a*d^3*f^8*x^3 - 10*b*d^3*f^7*x^3*e + 45*b*c^2
*d*f^8*x + 45*a*c*d^2*f^8*x - 90*b*c*d^2*f^7*x*e - 30*a*d^3*f^7*x*e + 45*b*
d^3*f^6*x*e^2)/f^10
```

3.21 $\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^3}\,dx$

Optimal. Leaf size=291

$$\frac{dx\left(3af\left(-3c^2f^2-4cdef+15d^2e^2\right)-be\left(3c^2f^2-100cdef+105d^2e^2\right)\right)}{24e^2f^4}+\frac{(de-cf)\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\left(be\left(-c^2f^2-10cdef+35\right.\right.}{8e^{5/2}f^{9/2}}$$

```
[Out] (d*(3*a*f*(15*d^2*e^2 - 4*c*d*e*f - 3*c^2*f^2) - b*e*(105*d^2*e^2 - 100*c*d
*e*f + 3*c^2*f^2))*x)/(24*e^2*f^4) + (d*(b*e*(35*d*e - 3*c*f) - 3*a*f*(5*d*
e + 3*c*f))*x*(c + d*x^2))/(24*e^2*f^3) - ((b*e - a*f)*x*(c + d*x^2)^3)/(4*
e*f*(e + f*x^2)^2) - ((b*e*(7*d*e - c*f) - 3*a*f*(d*e + c*f))*x*(c + d*x^2)
^2)/(8*e^2*f^2*(e + f*x^2)) + ((d*e - c*f)*(b*e*(35*d^2*e^2 - 10*c*d*e*f -
c^2*f^2) - 3*a*f*(5*d^2*e^2 + 2*c*d*e*f + c^2*f^2))*ArcTan[(Sqrt[f]*x)/Sqrt
[e]])/(8*e^(5/2)*f^(9/2))
```

Rubi [A] time = 0.415631, antiderivative size = 291, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {526, 528, 388, 205}

$$\frac{dx\left(3af\left(-3c^2f^2-4cdef+15d^2e^2\right)-be\left(3c^2f^2-100cdef+105d^2e^2\right)\right)}{24e^2f^4}+\frac{(de-cf)\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\left(be\left(-c^2f^2-10cdef+35\right.\right.}{8e^{5/2}f^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(c + d*x^2)^3)/(e + f*x^2)^3,x]
```

```
[Out] (d*(3*a*f*(15*d^2*e^2 - 4*c*d*e*f - 3*c^2*f^2) - b*e*(105*d^2*e^2 - 100*c*d
*e*f + 3*c^2*f^2))*x)/(24*e^2*f^4) + (d*(b*e*(35*d*e - 3*c*f) - 3*a*f*(5*d*
e + 3*c*f))*x*(c + d*x^2))/(24*e^2*f^3) - ((b*e - a*f)*x*(c + d*x^2)^3)/(4*
e*f*(e + f*x^2)^2) - ((b*e*(7*d*e - c*f) - 3*a*f*(d*e + c*f))*x*(c + d*x^2)
^2)/(8*e^2*f^2*(e + f*x^2)) + ((d*e - c*f)*(b*e*(35*d^2*e^2 - 10*c*d*e*f -
c^2*f^2) - 3*a*f*(5*d^2*e^2 + 2*c*d*e*f + c^2*f^2))*ArcTan[(Sqrt[f]*x)/Sqrt
[e]])/(8*e^(5/2)*f^(9/2))
```

Rule 526

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^q)/(a*b*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p +
 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)\left(c+dx^2\right)^3}{\left(e+fx^2\right)^3}\,dx = -\frac{(be-af)x\left(c+dx^2\right)^3}{4ef\left(e+fx^2\right)^2} - \frac{\int \frac{\left(c+dx^2\right)^2\left(-c(be+3af)-d(7be-3af)x^2\right)}{\left(e+fx^2\right)^2}\,dx}{4ef}$$

$$= -\frac{(be-af)x\left(c+dx^2\right)^3}{4ef\left(e+fx^2\right)^2} - \frac{(be(7de-cf)-3af(de+cf))x\left(c+dx^2\right)^2}{8e^2f^2\left(e+fx^2\right)} + \int \frac{\left(c+dx^2\right)\left(-c(3af(de-\cdots\right.}{\cdots}$$

$$= \frac{d(be(35de-3cf)-3af(5de+3cf))x\left(c+dx^2\right)}{24e^2f^3} - \frac{(be-af)x\left(c+dx^2\right)^3}{4ef\left(e+fx^2\right)^2} - \frac{(be(7de-cf)\cdots}{8\cdots}$$

$$= \frac{d\left(3af\left(15d^2e^2-4cdef-3c^2f^2\right)-be\left(105d^2e^2-100cdef+3c^2f^2\right)\right)x}{24e^2f^4} + \frac{d(be(35de-3c\cdots}{\cdots}$$

$$= \frac{d\left(3af\left(15d^2e^2-4cdef-3c^2f^2\right)-be\left(105d^2e^2-100cdef+3c^2f^2\right)\right)x}{24e^2f^4} + \frac{d(be(35de-3c\cdots}{\cdots}$$

Mathematica [A] time = 0.157301, size = 219, normalized size = 0.75

$$\frac{(de-cf)\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\left(be\left(-c^2f^2-10cdef+35d^2e^2\right)-3af\left(c^2f^2+2cdef+5d^2e^2\right)\right)}{8e^{5/2}f^{9/2}} + \frac{d^2x(adf+3bcf-3bde)}{f^4} - \frac{x(d\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(c + d*x^2)^3)/(e + f*x^2)^3,x]
```

```
[Out] (d^2*(-3*b*d*e + 3*b*c*f + a*d*f)*x)/f^4 + (b*d^3*x^3)/(3*f^3) + ((b*e - a*
f)*(d*e - c*f)^3*x)/(4*e*f^4*(e + f*x^2)^2) - ((d*e - c*f)^2*(b*e*(13*d*e -
 c*f) - 3*a*f*(3*d*e + c*f))*x)/(8*e^2*f^4*(e + f*x^2)) + ((d*e - c*f)*(b*e
*(35*d^2*e^2 - 10*c*d*e*f - c^2*f^2) - 3*a*f*(5*d^2*e^2 + 2*c*d*e*f + c^2*f
^2))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(8*e^(5/2)*f^(9/2))
```

Maple [B] time = 0.013, size = 589, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x^2+c)^3/(f*x^2+e)^3,x)
```

```
[Out] 21/8/f^3/(f*x^2+e)^2*b*c*d^2*e^2*x+3/8/f/e/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/
2))*a*c^2*d-45/8/f^3*e/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*b*c*d^2-15/8/f/(
f*x^2+e)^2*x^3*b*c^2*d+1/3*d^3/f^3*x^3*b+d^3/f^3*a*x+3*d^2/f^3*b*c*x-3*d^3/
f^4*b*e*x-1/8/f/(f*x^2+e)^2*b*c^3*x+3/8/e^2/(e*f)^(1/2)*arctan(x*f/(e*f)^(1
/2))*a*c^3+1/8/(f*x^2+e)^2/e*x^3*b*c^3+5/8/(f*x^2+e)^2/e*x*a*c^3-9/8/f^2/(f
*x^2+e)^2*a*c*d^2*e*x-9/8/f^2/(f*x^2+e)^2*b*c^2*d*e*x+27/8/f^2/(f*x^2+e)^2*
x^3*b*c*d^2*e-13/8/f^3/(f*x^2+e)^2*x^3*b*d^3*e^2-3/8/f/(f*x^2+e)^2*a*c^2*d*
x+7/8/f^3/(f*x^2+e)^2*a*d^3*e^2*x-11/8/f^4/(f*x^2+e)^2*b*d^3*e^3*x+9/8/f^2/
(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*a*c*d^2-15/8/f^3*e/(e*f)^(1/2)*arctan(x
*f/(e*f)^(1/2))*a*d^3+1/8/f/e/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*b*c^3+9/8
/f^2/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*b*c^2*d+35/8/f^4*e^2/(e*f)^(1/2)*a
rctan(x*f/(e*f)^(1/2))*b*d^3+3/8/(f*x^2+e)^2/e*x^3*a*c^2*d+3/8*f/(f*x^2+e)^
2/e^2*x^3*a*c^3-15/8/f/(f*x^2+e)^2*x^3*a*c*d^2+9/8/f^2/(f*x^2+e)^2*x^3*a*d^
3*e
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^3/(f*x^2+e)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.6465, size = 2291, normalized size = 7.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^3/(f*x^2+e)^3,x, algorithm="fricas")
```

```
[Out] [1/48*(16*b*d^3*e^3*f^4*x^7 - 16*(7*b*d^3*e^4*f^3 - 3*(3*b*c*d^2 + a*d^3)*e
^3*f^4)*x^5 - 2*(175*b*d^3*e^5*f^2 - 9*a*c^3*e*f^6 - 75*(3*b*c*d^2 + a*d^3)
*e^4*f^3 + 45*(b*c^2*d + a*c*d^2)*e^3*f^4 - 3*(b*c^3 + 3*a*c^2*d)*e^2*f^5)*
x^3 - 3*(35*b*d^3*e^6 + 3*a*c^3*e^2*f^4 - 15*(3*b*c*d^2 + a*d^3)*e^5*f + 9*
(b*c^2*d + a*c*d^2)*e^4*f^2 + (b*c^3 + 3*a*c^2*d)*e^3*f^3 + (35*b*d^3*e^4*f
^2 + 3*a*c^3*f^6 - 15*(3*b*c*d^2 + a*d^3)*e^3*f^3 + 9*(b*c^2*d + a*c*d^2)*e
^2*f^4 + (b*c^3 + 3*a*c^2*d)*e*f^5)*x^4 + 2*(35*b*d^3*e^5*f + 3*a*c^3*e*f^5
 - 15*(3*b*c*d^2 + a*d^3)*e^4*f^2 + 9*(b*c^2*d + a*c*d^2)*e^3*f^3 + (b*c^3
+ 3*a*c^2*d)*e^2*f^4)*x^2)*sqrt(-e*f)*log((f*x^2 - 2*sqrt(-e*f)*x - e)/(f*x
^2 + e)) - 6*(35*b*d^3*e^6*f - 5*a*c^3*e^2*f^5 - 15*(3*b*c*d^2 + a*d^3)*e^5
*f^2 + 9*(b*c^2*d + a*c*d^2)*e^4*f^3 + (b*c^3 + 3*a*c^2*d)*e^3*f^4)*x)/(e^3
*f^7*x^4 + 2*e^4*f^6*x^2 + e^5*f^5), 1/24*(8*b*d^3*e^3*f^4*x^7 - 8*(7*b*d^3
*e^4*f^3 - 3*(3*b*c*d^2 + a*d^3)*e^3*f^4)*x^5 - (175*b*d^3*e^5*f^2 - 9*a*c^
3*e*f^6 - 75*(3*b*c*d^2 + a*d^3)*e^4*f^3 + 45*(b*c^2*d + a*c*d^2)*e^3*f^4 -
 3*(b*c^3 + 3*a*c^2*d)*e^2*f^5)*x^3 + 3*(35*b*d^3*e^6 + 3*a*c^3*e^2*f^4 - 1
5*(3*b*c*d^2 + a*d^3)*e^5*f + 9*(b*c^2*d + a*c*d^2)*e^4*f^2 + (b*c^3 + 3*a*
c^2*d)*e^3*f^3 + (35*b*d^3*e^4*f^2 + 3*a*c^3*f^6 - 15*(3*b*c*d^2 + a*d^3)*e
^3*f^3 + 9*(b*c^2*d + a*c*d^2)*e^2*f^4 + (b*c^3 + 3*a*c^2*d)*e*f^5)*x^4 + 2
*(35*b*d^3*e^5*f + 3*a*c^3*e*f^5 - 15*(3*b*c*d^2 + a*d^3)*e^4*f^2 + 9*(b*c^
```

```
2*d + a*c*d^2)*e^3*f^3 + (b*c^3 + 3*a*c^2*d)*e^2*f^4)*x^2)*sqrt(e*f)*arctan
(sqrt(e*f)*x/e) - 3*(35*b*d^3*e^6*f - 5*a*c^3*e^2*f^5 - 15*(3*b*c*d^2 + a*d
^3)*e^5*f^2 + 9*(b*c^2*d + a*c*d^2)*e^4*f^3 + (b*c^3 + 3*a*c^2*d)*e^3*f^4)*
x)/(e^3*f^7*x^4 + 2*e^4*f^6*x^2 + e^5*f^5)]
```

Sympy [B] time = 59.853, size = 862, normalized size = 2.96

$$\frac{bd^3x^3}{3f^3} - \frac{\sqrt{-\frac{1}{e^5f^9}}\left(cf-de\right)\left(3ac^2f^3+6acdef^2+15ad^2e^2f+bc^2ef^2+10bcde^2f-35bd^2e^3\right)\log\left(-\frac{e^3f^4\sqrt{-\frac{1}{e^5f^9}}\left(cf-de\right)\left(3ac\right.}{3ac^3f^4+3ac^2def^3+9acd^2e^2}\right.}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+c)**3/(f*x**2+e)**3,x)
```

```
[Out] b*d**3*x**3/(3*f**3) - sqrt(-1/(e**5*f**9))*(c*f - d*e)*(3*a*c**2*f**3 + 6*
a*c*d*e*f**2 + 15*a*d**2*e**2*f + b*c**2*e*f**2 + 10*b*c*d*e**2*f - 35*b*d*
*2*e**3)*log(-e**3*f**4*sqrt(-1/(e**5*f**9))*(c*f - d*e)*(3*a*c**2*f**3 + 6
*a*c*d*e*f**2 + 15*a*d**2*e**2*f + b*c**2*e*f**2 + 10*b*c*d*e**2*f - 35*b*d
**2*e**3)/(3*a*c**3*f**4 + 3*a*c**2*d*e*f**3 + 9*a*c*d**2*e**2*f**2 - 15*a*
d**3*e**3*f + b*c**3*e*f**3 + 9*b*c**2*d*e**2*f**2 - 45*b*c*d**2*e**3*f + 3
5*b*d**3*e**4) + x)/16 + sqrt(-1/(e**5*f**9))*(c*f - d*e)*(3*a*c**2*f**3 +
6*a*c*d*e*f**2 + 15*a*d**2*e**2*f + b*c**2*e*f**2 + 10*b*c*d*e**2*f - 35*b*
d**2*e**3)*log(e**3*f**4*sqrt(-1/(e**5*f**9))*(c*f - d*e)*(3*a*c**2*f**3 +
6*a*c*d*e*f**2 + 15*a*d**2*e**2*f + b*c**2*e*f**2 + 10*b*c*d*e**2*f - 35*b*
d**2*e**3)/(3*a*c**3*f**4 + 3*a*c**2*d*e*f**3 + 9*a*c*d**2*e**2*f**2 - 15*a
*d**3*e**3*f + b*c**3*e*f**3 + 9*b*c**2*d*e**2*f**2 - 45*b*c*d**2*e**3*f +
35*b*d**3*e**4) + x)/16 + (x**3*(3*a*c**3*f**5 + 3*a*c**2*d*e*f**4 - 15*a*c
*d**2*e**2*f**3 + 9*a*d**3*e**3*f**2 + b*c**3*e*f**4 - 15*b*c**2*d*e**2*f**
3 + 27*b*c*d**2*e**3*f**2 - 13*b*d**3*e**4*f) + x*(5*a*c**3*e*f**4 - 3*a*c*
*2*d*e**2*f**3 - 9*a*c*d**2*e**3*f**2 + 7*a*d**3*e**4*f - b*c**3*e**2*f**3
- 9*b*c**2*d*e**3*f**2 + 21*b*c*d**2*e**4*f - 11*b*d**3*e**5))/(8*e**4*f**4
 + 16*e**3*f**5*x**2 + 8*e**2*f**6*x**4) + x*(a*d**3*f + 3*b*c*d**2*f - 3*b
*d**3*e)/f**4
```

Giac [A] time = 1.1868, size = 501, normalized size = 1.72

$$\frac{\left(3\,ac^3f^4+bc^3f^3e+3\,ac^2df^3e+9\,bc^2df^2e^2+9\,acd^2f^2e^2-45\,bcd^2fe^3-15\,ad^3fe^3+35\,bd^3e^4\right)\arctan\left(\sqrt{f}xe^{\left(-\frac{1}{2}\right)}\right)e^{\left(-\right.}}{8\,f^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^3/(f*x^2+e)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*a*c^3*f^4 + b*c^3*f^3*e + 3*a*c^2*d*f^3*e + 9*b*c^2*d*f^2*e^2 + 9*a*
c*d^2*f^2*e^2 - 45*b*c*d^2*f*e^3 - 15*a*d^3*f*e^3 + 35*b*d^3*e^4)*arctan(sq
rt(f)*x*e^(-1/2))*e^(-5/2)/f^(9/2) + 1/8*(3*a*c^3*f^5*x^3 + b*c^3*f^4*x^3*e
 + 3*a*c^2*d*f^4*x^3*e - 15*b*c^2*d*f^3*x^3*e^2 - 15*a*c*d^2*f^3*x^3*e^2 +
27*b*c*d^2*f^2*x^3*e^3 + 9*a*d^3*f^2*x^3*e^3 + 5*a*c^3*f^4*x*e - 13*b*d^3*f
*x^3*e^4 - b*c^3*f^3*x*e^2 - 3*a*c^2*d*f^3*x*e^2 - 9*b*c^2*d*f^2*x*e^3 - 9*
a*c*d^2*f^2*x*e^3 + 21*b*c*d^2*f*x*e^4 + 7*a*d^3*f*x*e^4 - 11*b*d^3*x*e^5)*
e^(-2)/((f*x^2 + e)^2*f^4) + 1/3*(b*d^3*f^6*x^3 + 9*b*c*d^2*f^6*x + 3*a*d^3
```

```
*f^6*x - 9*b*d^3*f^5*x*e)/f^9
```

3.22 $\int \frac{\left(a+bx^2\right)\left(c+dx^2\right)^3}{\left(e+fx^2\right)^4}\,dx$

Optimal. Leaf size=348

$$-\frac{x\left(c+dx^2\right)\left(be\left(-3c^2f^2-8cdef+35d^2e^2\right)-af\left(15c^2f^2+4cdef+5d^2e^2\right)\right)}{48e^3f^3\left(e+fx^2\right)}+\frac{dx\left(be\left(-3c^2f^2-10cdef+105d^2e^2\right)-\right.}{48e^3f^4}$$

```
[Out] (d*(b*e*(105*d^2*e^2 - 10*c*d*e*f - 3*c^2*f^2) - a*f*(15*d^2*e^2 + 14*c*d*e
*f + 15*c^2*f^2))*x)/(48*e^3*f^4) - ((b*e - a*f)*x*(c + d*x^2)^3)/(6*e*f*(e
 + f*x^2)^3) - ((b*e*(7*d*e - c*f) - a*f*(d*e + 5*c*f))*x*(c + d*x^2)^2)/(2
4*e^2*f^2*(e + f*x^2)^2) - ((b*e*(35*d^2*e^2 - 8*c*d*e*f - 3*c^2*f^2) - a*f
*(5*d^2*e^2 + 4*c*d*e*f + 15*c^2*f^2))*x*(c + d*x^2))/(48*e^3*f^3*(e + f*x^
2)) - ((b*e*(35*d^3*e^3 - 15*c*d^2*e^2*f - 3*c^2*d*e*f^2 - c^3*f^3) - a*f*(
5*d^3*e^3 + 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 + 5*c^3*f^3))*ArcTan[(Sqrt[f]*x)/
Sqrt[e]])/(16*e^(7/2)*f^(9/2))
```

Rubi [A] time = 0.449597, antiderivative size = 348, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {526, 388, 205}

$$-\frac{x\left(c+dx^2\right)\left(be\left(-3c^2f^2-8cdef+35d^2e^2\right)-af\left(15c^2f^2+4cdef+5d^2e^2\right)\right)}{48e^3f^3\left(e+fx^2\right)}+\frac{dx\left(be\left(-3c^2f^2-10cdef+105d^2e^2\right)-\right.}{48e^3f^4}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(c + d*x^2)^3)/(e + f*x^2)^4,x]
```


```
[Out] (d*(b*e*(105*d^2*e^2 - 10*c*d*e*f - 3*c^2*f^2) - a*f*(15*d^2*e^2 + 14*c*d*e
*f + 15*c^2*f^2))*x)/(48*e^3*f^4) - ((b*e - a*f)*x*(c + d*x^2)^3)/(6*e*f*(e
 + f*x^2)^3) - ((b*e*(7*d*e - c*f) - a*f*(d*e + 5*c*f))*x*(c + d*x^2)^2)/(2
4*e^2*f^2*(e + f*x^2)^2) - ((b*e*(35*d^2*e^2 - 8*c*d*e*f - 3*c^2*f^2) - a*f
*(5*d^2*e^2 + 4*c*d*e*f + 15*c^2*f^2))*x*(c + d*x^2))/(48*e^3*f^3*(e + f*x^
2)) - ((b*e*(35*d^3*e^3 - 15*c*d^2*e^2*f - 3*c^2*d*e*f^2 - c^3*f^3) - a*f*(
5*d^3*e^3 + 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 + 5*c^3*f^3))*ArcTan[(Sqrt[f]*x)/
Sqrt[e]])/(16*e^(7/2)*f^(9/2))
```


Rule 526

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^q)/(a*b*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p +
 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^4}\,dx = -\frac{(be-af)x(c+dx^2)^3}{6ef(e+fx^2)^3} - \frac{\int \frac{(c+dx^2)^2(-c(be+5af)-d(7be-af)x^2)}{(e+fx^2)^3}\,dx}{6ef}$$

$$= -\frac{(be-af)x(c+dx^2)^3}{6ef(e+fx^2)^3} - \frac{(be(7de-cf)-af(de+5cf))x(c+dx^2)^2}{24e^2f^2(e+fx^2)^2} + \frac{\int \frac{(c+dx^2)(c(de(7be-af)+3}{}}{}$$

$$= -\frac{(be-af)x(c+dx^2)^3}{6ef(e+fx^2)^3} - \frac{(be(7de-cf)-af(de+5cf))x(c+dx^2)^2}{24e^2f^2(e+fx^2)^2} - \frac{(be(35d^2e^2-8cdef}{}$$

$$= \frac{d(be(105d^2e^2-10cdef-3c^2f^2)-af(15d^2e^2+14cdef+15c^2f^2))x}{48e^3f^4} - \frac{(be-af)x(c+dx^2}{6ef(e+fx^2)^3}$$

$$= \frac{d(be(105d^2e^2-10cdef-3c^2f^2)-af(15d^2e^2+14cdef+15c^2f^2))x}{48e^3f^4} - \frac{(be-af)x(c+dx^2}{6ef(e+fx^2)^3}$$

Mathematica [A] time = 0.209906, size = 295, normalized size = 0.85

$$\frac{x(de-cf)(be(-c^2f^2-4cdef+29d^2e^2)-af(5c^2f^2+8cdef+11d^2e^2))}{16e^3f^4(e+fx^2)} - \frac{\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(be(-3c^2def^2-c^3f^3-15cd^2e^2f}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(c + d*x^2)^3)/(e + f*x^2)^4,x]
```

```
[Out] (b*d^3*x)/f^4 + ((b*e - a*f)*(d*e - c*f)^3*x)/(6*e*f^4*(e + f*x^2)^3) - ((d
*e - c*f)^2*(b*e*(19*d*e - c*f) - a*f*(13*d*e + 5*c*f))*x)/(24*e^2*f^4*(e +
 f*x^2)^2) + ((d*e - c*f)*(b*e*(29*d^2*e^2 - 4*c*d*e*f - c^2*f^2) - a*f*(11
*d^2*e^2 + 8*c*d*e*f + 5*c^2*f^2))*x)/(16*e^3*f^4*(e + f*x^2)) - ((b*e*(35*
d^3*e^3 - 15*c*d^2*e^2*f - 3*c^2*d*e*f^2 - c^3*f^3) - a*f*(5*d^3*e^3 + 3*c*
d^2*e^2*f + 3*c^2*d*e*f^2 + 5*c^3*f^3))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(16*e^
(7/2)*f^(9/2))
```

Maple [B] time = 0.013, size = 735, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x^2+c)^3/(f*x^2+e)^4,x)
```

```
[Out] b*d^3/f^4*x+15/16/f^3/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*b*c*d^2+1/16/f/e^
2/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*b*c^3-35/16/f^4*e/(e*f)^(1/2)*arctan(
```

```
x*f/(e*f)^(1/2))*b*d^3+5/16*f^2/(f*x^2+e)^3/e^3*x^5*a*c^3+1/16*f/(f*x^2+e)^
3/e^2*x^5*b*c^3-33/16/f/(f*x^2+e)^3*x^5*b*c*d^2+29/16/f^2/(f*x^2+e)^3*x^5*b
*d^3*e+5/6*f/(f*x^2+e)^3/e^2*x^3*a*c^3-1/2/f/(f*x^2+e)^3*x^3*a*c*d^2-5/6/f^
2/(f*x^2+e)^3*x^3*a*d^3*e-1/2/f/(f*x^2+e)^3*x^3*b*c^2*d-1/16/f/(f*x^2+e)^3*
b*c^3*x-11/16/f/(f*x^2+e)^3*x^5*a*d^3+5/16/f^3/(e*f)^(1/2)*arctan(x*f/(e*f)
^(1/2))*a*d^3+5/16/e^3/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*a*c^3+1/6/(f*x^2
+e)^3/e*x^3*b*c^3+11/16/(f*x^2+e)^3/e*x*a*c^3+17/6/f^3/(f*x^2+e)^3*x^3*b*d^
3*e^2-3/16/f/(f*x^2+e)^3*a*c^2*d*x-5/16/f^3/(f*x^2+e)^3*a*d^3*e^2*x+3/16/(f
*x^2+e)^3/e*x^5*a*c*d^2+3/16/(f*x^2+e)^3/e*x^5*b*c^2*d+1/2/(f*x^2+e)^3/e*x^
3*a*c^2*d+19/16/f^4/(f*x^2+e)^3*b*d^3*e^3*x-3/16/f^2/(f*x^2+e)^3*b*c^2*d*e*
x-15/16/f^3/(f*x^2+e)^3*b*c*d^2*e^2*x+3/16*f/(f*x^2+e)^3/e^2*x^5*a*c^2*d-5/
2/f^2/(f*x^2+e)^3*x^3*b*c*d^2*e-3/16/f^2/(f*x^2+e)^3*a*c*d^2*e*x+3/16/f/e^2
/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*a*c^2*d+3/16/f^2/e/(e*f)^(1/2)*arctan(
x*f/(e*f)^(1/2))*a*c*d^2+3/16/f^2/e/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*b*c
^2*d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^3/(f*x^2+e)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.71072, size = 2913, normalized size = 8.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^3/(f*x^2+e)^4,x, algorithm="fricas")
```

```
[Out] [1/96*(96*b*d^3*e^4*f^4*x^7 + 6*(77*b*d^3*e^5*f^3 + 5*a*c^3*e*f^7 - 11*(3*b
*c*d^2 + a*d^3)*e^4*f^4 + 3*(b*c^2*d + a*c*d^2)*e^3*f^5 + (b*c^3 + 3*a*c^2*
d)*e^2*f^6)*x^5 + 16*(35*b*d^3*e^6*f^2 + 5*a*c^3*e^2*f^6 - 5*(3*b*c*d^2 + a
*d^3)*e^5*f^3 - 3*(b*c^2*d + a*c*d^2)*e^4*f^4 + (b*c^3 + 3*a*c^2*d)*e^3*f^5
)*x^3 + 3*(35*b*d^3*e^7 - 5*a*c^3*e^3*f^4 - 5*(3*b*c*d^2 + a*d^3)*e^6*f - 3
*(b*c^2*d + a*c*d^2)*e^5*f^2 - (b*c^3 + 3*a*c^2*d)*e^4*f^3 + (35*b*d^3*e^4*
f^3 - 5*a*c^3*f^7 - 5*(3*b*c*d^2 + a*d^3)*e^3*f^4 - 3*(b*c^2*d + a*c*d^2)*e
^2*f^5 - (b*c^3 + 3*a*c^2*d)*e*f^6)*x^6 + 3*(35*b*d^3*e^5*f^2 - 5*a*c^3*e*f
^6 - 5*(3*b*c*d^2 + a*d^3)*e^4*f^3 - 3*(b*c^2*d + a*c*d^2)*e^3*f^4 - (b*c^3
 + 3*a*c^2*d)*e^2*f^5)*x^4 + 3*(35*b*d^3*e^6*f - 5*a*c^3*e^2*f^5 - 5*(3*b*c
*d^2 + a*d^3)*e^5*f^2 - 3*(b*c^2*d + a*c*d^2)*e^4*f^3 - (b*c^3 + 3*a*c^2*d)
*e^3*f^4)*x^2)*sqrt(-e*f)*log((f*x^2 - 2*sqrt(-e*f)*x - e)/(f*x^2 + e)) + 6
*(35*b*d^3*e^7*f + 11*a*c^3*e^3*f^5 - 5*(3*b*c*d^2 + a*d^3)*e^6*f^2 - 3*(b*
c^2*d + a*c*d^2)*e^5*f^3 - (b*c^3 + 3*a*c^2*d)*e^4*f^4)*x)/(e^4*f^8*x^6 + 3
*e^5*f^7*x^4 + 3*e^6*f^6*x^2 + e^7*f^5), 1/48*(48*b*d^3*e^4*f^4*x^7 + 3*(77
*b*d^3*e^5*f^3 + 5*a*c^3*e*f^7 - 11*(3*b*c*d^2 + a*d^3)*e^4*f^4 + 3*(b*c^2*
d + a*c*d^2)*e^3*f^5 + (b*c^3 + 3*a*c^2*d)*e^2*f^6)*x^5 + 8*(35*b*d^3*e^6*f
^2 + 5*a*c^3*e^2*f^6 - 5*(3*b*c*d^2 + a*d^3)*e^5*f^3 - 3*(b*c^2*d + a*c*d^2
)*e^4*f^4 + (b*c^3 + 3*a*c^2*d)*e^3*f^5)*x^3 - 3*(35*b*d^3*e^7 - 5*a*c^3*e^
3*f^4 - 5*(3*b*c*d^2 + a*d^3)*e^6*f - 3*(b*c^2*d + a*c*d^2)*e^5*f^2 - (b*c^
3 + 3*a*c^2*d)*e^4*f^3 + (35*b*d^3*e^4*f^3 - 5*a*c^3*f^7 - 5*(3*b*c*d^2 + a
```

```
*d^3)*e^3*f^4 - 3*(b*c^2*d + a*c*d^2)*e^2*f^5 - (b*c^3 + 3*a*c^2*d)*e*f^6)*
x^6 + 3*(35*b*d^3*e^5*f^2 - 5*a*c^3*e*f^6 - 5*(3*b*c*d^2 + a*d^3)*e^4*f^3 -
 3*(b*c^2*d + a*c*d^2)*e^3*f^4 - (b*c^3 + 3*a*c^2*d)*e^2*f^5)*x^4 + 3*(35*b
*d^3*e^6*f - 5*a*c^3*e^2*f^5 - 5*(3*b*c*d^2 + a*d^3)*e^5*f^2 - 3*(b*c^2*d +
 a*c*d^2)*e^4*f^3 - (b*c^3 + 3*a*c^2*d)*e^3*f^4)*x^2)*sqrt(e*f)*arctan(sqrt
(e*f)*x/e) + 3*(35*b*d^3*e^7*f + 11*a*c^3*e^3*f^5 - 5*(3*b*c*d^2 + a*d^3)*e
^6*f^2 - 3*(b*c^2*d + a*c*d^2)*e^5*f^3 - (b*c^3 + 3*a*c^2*d)*e^4*f^4)*x)/(e
^4*f^8*x^6 + 3*e^5*f^7*x^4 + 3*e^6*f^6*x^2 + e^7*f^5)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+c)**3/(f*x**2+e)**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20757, size = 603, normalized size = 1.73

$$\frac{bd^3x}{f^4} + \frac{\left(5\,ac^3f^4 + bc^3f^3e + 3\,ac^2df^3e + 3\,bc^2df^2e^2 + 3\,acd^2f^2e^2 + 15\,bcd^2fe^3 + 5\,ad^3fe^3 - 35\,bd^3e^4\right)\arctan\left(\sqrt{f}xe^{\left(-\frac{1}{2}\right)}\right)}{16\,f^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^3/(f*x^2+e)^4,x, algorithm="giac")
```

```
[Out] b*d^3*x/f^4 + 1/16*(5*a*c^3*f^4 + b*c^3*f^3*e + 3*a*c^2*d*f^3*e + 3*b*c^2*d
*f^2*e^2 + 3*a*c*d^2*f^2*e^2 + 15*b*c*d^2*f*e^3 + 5*a*d^3*f*e^3 - 35*b*d^3*
e^4)*arctan(sqrt(f)*x*e^(-1/2))*e^(-7/2)/f^(9/2) + 1/48*(15*a*c^3*f^6*x^5 +
 3*b*c^3*f^5*x^5*e + 9*a*c^2*d*f^5*x^5*e + 9*b*c^2*d*f^4*x^5*e^2 + 9*a*c*d^
2*f^4*x^5*e^2 - 99*b*c*d^2*f^3*x^5*e^3 - 33*a*d^3*f^3*x^5*e^3 + 40*a*c^3*f^
5*x^3*e + 87*b*d^3*f^2*x^5*e^4 + 8*b*c^3*f^4*x^3*e^2 + 24*a*c^2*d*f^4*x^3*e
^2 - 24*b*c^2*d*f^3*x^3*e^3 - 24*a*c*d^2*f^3*x^3*e^3 - 120*b*c*d^2*f^2*x^3*
e^4 - 40*a*d^3*f^2*x^3*e^4 + 33*a*c^3*f^4*x*e^2 + 136*b*d^3*f*x^3*e^5 - 3*b
*c^3*f^3*x*e^3 - 9*a*c^2*d*f^3*x*e^3 - 9*b*c^2*d*f^2*x*e^4 - 9*a*c*d^2*f^2*
x*e^4 - 45*b*c*d^2*f*x*e^5 - 15*a*d^3*f*x*e^5 + 57*b*d^3*x*e^6)*e^(-3)/((f*
x^2 + e)^3*f^4)
```

3.23 $\int \left(a+bx^2\right) \left(c+dx^2\right)^{3/2} \sqrt{e+fx^2}\,dx$

Optimal. Leaf size=544

$$-\frac{e^{3/2}\sqrt{c+dx^2}\left(7adf(de-9cf)-b\left(-3c^2f^2-9cdef+4d^2e^2\right)\right)\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{105df^{5/2}\sqrt{e+fx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}+\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}}{}$$

```
[Out] -((7*a*d*f*(2*d^2*e^2 - 7*c*d*e*f - 3*c^2*f^2) - b*(8*d^3*e^3 - 19*c*d^2*e^
2*f + 9*c^2*d*e*f^2 - 6*c^3*f^3))*x*Sqrt[c + d*x^2])/(105*d^2*f^2*Sqrt[e +
f*x^2]) + ((7*a*d*f*(d*e + 3*c*f) - b*(4*d^2*e^2 - 6*c*d*e*f + 6*c^2*f^2))*
x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(105*d*f^2) + ((b*d*e - 2*b*c*f + 7*a*d*
f)*x*(c + d*x^2)^(3/2)*Sqrt[e + f*x^2])/(35*d*f) + (b*x*(c + d*x^2)^(5/2)*S
qrt[e + f*x^2])/(7*d) + (Sqrt[e]*(7*a*d*f*(2*d^2*e^2 - 7*c*d*e*f - 3*c^2*f^
2) - b*(8*d^3*e^3 - 19*c*d^2*e^2*f + 9*c^2*d*e*f^2 - 6*c^3*f^3))*Sqrt[c + d
*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(105*d^2*f^(
5/2)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) - (e^(3/2)*(7*a
*d*f*(d*e - 9*c*f) - b*(4*d^2*e^2 - 9*c*d*e*f - 3*c^2*f^2))*Sqrt[c + d*x^2]
*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(105*d*f^(5/2)*Sq
rt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rubi [A] time = 0.681249, antiderivative size = 544, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {528, 531, 418, 492, 411}

$$\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}\left(7adf(3cf+de)-b\left(6c^2f^2-6cdef+4d^2e^2\right)\right)}{105df^2}-\frac{x\sqrt{c+dx^2}\left(7adf\left(-3c^2f^2-7cdef+2d^2e^2\right)-b\right.}{105d^2f^2\sqrt{e+}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)*(c + d*x^2)^(3/2)*Sqrt[e + f*x^2],x]
```


```
[Out] -((7*a*d*f*(2*d^2*e^2 - 7*c*d*e*f - 3*c^2*f^2) - b*(8*d^3*e^3 - 19*c*d^2*e^
2*f + 9*c^2*d*e*f^2 - 6*c^3*f^3))*x*Sqrt[c + d*x^2])/(105*d^2*f^2*Sqrt[e +
f*x^2]) + ((7*a*d*f*(d*e + 3*c*f) - b*(4*d^2*e^2 - 6*c*d*e*f + 6*c^2*f^2))*
x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(105*d*f^2) + ((b*d*e - 2*b*c*f + 7*a*d*
f)*x*(c + d*x^2)^(3/2)*Sqrt[e + f*x^2])/(35*d*f) + (b*x*(c + d*x^2)^(5/2)*S
qrt[e + f*x^2])/(7*d) + (Sqrt[e]*(7*a*d*f*(2*d^2*e^2 - 7*c*d*e*f - 3*c^2*f^
2) - b*(8*d^3*e^3 - 19*c*d^2*e^2*f + 9*c^2*d*e*f^2 - 6*c^3*f^3))*Sqrt[c + d
*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(105*d^2*f^(
5/2)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) - (e^(3/2)*(7*a
*d*f*(d*e - 9*c*f) - b*(4*d^2*e^2 - 9*c*d*e*f - 3*c^2*f^2))*Sqrt[c + d*x^2]
*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(105*d*f^(5/2)*Sq
rt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```


Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
```

```
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \left(a+bx^2\right)\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}\,dx = \frac{bx\left(c+dx^2\right)^{5/2}\sqrt{e+fx^2}}{7d}+\frac{\int \frac{\left(c+dx^2\right)^{3/2}\left(-(bc-7ad)e+(bde-2bcf+7adf)x^2\right)}{\sqrt{e+fx^2}}\,dx}{7d}$$

$$= \frac{(bde-2bcf+7adf)x\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}}{35df}+\frac{bx\left(c+dx^2\right)^{5/2}\sqrt{e+fx^2}}{7d}+\int \frac{\sqrt{c+\cdots}}{\cdots}$$

$$= \frac{\left(7adf(de+3cf)-b\left(4d^2e^2-6cdef+6c^2f^2\right)\right)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{105df^2}+\frac{(bde-2bc\cdots}{\cdots}$$

$$= \frac{\left(7adf(de+3cf)-b\left(4d^2e^2-6cdef+6c^2f^2\right)\right)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{105df^2}+\frac{(bde-2bc\cdots}{\cdots}$$

$$= -\frac{\left(7adf\left(2d^2e^2-7cdef-3c^2f^2\right)-b\left(8d^3e^3-19cd^2e^2f+9c^2def^2-6c^3f^3\right)\right)x\sqrt{c\cdots}}{105d^2f^2\sqrt{e+fx^2}}$$

$$= -\frac{\left(7adf\left(2d^2e^2-7cdef-3c^2f^2\right)-b\left(8d^3e^3-19cd^2e^2f+9c^2def^2-6c^3f^3\right)\right)x\sqrt{c\cdots}}{105d^2f^2\sqrt{e+fx^2}}$$

Mathematica [C] time = 1.13472, size = 373, normalized size = 0.69

$$-ie\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}(cf-de)\left(b\left(3c^2f^2-15cdef+8d^2e^2\right)-14adf(de-3cf)\right)\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{\frac{d}{c}}\right),\frac{cf}{de}\right)+fx\sqrt{\ }$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)*(c + d*x^2)^(3/2)*Sqrt[e + f*x^2],x]
```

```
[Out] (Sqrt[d/c]*f*x*(c + d*x^2)*(e + f*x^2)*(7*a*d*f*(6*c*f + d*(e + 3*f*x^2)) +
 b*(3*c^2*f^2 + 3*c*d*f*(3*e + 8*f*x^2) + d^2*(-4*e^2 + 3*e*f*x^2 + 15*f^2*
x^4))) + I*e*(7*a*d*f*(2*d^2*e^2 - 7*c*d*e*f - 3*c^2*f^2) + b*(-8*d^3*e^3 +
 19*c*d^2*e^2*f - 9*c^2*d*e*f^2 + 6*c^3*f^3))*Sqrt[1 + (d*x^2)/c]*Sqrt[1 +
(f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - I*e*(-(d*e) + c
*f)*(-14*a*d*f*(d*e - 3*c*f) + b*(8*d^2*e^2 - 15*c*d*e*f + 3*c^2*f^2))*Sqrt
[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)
/(d*e)])/(105*d*Sqrt[d/c]*f^3*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Maple [B] time = 0.039, size = 1332, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x^2+c)^(3/2)*(f*x^2+e)^(1/2),x)
```

```
[Out] 1/105*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)*(27*(-d/c)^(1/2)*x^5*b*c^2*d*f^4-6*((
d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1
/2))*b*c^3*e*f^3+14*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d
/c)^(1/2),(c*f/d/e)^(1/2))*a*d^3*e^3*f-4*(-d/c)^(1/2)*x*b*c*d^2*e^3*f+9*((d
*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/
2))*b*c^2*d*e^2*f^2+15*(-d/c)^(1/2)*x^9*b*d^3*f^4-18*((d*x^2+c)/c)^(1/2)*((
f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^2*d*e^2*f^2
+49*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d
/e)^(1/2))*a*c*d^2*e^2*f^2+23*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Ellip
ticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c*d^2*e^3*f+21*((d*x^2+c)/c)^(1/2)*(
(f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c^2*d*e*f^3+
42*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/
e)^(1/2))*a*c^2*d*e*f^3+3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF
(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^3*e*f^3-14*((d*x^2+c)/c)^(1/2)*((f*x^2
+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d^3*e^3*f+63*(-d/c
)^(1/2)*x^5*a*c*d^2*f^4+28*(-d/c)^(1/2)*x^5*a*d^3*e*f^3-8*((d*x^2+c)/c)^(1/
2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*d^3*e^4+
8*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e
)^(1/2))*b*d^3*e^4+51*(-d/c)^(1/2)*x^5*b*c*d^2*e*f^3+70*(-d/c)^(1/2)*x^3*a*
c*d^2*e*f^3+36*(-d/c)^(1/2)*x^3*b*c^2*d*e*f^3-19*((d*x^2+c)/c)^(1/2)*((f*x^
2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c*d^2*e^3*f+21*(-
d/c)^(1/2)*x^7*a*d^3*f^4+3*(-d/c)^(1/2)*x^3*b*c^3*f^4+9*(-d/c)^(1/2)*x*b*c^
2*d*e^2*f^2+8*(-d/c)^(1/2)*x^3*b*c*d^2*e^2*f^2+42*(-d/c)^(1/2)*x*a*c^2*d*e*
f^3+7*(-d/c)^(1/2)*x*a*c*d^2*e^2*f^2-(-d/c)^(1/2)*x^5*b*d^3*e^2*f^2+42*(-d/
c)^(1/2)*x^3*a*c^2*d*f^4+7*(-d/c)^(1/2)*x^3*a*d^3*e^2*f^2-4*(-d/c)^(1/2)*x^
3*b*d^3*e^3*f+3*(-d/c)^(1/2)*x*b*c^3*e*f^3+39*(-d/c)^(1/2)*x^7*b*c*d^2*f^4+
18*(-d/c)^(1/2)*x^7*b*d^3*e*f^3-56*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*
EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c*d^2*e^2*f^2)/d/(d*f*x^4+c*f*x
^2+d*e*x^2+c*e)/f^3/(-d/c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)(dx^2 + c)^{\frac{3}{2}} \sqrt{fx^2 + e}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^(3/2)*(f*x^2+e)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)*(d*x^2 + c)^(3/2)*sqrt(f*x^2 + e), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bdx^4 + (bc + ad)x^2 + ac\right)\sqrt{dx^2 + c}\sqrt{fx^2 + e}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^(3/2)*(f*x^2+e)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*d*x^4 + (b*c + a*d)*x^2 + a*c)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e),
 x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + bx^2)(c + dx^2)^{\frac{3}{2}} \sqrt{e + fx^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+c)**(3/2)*(f*x**2+e)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)*(c + d*x**2)**(3/2)*sqrt(e + f*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)(dx^2 + c)^{\frac{3}{2}} \sqrt{fx^2 + e}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^(3/2)*(f*x^2+e)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)*(d*x^2 + c)^(3/2)*sqrt(f*x^2 + e), x)
```

3.24 $\int \left(a+bx^2\right)\sqrt{c+dx^2}\sqrt{e+fx^2}\,dx$

Optimal. Leaf size=381

$$-\frac{e^{3/2}\sqrt{c+dx^2}(-10adf+bcf+bde)\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{15df^{3/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{x\sqrt{c+dx^2}\left(5adf(cf+de)-2b\left(c^2f^2-cdef+\right.\right.}{15d^2f\sqrt{e+fx^2}}$$

```
[Out] ((5*a*d*f*(d*e + c*f) - 2*b*(d^2*e^2 - c*d*e*f + c^2*f^2))*x*Sqrt[c + d*x^2
])/(15*d^2*f*Sqrt[e + f*x^2]) + ((b*d*e - 2*b*c*f + 5*a*d*f)*x*Sqrt[c + d*x
^2]*Sqrt[e + f*x^2])/(15*d*f) + (b*x*(c + d*x^2)^(3/2)*Sqrt[e + f*x^2])/(5*
d) - (Sqrt[e]*(5*a*d*f*(d*e + c*f) - 2*b*(d^2*e^2 - c*d*e*f + c^2*f^2))*Sqr
t[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(15*d
^2*f^(3/2)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) - (e^(3/2
)*(b*d*e + b*c*f - 10*a*d*f)*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/S
qrt[e]], 1 - (d*e)/(c*f)])/(15*d*f^(3/2)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2
))]*Sqrt[e + f*x^2])
```

Rubi [A] time = 0.377796, antiderivative size = 381, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {528, 531, 418, 492, 411}

$$\frac{x\sqrt{c+dx^2}\left(5adf(cf+de)-2b\left(c^2f^2-cdef+d^2e^2\right)\right)}{15d^2f\sqrt{e+fx^2}}-\frac{\sqrt{e}\sqrt{c+dx^2}\left(5adf(cf+de)-2b\left(c^2f^2-cdef+d^2e^2\right)\right)E\left(\tan\right.}{15d^2f^{3/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2],x]
```


```
[Out] ((5*a*d*f*(d*e + c*f) - 2*b*(d^2*e^2 - c*d*e*f + c^2*f^2))*x*Sqrt[c + d*x^2
])/(15*d^2*f*Sqrt[e + f*x^2]) + ((b*d*e - 2*b*c*f + 5*a*d*f)*x*Sqrt[c + d*x
^2]*Sqrt[e + f*x^2])/(15*d*f) + (b*x*(c + d*x^2)^(3/2)*Sqrt[e + f*x^2])/(5*
d) - (Sqrt[e]*(5*a*d*f*(d*e + c*f) - 2*b*(d^2*e^2 - c*d*e*f + c^2*f^2))*Sqr
t[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(15*d
^2*f^(3/2)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) - (e^(3/2
)*(b*d*e + b*c*f - 10*a*d*f)*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/S
qrt[e]], 1 - (d*e)/(c*f)])/(15*d*f^(3/2)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2
))]*Sqrt[e + f*x^2])
```


Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
```

```
d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \left(a+bx^2\right)\sqrt{c+dx^2}\sqrt{e+fx^2}\,dx = \frac{bx\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}}{5d}+\frac{\int \frac{\sqrt{c+dx^2}\left(-(bc-5ad)e+(bde-2bcf+5adf)x^2\right)}{\sqrt{e+fx^2}}\,dx}{5d}$$

$$= \frac{(bde-2bcf+5adf)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{15df}+\frac{bx\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}}{5d}+\frac{\int \frac{-ce(bde+bcf}{}}{}$$

$$= \frac{(bde-2bcf+5adf)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{15df}+\frac{bx\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}}{5d}-\frac{(ce(bde+bc}{}$$

$$= \frac{\left(5adf(de+cf)-2b\left(d^2e^2-cdef+c^2f^2\right)\right)x\sqrt{c+dx^2}}{15d^2f\sqrt{e+fx^2}}+\frac{(bde-2bcf+5adf)x\sqrt{c+}}{15df}$$

$$= \frac{\left(5adf(de+cf)-2b\left(d^2e^2-cdef+c^2f^2\right)\right)x\sqrt{c+dx^2}}{15d^2f\sqrt{e+fx^2}}+\frac{(bde-2bcf+5adf)x\sqrt{c+}}{15df}$$

Mathematica [C] time = 0.75238, size = 267, normalized size = 0.7

$$\frac{-ie\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}(cf-de)(5adf+bcf-2bde)\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{\frac{d}{c}}\right),\frac{cf}{de}\right)+ie\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\left(2b\left(c^2f^2-cde\right.\right.}{15df^2\sqrt{\frac{d}{c}}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2],x]
```

```
[Out] (Sqrt[d/c]*f*x*(c + d*x^2)*(e + f*x^2)*(b*c*f + 5*a*d*f + b*d*(e + 3*f*x^2)
) + I*e*(-5*a*d*f*(d*e + c*f) + 2*b*(d^2*e^2 - c*d*e*f + c^2*f^2))*Sqrt[1 +
```

```
(d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*
e)] - I*e*(-(d*e) + c*f)*(-2*b*d*e + b*c*f + 5*a*d*f)*Sqrt[1 + (d*x^2)/c]*S
qrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/(15*d*Sq
rt[d/c]*f^2*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Maple [B] time = 0.015, size = 865, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2),x)
```

```
[Out] 1/15*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)*(3*(-d/c)^(1/2)*x^7*b*d^2*f^3+5*(-d/c)
^(1/2)*x^5*a*d^2*f^3+4*(-d/c)^(1/2)*x^5*b*c*d*f^3+4*(-d/c)^(1/2)*x^5*b*d^2*
e*f^2+5*(-d/c)^(1/2)*x^3*a*c*d*f^3+5*(-d/c)^(1/2)*x^3*a*d^2*e*f^2+(-d/c)^(1
/2)*x^3*b*c^2*f^3+5*(-d/c)^(1/2)*x^3*b*c*d*e*f^2+(-d/c)^(1/2)*x^3*b*d^2*e^2
*f+5*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/
d/e)^(1/2))*a*c*d*e*f^2-5*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF
(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d^2*e^2*f+((d*x^2+c)/c)^(1/2)*((f*x^2+e)
/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^2*e*f^2-3*((d*x^2+c
)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*
c*d*e^2*f+2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2
),(c*f/d/e)^(1/2))*b*d^2*e^3+5*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Elli
pticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c*d*e*f^2+5*((d*x^2+c)/c)^(1/2)*((f
*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d^2*e^2*f-2*((
d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1
/2))*b*c^2*e*f^2+2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/
c)^(1/2),(c*f/d/e)^(1/2))*b*c*d*e^2*f-2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(
1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*d^2*e^3+5*(-d/c)^(1/2)*x*a
*c*d*e*f^2+(-d/c)^(1/2)*x*b*c^2*e*f^2+(-d/c)^(1/2)*x*b*c*d*e^2*f)/(d*f*x^4+
c*f*x^2+d*e*x^2+c*e)/d/f^2/(-d/c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2 + a\right)\sqrt{dx^2 + c}\sqrt{fx^2 + e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(bx^2 + a\right)\sqrt{dx^2 + c}\sqrt{fx^2 + e}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+bx^2\right)\sqrt{c+dx^2}\sqrt{e+fx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+c)**(1/2)*(f*x**2+e)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)*sqrt(c + d*x**2)*sqrt(e + f*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)\sqrt{dx^2+c}\sqrt{fx^2+e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e), x)
```

3.25 $\int \frac{(a+bx^2)\sqrt{e+fx^2}}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=283

$$-\frac{e^{3/2}\sqrt{c+dx^2}(bc-3ad)\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{3cd\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{x\sqrt{c+dx^2}(3adf-2bcf+bde)}{3d^2\sqrt{e+fx^2}}-\frac{\sqrt{e}\sqrt{c+dx^2}(3adf-2bc}{3d^2\sqrt{f}\sqrt{}}$$

```
[Out] ((b*d*e - 2*b*c*f + 3*a*d*f)*x*Sqrt[c + d*x^2])/(3*d^2*Sqrt[e + f*x^2]) + (
b*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(3*d) - (Sqrt[e]*(b*d*e - 2*b*c*f + 3*
a*d*f)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*
f)])/(3*d^2*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
- ((b*c - 3*a*d)*e^(3/2)*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[
e]], 1 - (d*e)/(c*f)])/(3*c*d*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]
*Sqrt[e + f*x^2])
```

Rubi [A] time = 0.181188, antiderivative size = 283, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {528, 531, 418, 492, 411}

$$\frac{x\sqrt{c+dx^2}(3adf-2bcf+bde)}{3d^2\sqrt{e+fx^2}}-\frac{\sqrt{e}\sqrt{c+dx^2}(3adf-2bcf+bde)E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3d^2\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{e^{3/2}\sqrt{c+dx^2}(bc-3ad)F}{3cd\sqrt{f}\sqrt{e+f}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*Sqrt[e + f*x^2])/Sqrt[c + d*x^2],x]
```

```
[Out] ((b*d*e - 2*b*c*f + 3*a*d*f)*x*Sqrt[c + d*x^2])/(3*d^2*Sqrt[e + f*x^2]) + (
b*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(3*d) - (Sqrt[e]*(b*d*e - 2*b*c*f + 3*
a*d*f)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*
f)])/(3*d^2*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
- ((b*c - 3*a*d)*e^(3/2)*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[
e]], 1 - (d*e)/(c*f)])/(3*c*d*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]
*Sqrt[e + f*x^2])
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)\sqrt{e+fx^2}}{\sqrt{c+dx^2}}\,dx &= \frac{bx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3d} + \frac{\int \frac{-(bc-3ad)e+(bde-2bcf+3adf)x^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{3d} \\
&= \frac{bx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3d} - \frac{((bc-3ad)e)\int \frac{1}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{3d} + \frac{(bde-2bcf+3adf)\int \frac{x^2}{\sqrt{c+dx^2}\sqrt{e+f\ldots}}}{3d} \\
&= \frac{(bde-2bcf+3adf)x\sqrt{c+dx^2}}{3d^2\sqrt{e+fx^2}} + \frac{bx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3d} - \frac{(bc-3ad)e^{3/2}\sqrt{c+dx^2}F\left(\tan^{-1}\left(\ldots\right.\right.}{3cd\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+f\ldots}} \\
&= \frac{(bde-2bcf+3adf)x\sqrt{c+dx^2}}{3d^2\sqrt{e+fx^2}} + \frac{bx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3d} - \frac{\sqrt{e}(bde-2bcf+3adf)\sqrt{c+dx^2}E\ldots}{3d^2\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{\ldots}}
\end{aligned}$$

Mathematica [C] time = 0.394991, size = 212, normalized size = 0.75

$$\frac{-ibe\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}(cf-de)\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{\frac{d}{c}}\right),\frac{cf}{de}\right)+ie\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}(-3adf+2bcf-bde)E\left(i\sinh^{-1}\left(\ldots\right.\right.}{3df\sqrt{\frac{d}{c}}\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*Sqrt[e + f*x^2])/Sqrt[c + d*x^2],x]
```

```
[Out] (b*Sqrt[d/c]*f*x*(c + d*x^2)*(e + f*x^2) + I*e*(-(b*d*e) + 2*b*c*f - 3*a*d*
f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x]
, (c*f)/(d*e)] - I*b*e*(-(d*e) + c*f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/
e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/(3*d*Sqrt[d/c]*f*Sqrt[c
+ d*x^2]*Sqrt[e + f*x^2])
```

Maple [A] time = 0.017, size = 394, normalized size = 1.4

$$\frac{1}{\left(3\,dfx^4+3\,cfx^2+3\,dex^2+3\,ce\right)df}\sqrt{fx^2+e}\sqrt{dx^2+c}\left(\sqrt{-\frac{d}{c}}x^5bdf^2+\sqrt{-\frac{d}{c}}x^3bcf^2+\sqrt{-\frac{d}{c}}x^3bdef+\sqrt{\frac{dx^2+c}{c}}\sqrt{f\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(f*x^2+e)^(1/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/3*(f*x^2+e)^(1/2)*(d*x^2+c)^(1/2)*((-d/c)^(1/2)*x^5*b*d*f^2+(-d/c)^(1/2)*
x^3*b*c*f^2+(-d/c)^(1/2)*x^3*b*d*e*f+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2
)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c*e*f-((d*x^2+c)/c)^(1/2)*((f
*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*d*e^2+3*((d*x^
2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))
*a*d*e*f-2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2)
,(c*f/d/e)^(1/2))*b*c*e*f+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE
(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*d*e^2+(-d/c)^(1/2)*x*b*c*e*f)/(d*f*x^4+c
*f*x^2+d*e*x^2+c*e)/d/(-d/c)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)\sqrt{fx^2+e}}{\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(f*x^2+e)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)*sqrt(f*x^2 + e)/sqrt(d*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(bx^2+a\right)\sqrt{fx^2+e}}{\sqrt{dx^2+c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(f*x^2+e)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)*sqrt(f*x^2 + e)/sqrt(d*x^2 + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^2\right)\sqrt{e+fx^2}}{\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(f*x**2+e)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)*sqrt(e + f*x**2)/sqrt(c + d*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)\sqrt{fx^2 + e}}{\sqrt{dx^2 + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(f*x^2+e)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)*sqrt(f*x^2 + e)/sqrt(d*x^2 + c), x)
```

3.26 $\int \frac{(a+bx^2)\sqrt{e+fx^2}}{(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=271

$$\frac{be^{3/2}\sqrt{c+dx^2}\,\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{cd\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{fx\sqrt{c+dx^2}(2bc-ad)}{cd^2\sqrt{e+fx^2}}-\frac{\sqrt{e}\sqrt{f}\sqrt{c+dx^2}(2bc-ad)E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1\right.}{cd^2\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

```
[Out] ((2*b*c - a*d)*f*x*Sqrt[c + d*x^2])/(c*d^2*Sqrt[e + f*x^2]) - ((b*c - a*d)*
x*Sqrt[e + f*x^2])/(c*d*Sqrt[c + d*x^2]) - ((2*b*c - a*d)*Sqrt[e]*Sqrt[f]*S
qrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(c*
d^2*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (b*e^(3/2)*Sqr
t[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(c*d*
Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rubi [A] time = 0.17057, antiderivative size = 271, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {526, 531, 418, 492, 411}

$$\frac{fx\sqrt{c+dx^2}(2bc-ad)}{cd^2\sqrt{e+fx^2}}-\frac{\sqrt{e}\sqrt{f}\sqrt{c+dx^2}(2bc-ad)E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{cd^2\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{x\sqrt{e+fx^2}(bc-ad)}{cd\sqrt{c+dx^2}}+\frac{be^{3/2}\sqrt{c+dx^2}F\left(\text{t}\ldots\right.}{cd\sqrt{f}\sqrt{e+\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*Sqrt[e + f*x^2])/(c + d*x^2)^(3/2),x]
```



```
[Out] ((2*b*c - a*d)*f*x*Sqrt[c + d*x^2])/(c*d^2*Sqrt[e + f*x^2]) - ((b*c - a*d)*
x*Sqrt[e + f*x^2])/(c*d*Sqrt[c + d*x^2]) - ((2*b*c - a*d)*Sqrt[e]*Sqrt[f]*S
qrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(c*
d^2*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (b*e^(3/2)*Sqr
t[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(c*d*
Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```


Rule 526

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^q)/(a*b*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p
 + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p +
 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{(a+bx^2)\sqrt{e+fx^2}}{(c+dx^2)^{3/2}}\,dx = -\frac{(bc-ad)x\sqrt{e+fx^2}}{cd\sqrt{c+dx^2}} - \frac{\int \frac{-bce-(2bc-ad)fx^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{cd}$$

$$= -\frac{(bc-ad)x\sqrt{e+fx^2}}{cd\sqrt{c+dx^2}} + \frac{(be)\int \frac{1}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{d} + \frac{((2bc-ad)f)\int \frac{x^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{cd}$$

$$= \frac{(2bc-ad)fx\sqrt{c+dx^2}}{cd^2\sqrt{e+fx^2}} - \frac{(bc-ad)x\sqrt{e+fx^2}}{cd\sqrt{c+dx^2}} + \frac{be^{3/2}\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{cd\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} - \frac{((2b}{}$$

$$= \frac{(2bc-ad)fx\sqrt{c+dx^2}}{cd^2\sqrt{e+fx^2}} - \frac{(bc-ad)x\sqrt{e+fx^2}}{cd\sqrt{c+dx^2}} - \frac{(2bc-ad)\sqrt{e}\sqrt{f}\sqrt{c+dx^2}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\right.}{cd^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

Mathematica [C] time = 0.348876, size = 192, normalized size = 0.71

$$\frac{-(bc-ad)\left(x\sqrt{\frac{d}{c}}(e+fx^2) - ie\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\,\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{\frac{d}{c}}\right),\frac{cf}{de}\right)\right) - ie\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}(2bc-ad)E\left(i\,\text{si}\right.}{c^2\left(\frac{d}{c}\right)^{3/2}\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*Sqrt[e + f*x^2])/(c + d*x^2)^(3/2),x]
```

```
[Out] ((-I)*(2*b*c - a*d)*e*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*A
rcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - (b*c - a*d)*(Sqrt[d/c]*x*(e + f*x^2) -
I*e*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x
], (c*f)/(d*e)]))/(c^2*(d/c)^(3/2)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Maple [A] time = 0.038, size = 328, normalized size = 1.2

$$\frac{1}{d\left(dfx^4+cfx^2+dex^2+ce\right)c}\sqrt{fx^2+e}\sqrt{dx^2+c}\left(x^3adf\sqrt{-\frac{d}{c}}-x^3bcf\sqrt{-\frac{d}{c}}+\text{EllipticF}\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)ade\sqrt{\frac{dx^2+}{c}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(f*x^2+e)^(1/2)/(d*x^2+c)^(3/2),x)
```

```
[Out] (f*x^2+e)^(1/2)*(d*x^2+c)^(1/2)*(x^3*a*d*f*(-d/c)^(1/2)-x^3*b*c*f*(-d/c)^(1
/2)+EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d*e*((d*x^2+c)/c)^(1/2)*((f
*x^2+e)/e)^(1/2)-EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c*e*((d*x^2+c)
/c)^(1/2)*((f*x^2+e)/e)^(1/2)-EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d
*e*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+2*EllipticE(x*(-d/c)^(1/2),(c*f/
d/e)^(1/2))*b*c*e*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+x*a*d*e*(-d/c)^(1
/2)-x*b*c*e*(-d/c)^(1/2))/d/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)/c/(-d/c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)\sqrt{fx^2+e}}{\left(dx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(f*x^2+e)^(1/2)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)*sqrt(f*x^2 + e)/(d*x^2 + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)\sqrt{dx^2+c}\sqrt{fx^2+e}}{d^2x^4+2\,cdx^2+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(f*x^2+e)^(1/2)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(d^2*x^4 + 2*c*d*x^2 +
 c^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^2\right)\sqrt{e+fx^2}}{\left(c+dx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(f*x**2+e)**(1/2)/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral((a + b*x**2)*sqrt(e + f*x**2)/(c + d*x**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)\sqrt{fx^2+e}}{(dx^2+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(f*x^2+e)^(1/2)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)*sqrt(f*x^2 + e)/(d*x^2 + c)^(3/2), x)
```

3.27 $\int \frac{(a+bx^2)\sqrt{e+fx^2}}{(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=274

$$\frac{e^{3/2}\sqrt{f}\sqrt{c+dx^2}(bc-ad)\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{3c^2d\sqrt{e+fx^2}(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{\sqrt{e+fx^2}(de(2ad+bc)-cf(ad+2bc))E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\right.}{3c^{3/2}d^{3/2}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$

```
[Out] -((b*c - a*d)*x*Sqrt[e + f*x^2])/(3*c*d*(c + d*x^2)^(3/2)) + ((d*(b*c + 2*a
*d)*e - c*(2*b*c + a*d)*f)*Sqrt[e + f*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqr
t[c]], 1 - (c*f)/(d*e)])/(3*c^(3/2)*d^(3/2)*(d*e - c*f)*Sqrt[c + d*x^2]*Sqr
t[(c*(e + f*x^2))/(e*(c + d*x^2))]) + ((b*c - a*d)*e^(3/2)*Sqrt[f]*Sqrt[c +
 d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(3*c^2*d*(
d*e - c*f)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rubi [A] time = 0.20503, antiderivative size = 274, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {526, 525, 418, 411}

$$\frac{\sqrt{e+fx^2}(de(2ad+bc)-cf(ad+2bc))E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{3c^{3/2}d^{3/2}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}+\frac{e^{3/2}\sqrt{f}\sqrt{c+dx^2}(bc-ad)F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3c^2d\sqrt{e+fx^2}(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{x}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*Sqrt[e + f*x^2])/(c + d*x^2)^(5/2),x]
```


```
[Out] -((b*c - a*d)*x*Sqrt[e + f*x^2])/(3*c*d*(c + d*x^2)^(3/2)) + ((d*(b*c + 2*a
*d)*e - c*(2*b*c + a*d)*f)*Sqrt[e + f*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqr
t[c]], 1 - (c*f)/(d*e)])/(3*c^(3/2)*d^(3/2)*(d*e - c*f)*Sqrt[c + d*x^2]*Sqr
t[(c*(e + f*x^2))/(e*(c + d*x^2))]) + ((b*c - a*d)*e^(3/2)*Sqrt[f]*Sqrt[c +
 d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(3*c^2*d*(
d*e - c*f)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```


Rule 526

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^q)/(a*b*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p +
 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 525

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{(a+bx^2)\sqrt{e+fx^2}}{(c+dx^2)^{5/2}}\,dx = -\frac{(bc-ad)x\sqrt{e+fx^2}}{3cd(c+dx^2)^{3/2}} - \frac{\int \frac{-(bc+2ad)e-(2bc+ad)fx^2}{(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx}{3cd}$$

$$= -\frac{(bc-ad)x\sqrt{e+fx^2}}{3cd(c+dx^2)^{3/2}} + \frac{((bc-ad)ef)\int \frac{1}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{3cd(de-cf)} + \frac{(d(bc+2ad)e-c(2bc+ad)f)\int \ldots}{3cd(de-cf)}$$

$$= -\frac{(bc-ad)x\sqrt{e+fx^2}}{3cd(c+dx^2)^{3/2}} + \frac{(d(bc+2ad)e-c(2bc+ad)f)\sqrt{e+fx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{3c^{3/2}d^{3/2}(de-cf)\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}} + \ldots$$

Mathematica [C] time = 0.973162, size = 297, normalized size = 1.08

$$\frac{-ie(c+dx^2)\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}(2ad+bc)(cf-de)\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{\frac{d}{c}}\right),\frac{cf}{de}\right)+x\sqrt{\frac{d}{c}}(e+fx^2)\left(ad\left(2c^2f-3cde+c\ldots\right.\right.}{3c^3\left(\frac{d}{c}\right)^{3/2}(c+\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*Sqrt[e + f*x^2])/(c + d*x^2)^(5/2),x]
```

```
[Out] (Sqrt[d/c]*x*(e + f*x^2)*(a*d*(-3*c*d*e + 2*c^2*f - 2*d^2*e*x^2 + c*d*f*x^2
) + b*c*(c^2*f - d^2*e*x^2 + 2*c*d*f*x^2)) + I*e*(a*d*(-2*d*e + c*f) + b*c*
(-(d*e) + 2*c*f))*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*Ellip
ticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - I*(b*c + 2*a*d)*e*(-(d*e) + c*f
)*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[S
qrt[d/c]*x], (c*f)/(d*e)])/(3*c^3*(d/c)^(3/2)*(-(d*e) + c*f)*(c + d*x^2)^(3
/2)*Sqrt[e + f*x^2])
```

Maple [B] time = 0.042, size = 1236, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(f*x^2+e)^(1/2)/(d*x^2+c)^(5/2),x)
```

```
[Out] 1/3*(2*x*a*c^2*d*e*f*(-d/c)^(1/2)+EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))
*b*c^3*e*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-EllipticE(x*(-d/c)^(1/2)
,(c*f/d/e)^(1/2))*a*c^2*d*e*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-2*x^3
*a*c*d^2*e*f*(-d/c)^(1/2)+2*x^3*b*c^2*d*e*f*(-d/c)^(1/2)-3*x*a*c*d^2*e^2*(-
d/c)^(1/2)+x*b*c^3*e*f*(-d/c)^(1/2)+x^5*a*c*d^2*f^2*(-d/c)^(1/2)-2*x^5*a*d^
3*e*f*(-d/c)^(1/2)+2*x^5*b*c^2*d*f^2*(-d/c)^(1/2)+2*x^3*a*c^2*d*f^2*(-d/c)^
(1/2)-2*x^3*a*d^3*e^2*(-d/c)^(1/2)-x^3*b*c*d^2*e^2*(-d/c)^(1/2)+EllipticF(x
*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*b*c^2*d*e*f*((d*x^2+c)/c)^(1/2)*((f*x^2+
e)/e)^(1/2)-EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a*c*d^2*e*f*((d*x
^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-x^5*b*c*d^2*e*f*(-d/c)^(1/2)+x^3*b*c^3*f
^2*(-d/c)^(1/2)-2*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a*d^3*e^2*(
(d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-2*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)
^(1/2))*a*c*d^2*e^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+2*EllipticE(x*(
-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a*d^3*e^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e
)^(1/2)-EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^2*d*e^2*((d*x^2+c)/c)
^(1/2)*((f*x^2+e)/e)^(1/2)+2*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c*
d^2*e^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-2*EllipticE(x*(-d/c)^(1/2),
(c*f/d/e)^(1/2))*b*c^3*e*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+Elliptic
E(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^2*d*e^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e
)/e)^(1/2)-EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*b*c*d^2*e^2*((d*x^
2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))
*x^2*b*c*d^2*e^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+2*EllipticF(x*(-d/
c)^(1/2),(c*f/d/e)^(1/2))*a*c^2*d*e*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/
2)-2*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*b*c^2*d*e*f*((d*x^2+c)/c
)^(1/2)*((f*x^2+e)/e)^(1/2)+2*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2
*a*c*d^2*e*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2))/(f*x^2+e)^(1/2)/(-d/c
)^(1/2)/(c*f-d*e)/c^2/d/(d*x^2+c)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)\sqrt{fx^2+e}}{(dx^2+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(f*x^2+e)^(1/2)/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)*sqrt(f*x^2 + e)/(d*x^2 + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2+a)\sqrt{dx^2+c}\sqrt{fx^2+e}}{d^3x^6+3\,cd^2x^4+3\,c^2dx^2+c^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(f*x^2+e)^(1/2)/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(d^3*x^6 + 3*c*d^2*x^4
 + 3*c^2*d*x^2 + c^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + bx^2)\sqrt{e + fx^2}}{(c + dx^2)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(f*x**2+e)**(1/2)/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral((a + b*x**2)*sqrt(e + f*x**2)/(c + d*x**2)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)\sqrt{fx^2 + e}}{(dx^2 + c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(f*x^2+e)^(1/2)/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)*sqrt(f*x^2 + e)/(d*x^2 + c)^(5/2), x)
```

3.28 $\int \frac{(a+bx^2)\sqrt{e+fx^2}}{(c+dx^2)^{7/2}}\,dx$

Optimal. Leaf size=385

$$-\frac{e^{3/2}\sqrt{f}\sqrt{c+dx^2}(2ad(2de-3cf)+bc(cf+de))\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{15c^3d\sqrt{e+fx^2}(de-cf)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{\sqrt{e+fx^2}\left(ad\left(3c^2f^2-13cdef+8\right.\right.}{15c^{5/2}d^{3}}$$

```
[Out] -((b*c - a*d)*x*Sqrt[e + f*x^2])/(5*c*d*(c + d*x^2)^(5/2)) + ((a*d*(4*d*e -
 3*c*f) + b*c*(d*e - 2*c*f))*x*Sqrt[e + f*x^2])/(15*c^2*d*(d*e - c*f)*(c +
d*x^2)^(3/2)) + ((2*b*c*(d^2*e^2 - c*d*e*f + c^2*f^2) + a*d*(8*d^2*e^2 - 13
*c*d*e*f + 3*c^2*f^2))*Sqrt[e + f*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]
], 1 - (c*f)/(d*e)])/(15*c^(5/2)*d^(3/2)*(d*e - c*f)^2*Sqrt[c + d*x^2]*Sqrt
[(c*(e + f*x^2))/(e*(c + d*x^2))]) - (e^(3/2)*Sqrt[f]*(2*a*d*(2*d*e - 3*c*f
) + b*c*(d*e + c*f))*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]],
 1 - (d*e)/(c*f)])/(15*c^3*d*(d*e - c*f)^2*Sqrt[(e*(c + d*x^2))/(c*(e + f*x
^2))]*Sqrt[e + f*x^2])
```

Rubi [A] time = 0.380205, antiderivative size = 385, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {526, 527, 525, 418, 411}

$$\frac{\sqrt{e+fx^2}\left(ad\left(3c^2f^2-13cdef+8d^2e^2\right)+2bc\left(c^2f^2-cdef+d^2e^2\right)\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{15c^{5/2}d^{3/2}\sqrt{c+dx^2}(de-cf)^2\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}-\frac{e^{3/2}\sqrt{f}\sqrt{c+dx^2}(2ad(2}{15c^3d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*Sqrt[e + f*x^2])/(c + d*x^2)^(7/2),x]
```


```
[Out] -((b*c - a*d)*x*Sqrt[e + f*x^2])/(5*c*d*(c + d*x^2)^(5/2)) + ((a*d*(4*d*e -
 3*c*f) + b*c*(d*e - 2*c*f))*x*Sqrt[e + f*x^2])/(15*c^2*d*(d*e - c*f)*(c +
d*x^2)^(3/2)) + ((2*b*c*(d^2*e^2 - c*d*e*f + c^2*f^2) + a*d*(8*d^2*e^2 - 13
*c*d*e*f + 3*c^2*f^2))*Sqrt[e + f*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]
], 1 - (c*f)/(d*e)])/(15*c^(5/2)*d^(3/2)*(d*e - c*f)^2*Sqrt[c + d*x^2]*Sqrt
[(c*(e + f*x^2))/(e*(c + d*x^2))]) - (e^(3/2)*Sqrt[f]*(2*a*d*(2*d*e - 3*c*f
) + b*c*(d*e + c*f))*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]],
 1 - (d*e)/(c*f)])/(15*c^3*d*(d*e - c*f)^2*Sqrt[(e*(c + d*x^2))/(c*(e + f*x
^2))]*Sqrt[e + f*x^2])
```


Rule *526*

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^q)/(a*b*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p +
 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule *527*

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
```

```
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 525

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)\sqrt{e+fx^2}}{\left(c+dx^2\right)^{7/2}}\,dx = -\frac{(bc-ad)x\sqrt{e+fx^2}}{5cd\left(c+dx^2\right)^{5/2}} - \frac{\int \frac{-(bc+4ad)e-(2bc+3ad)fx^2}{\left(c+dx^2\right)^{5/2}\sqrt{e+fx^2}}\,dx}{5cd}$$

$$= -\frac{(bc-ad)x\sqrt{e+fx^2}}{5cd\left(c+dx^2\right)^{5/2}} + \frac{(ad(4de-3cf)+bc(de-2cf))x\sqrt{e+fx^2}}{15c^2d(de-cf)\left(c+dx^2\right)^{3/2}} + \frac{\int \frac{e(ad(8de-9cf)+bc(2de-cf))}{\left(c+dx^2\right.}}{15c^2d}$$

$$= -\frac{(bc-ad)x\sqrt{e+fx^2}}{5cd\left(c+dx^2\right)^{5/2}} + \frac{(ad(4de-3cf)+bc(de-2cf))x\sqrt{e+fx^2}}{15c^2d(de-cf)\left(c+dx^2\right)^{3/2}} - \frac{(ef(2ad(2de-3cf)+b}{15c^2}$$

$$= -\frac{(bc-ad)x\sqrt{e+fx^2}}{5cd\left(c+dx^2\right)^{5/2}} + \frac{(ad(4de-3cf)+bc(de-2cf))x\sqrt{e+fx^2}}{15c^2d(de-cf)\left(c+dx^2\right)^{3/2}} + \frac{\left(2bc\left(d^2e^2-cdef+c^2f\right.\right.}{}$$

Mathematica [C] time = 1.26903, size = 379, normalized size = 0.98

$$\frac{-x\sqrt{\frac{d}{c}}\left(e+fx^2\right)\left(-\left(c+dx^2\right)^2\left(ad\left(3c^2f^2-13cdef+8d^2e^2\right)+2bc\left(c^2f^2-cdef+d^2e^2\right)\right)+3c^2(bc-ad)(de-cf)^2-c\left(c+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*Sqrt[e + f*x^2])/(c + d*x^2)^(7/2),x]
```

```
[Out] (-(Sqrt[d/c]*x*(e + f*x^2)*(3*c^2*(b*c - a*d)*(d*e - c*f)^2 - c*(d*e - c*f)
*(a*d*(4*d*e - 3*c*f) + b*c*(d*e - 2*c*f))*(c + d*x^2) - (2*b*c*(d^2*e^2 -
c*d*e*f + c^2*f^2) + a*d*(8*d^2*e^2 - 13*c*d*e*f + 3*c^2*f^2))*(c + d*x^2)^
2)) + I*e*(c + d*x^2)^2*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*((2*b*c*(d^
2*e^2 - c*d*e*f + c^2*f^2) + a*d*(8*d^2*e^2 - 13*c*d*e*f + 3*c^2*f^2))*Elli
pticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - (-(d*e) + c*f)*(b*c*(-2*d*e +
c*f) + a*d*(-8*d*e + 9*c*f))*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]
))/(15*c^4*(d/c)^(3/2)*(d*e - c*f)^2*(c + d*x^2)^(5/2)*Sqrt[e + f*x^2])
```

Maple [B] time = 0.055, size = 2856, normalized size = 7.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(f*x^2+e)^(1/2)/(d*x^2+c)^(7/2),x)
```

```
[Out] 1/15*(2*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^4*d*e^2*f*((d*x^2+c)/
c)^(1/2)*((f*x^2+e)/e)^(1/2)+16*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x
^2*a*c*d^4*e^3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+4*EllipticF(x*(-d/c)
^(1/2),(c*f/d/e)^(1/2))*x^2*b*c^2*d^3*e^3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)
^(1/2)-16*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a*c*d^4*e^3*((d*x^2
+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+2*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2)
)*x^4*b*c*d^4*e^3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-2*EllipticE(x*(-d
/c)^(1/2),(c*f/d/e)^(1/2))*x^4*b*c*d^4*e^3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e
)^(1/2)-3*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^4*d*e^2*f*((d*x^2+c
)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-3*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*
a*c^4*d*e*f^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+13*EllipticE(x*(-d/c)
^(1/2),(c*f/d/e)^(1/2))*a*c^3*d^2*e^2*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(
1/2)+9*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c^4*d*e*f^2*((d*x^2+c)/c
)^(1/2)*((f*x^2+e)/e)^(1/2)-17*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*
c^3*d^2*e^2*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-3*EllipticF(x*(-d/c)^
(1/2),(c*f/d/e)^(1/2))*x^4*b*c^2*d^3*e^2*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e
)^(1/2)-3*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^4*a*c^2*d^3*e*f^2*((d
*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+13*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^
(1/2))*x^4*a*c*d^4*e^2*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+8*x^5*a*d^
5*e^3*(-d/c)^(1/2)+9*x^5*a*c^3*d^2*f^3*(-d/c)^(1/2)+2*x^7*b*c^3*d^2*f^3*(-d
/c)^(1/2)-4*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*b*c^2*d^3*e^3*((d
*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+6*x^5*b*c^4*d*f^3*(-d/c)^(1/2)+2*x^5*b
*c*d^4*e^3*(-d/c)^(1/2)+9*x^3*a*c^4*d*f^3*(-d/c)^(1/2)+20*x^3*a*c*d^4*e^3*(
-d/c)^(1/2)+5*x^3*b*c^2*d^3*e^3*(-d/c)^(1/2)+15*x*a*c^2*d^3*e^3*(-d/c)^(1/2
)+x*b*c^5*e*f^2*(-d/c)^(1/2)+3*x^7*a*c^2*d^3*f^3*(-d/c)^(1/2)+8*x^7*a*d^5*e
^2*f*(-d/c)^(1/2)+2*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^3*d^2*e^3
*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-8*EllipticE(x*(-d/c)^(1/2),(c*f/d/
e)^(1/2))*a*c^2*d^3*e^3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-2*EllipticE
(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^5*e*f^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)
/e)^(1/2)-2*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^3*d^2*e^3*((d*x^2
+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+2*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2)
)*x^2*b*c^4*d*e*f^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-6*EllipticE(x*(
-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a*c^3*d^2*e*f^2*((d*x^2+c)/c)^(1/2)*((f*x^
2+e)/e)^(1/2)+26*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a*c^2*d^3*e^
2*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+EllipticF(x*(-d/c)^(1/2),(c*f/d
/e)^(1/2))*b*c^5*e*f^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-13*x^7*a*c*d
^4*e*f^2*(-d/c)^(1/2)-2*x^7*b*c^2*d^3*e*f^2*(-d/c)^(1/2)+2*x^7*b*c*d^4*e^2*
f*(-d/c)^(1/2)-30*x^5*a*c^2*d^3*e*f^2*(-d/c)^(1/2)+7*x^5*a*c*d^4*e^2*f*(-d/
c)^(1/2)-5*x^5*b*c^3*d^2*e*f^2*(-d/c)^(1/2)+3*x^5*b*c^2*d^3*e^2*f*(-d/c)^(1
/2)-17*x^3*a*c^3*d^2*e*f^2*(-d/c)^(1/2)+9*x*a*c^4*d*e*f^2*(-d/c)^(1/2)-26*x
```

```
*a*c^3*d^2*e^2*f*(-d/c)^(1/2)+x*b*c^4*d*e^2*f*(-d/c)^(1/2)+8*EllipticF(x*(-
d/c)^(1/2),(c*f/d/e)^(1/2))*x^4*a*d^5*e^3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)
^(1/2)-18*x^3*a*c^2*d^3*e^2*f*(-d/c)^(1/2)+7*x^3*b*c^4*d*e*f^2*(-d/c)^(1/2)
-7*x^3*b*c^3*d^2*e^2*f*(-d/c)^(1/2)+x^3*b*c^5*f^3*(-d/c)^(1/2)-8*EllipticE(
x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^4*a*d^5*e^3*((d*x^2+c)/c)^(1/2)*((f*x^2+e
)/e)^(1/2)+8*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c^2*d^3*e^3*((d*x^
2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-2*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2
))*x^4*b*c^3*d^2*e*f^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+2*EllipticE(
x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^4*b*c^2*d^3*e^2*f*((d*x^2+c)/c)^(1/2)*((f
*x^2+e)/e)^(1/2)+18*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a*c^3*d^2
*e*f^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-34*EllipticF(x*(-d/c)^(1/2),
(c*f/d/e)^(1/2))*x^2*a*c^2*d^3*e^2*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2
)-4*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*b*c^4*d*e*f^2*((d*x^2+c)/
c)^(1/2)*((f*x^2+e)/e)^(1/2)+4*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^
2*b*c^3*d^2*e^2*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+9*EllipticF(x*(-d
/c)^(1/2),(c*f/d/e)^(1/2))*x^4*a*c^2*d^3*e*f^2*((d*x^2+c)/c)^(1/2)*((f*x^2+
e)/e)^(1/2)-17*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^4*a*c*d^4*e^2*f*
((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^
(1/2))*x^4*b*c^3*d^2*e*f^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-6*Ellipt
icF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*b*c^3*d^2*e^2*f*((d*x^2+c)/c)^(1/2)
*((f*x^2+e)/e)^(1/2))/(f*x^2+e)^(1/2)/(-d/c)^(1/2)/(c*f-d*e)^2/c^3/d/(d*x^2
+c)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)\sqrt{fx^2+e}}{(dx^2+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(f*x^2+e)^(1/2)/(d*x^2+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)*sqrt(f*x^2 + e)/(d*x^2 + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2+a)\sqrt{dx^2+c}\sqrt{fx^2+e}}{d^4x^8+4\,cd^3x^6+6\,c^2d^2x^4+4\,c^3dx^2+c^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(f*x^2+e)^(1/2)/(d*x^2+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(d^4*x^8 + 4*c*d^3*x^6
 + 6*c^2*d^2*x^4 + 4*c^3*d*x^2 + c^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(f*x**2+e)**(1/2)/(d*x**2+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)\sqrt{fx^2 + e}}{(dx^2 + c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(f*x^2+e)^(1/2)/(d*x^2+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)*sqrt(f*x^2 + e)/(d*x^2 + c)^(7/2), x)
```

3.29 $\int \left(a+bx^2\right)\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}\,dx$

Optimal. Leaf size=543

$$\frac{e^{3/2}\sqrt{c+dx^2}\left(7adf(9de-cf)-b\left(-4c^2f^2+9cdef+3d^2e^2\right)\right)\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{105d^2f^{3/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}\left(14a\right.}{}$$

```
[Out] ((7*a*d*f*(3*d^2*e^2 + 7*c*d*e*f - 2*c^2*f^2) - b*(6*d^3*e^3 - 9*c*d^2*e^2*
f + 19*c^2*d*e*f^2 - 8*c^3*f^3))*x*Sqrt[c + d*x^2])/(105*d^3*f*Sqrt[e + f*x
^2]) + ((14*a*d*f*(3*d*e - c*f) + b*(3*d^2*e^2 - 15*c*d*e*f + 8*c^2*f^2))*x
*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(105*d^2*f) + ((3*b*d*e - 4*b*c*f + 7*a*d
*f)*x*(c + d*x^2)^(3/2)*Sqrt[e + f*x^2])/(35*d^2) + (b*x*(c + d*x^2)^(3/2)*
(e + f*x^2)^(3/2))/(7*d) - (Sqrt[e]*(7*a*d*f*(3*d^2*e^2 + 7*c*d*e*f - 2*c^2
*f^2) - b*(6*d^3*e^3 - 9*c*d^2*e^2*f + 19*c^2*d*e*f^2 - 8*c^3*f^3))*Sqrt[c
+ d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(105*d^3*
f^(3/2)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (e^(3/2)*(
7*a*d*f*(9*d*e - c*f) - b*(3*d^2*e^2 + 9*c*d*e*f - 4*c^2*f^2))*Sqrt[c + d*x
^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(105*d^2*f^(3/
2)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rubi [A] time = 0.64488, antiderivative size = 543, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {528, 531, 418, 492, 411}

$$\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}\left(14adf(3de-cf)+b\left(8c^2f^2-15cdef+3d^2e^2\right)\right)}{105d^2f}+\frac{x\sqrt{c+dx^2}\left(7adf\left(-2c^2f^2+7cdef+3d^2e^2\right)-b\left(\right.\right.}{105d^3f\sqrt{e+f}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)*Sqrt[c + d*x^2]*(e + f*x^2)^(3/2),x]
```


```
[Out] ((7*a*d*f*(3*d^2*e^2 + 7*c*d*e*f - 2*c^2*f^2) - b*(6*d^3*e^3 - 9*c*d^2*e^2*
f + 19*c^2*d*e*f^2 - 8*c^3*f^3))*x*Sqrt[c + d*x^2])/(105*d^3*f*Sqrt[e + f*x
^2]) + ((14*a*d*f*(3*d*e - c*f) + b*(3*d^2*e^2 - 15*c*d*e*f + 8*c^2*f^2))*x
*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(105*d^2*f) + ((3*b*d*e - 4*b*c*f + 7*a*d
*f)*x*(c + d*x^2)^(3/2)*Sqrt[e + f*x^2])/(35*d^2) + (b*x*(c + d*x^2)^(3/2)*
(e + f*x^2)^(3/2))/(7*d) - (Sqrt[e]*(7*a*d*f*(3*d^2*e^2 + 7*c*d*e*f - 2*c^2
*f^2) - b*(6*d^3*e^3 - 9*c*d^2*e^2*f + 19*c^2*d*e*f^2 - 8*c^3*f^3))*Sqrt[c
+ d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(105*d^3*
f^(3/2)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (e^(3/2)*(
7*a*d*f*(9*d*e - c*f) - b*(3*d^2*e^2 + 9*c*d*e*f - 4*c^2*f^2))*Sqrt[c + d*x
^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(105*d^2*f^(3/
2)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```


Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
```

```
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \left(a+bx^2\right)\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}\,dx = \frac{bx\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^{3/2}}{7d} + \frac{\int \sqrt{c+dx^2}\sqrt{e+fx^2}\left(-(bc-7ad)e+(3bde-4\right.}{7d}$$

$$= \frac{(3bde-4bcf+7adf)x\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}}{35d^2} + \frac{bx\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^{3/2}}{7d} +$$

$$= \frac{\left(14adf(3de-cf)+b\left(3d^2e^2-15cdef+8c^2f^2\right)\right)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{105d^2f} + \frac{(3bde}{}$$

$$= \frac{\left(14adf(3de-cf)+b\left(3d^2e^2-15cdef+8c^2f^2\right)\right)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{105d^2f} + \frac{(3bde}{}$$

$$= \frac{\left(7adf\left(3d^2e^2+7cdef-2c^2f^2\right)-b\left(6d^3e^3-9cd^2e^2f+19c^2def^2-8c^3f^3\right)\right)x}{105d^3f\sqrt{e+fx^2}}$$

$$= \frac{\left(7adf\left(3d^2e^2+7cdef-2c^2f^2\right)-b\left(6d^3e^3-9cd^2e^2f+19c^2def^2-8c^3f^3\right)\right)x}{105d^3f\sqrt{e+fx^2}}$$

Mathematica [C] time = 1.14135, size = 372, normalized size = 0.69

$$\frac{ie\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}(cf-de)\left(b\left(4c^2f^2-6cdef+6d^2e^2\right)-7adf(cf+3de)\right)\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{\frac{d}{c}}\right),\frac{cf}{de}\right)+fx\left(-\sqrt{\frac{d}{c}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)*Sqrt[c + d*x^2]*(e + f*x^2)^(3/2),x]
```

```
[Out] (-(Sqrt[d/c]*f*x*(c + d*x^2)*(e + f*x^2)*(4*b*c^2*f^2 - 3*b*c*d*f*(3*e + f*
x^2) - 7*a*d*f*(6*d*e + c*f + 3*d*f*x^2) - 3*b*d^2*(e^2 + 8*e*f*x^2 + 5*f^2
*x^4))) - I*e*(7*a*d*f*(3*d^2*e^2 + 7*c*d*e*f - 2*c^2*f^2) + b*(-6*d^3*e^3
+ 9*c*d^2*e^2*f - 19*c^2*d*e*f^2 + 8*c^3*f^3))*Sqrt[1 + (d*x^2)/c]*Sqrt[1 +
 (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + I*e*(-(d*e) +
c*f)*(-7*a*d*f*(3*d*e + c*f) + b*(6*d^2*e^2 - 6*c*d*e*f + 4*c^2*f^2))*Sqrt[
1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/
(d*e)])/(105*c^2*(d/c)^(5/2)*f^2*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Maple [B] time = 0.016, size = 1332, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x^2+c)^(1/2)*(f*x^2+e)^(3/2),x)
```

```
[Out] 1/105*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)*(-(-d/c)^(1/2)*x^5*b*c^2*d*f^4+8*((d*
x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2
))*b*c^3*e*f^3-21*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c
)^(1/2),(c*f/d/e)^(1/2))*a*d^3*e^3*f+3*(-d/c)^(1/2)*x*b*c*d^2*e^3*f-19*((d*
x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2
))*b*c^2*d*e^2*f^2+15*(-d/c)^(1/2)*x^9*b*d^3*f^4+10*((d*x^2+c)/c)^(1/2)*((f
*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^2*d*e^2*f^2+
49*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/
e)^(1/2))*a*c*d^2*e^2*f^2-12*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Ellipt
icF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c*d^2*e^3*f-14*((d*x^2+c)/c)^(1/2)*((
f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c^2*d*e*f^3+7
*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)
^(1/2))*a*c^2*d*e*f^3-4*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x
*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^3*e*f^3+21*((d*x^2+c)/c)^(1/2)*((f*x^2+e
)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d^3*e^3*f+28*(-d/c)^
(1/2)*x^5*a*c*d^2*f^4+63*(-d/c)^(1/2)*x^5*a*d^3*e*f^3+6*((d*x^2+c)/c)^(1/2)
*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*d^3*e^4-6*
((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^
(1/2))*b*d^3*e^4+51*(-d/c)^(1/2)*x^5*b*c*d^2*e*f^3+70*(-d/c)^(1/2)*x^3*a*c*
d^2*e*f^3+8*(-d/c)^(1/2)*x^3*b*c^2*d*e*f^3+9*((d*x^2+c)/c)^(1/2)*((f*x^2+e)
/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c*d^2*e^3*f+21*(-d/c)
^(1/2)*x^7*a*d^3*f^4-4*(-d/c)^(1/2)*x^3*b*c^3*f^4+9*(-d/c)^(1/2)*x*b*c^2*d*
e^2*f^2+36*(-d/c)^(1/2)*x^3*b*c*d^2*e^2*f^2+7*(-d/c)^(1/2)*x*a*c^2*d*e*f^3+
42*(-d/c)^(1/2)*x*a*c*d^2*e^2*f^2+27*(-d/c)^(1/2)*x^5*b*d^3*e^2*f^2+7*(-d/c
)^(1/2)*x^3*a*c^2*d*f^4+42*(-d/c)^(1/2)*x^3*a*d^3*e^2*f^2+3*(-d/c)^(1/2)*x^
3*b*d^3*e^3*f-4*(-d/c)^(1/2)*x*b*c^3*e*f^3+18*(-d/c)^(1/2)*x^7*b*c*d^2*f^4+
39*(-d/c)^(1/2)*x^7*b*d^3*e*f^3+14*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*
EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c*d^2*e^2*f^2)/f^2/(d*f*x^4+c*f
```

```
*x^2+d*e*x^2+c*e)/d^2/(-d/c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)\sqrt{dx^2 + c}(fx^2 + e)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^(1/2)*(f*x^2+e)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((bfx^4 + (be + af)x^2 + ae)\sqrt{dx^2 + c}\sqrt{fx^2 + e}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^(1/2)*(f*x^2+e)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*f*x^4 + (b*e + a*f)*x^2 + a*e)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e),
 x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + bx^2) \sqrt{c + dx^2} (e + fx^2)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+c)**(1/2)*(f*x**2+e)**(3/2),x)
```

```
[Out] Integral((a + b*x**2)*sqrt(c + d*x**2)*(e + f*x**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)\sqrt{dx^2 + c}(fx^2 + e)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^(1/2)*(f*x^2+e)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2), x)
```

3.30 $\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=400

$$\frac{e^{3/2}\sqrt{c+dx^2}\left(5ad(3de-cf)-b\left(6cde-4c^2f\right)\right)\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{15cd^2\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{x\sqrt{c+dx^2}\left(10adf(2de-cf)+b\left(8c^2f\right.\right.}{15d^3\sqrt{e+fx^2}}$$

```
[Out] ((10*a*d*f*(2*d*e - c*f) + b*(3*d^2*e^2 - 13*c*d*e*f + 8*c^2*f^2))*x*Sqrt[c
 + d*x^2])/(15*d^3*Sqrt[e + f*x^2]) + ((3*b*d*e - 4*b*c*f + 5*a*d*f)*x*Sqrt
[c + d*x^2]*Sqrt[e + f*x^2])/(15*d^2) + (b*x*Sqrt[c + d*x^2]*(e + f*x^2)^(3
/2))/(5*d) - (Sqrt[e]*(10*a*d*f*(2*d*e - c*f) + b*(3*d^2*e^2 - 13*c*d*e*f +
 8*c^2*f^2))*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*
e)/(c*f)])/(15*d^3*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f
*x^2]) + (e^(3/2)*(5*a*d*(3*d*e - c*f) - b*(6*c*d*e - 4*c^2*f))*Sqrt[c + d*
x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(15*c*d^2*Sqr
t[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rubi [A] time = 0.43864, antiderivative size = 400, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {528, 531, 418, 492, 411}

$$\frac{x\sqrt{c+dx^2}\left(10adf(2de-cf)+b\left(8c^2f^2-13cdef+3d^2e^2\right)\right)}{15d^3\sqrt{e+fx^2}}-\frac{\sqrt{e}\sqrt{c+dx^2}\left(10adf(2de-cf)+b\left(8c^2f^2-13cdef+3d^2e^2\right.\right.}{15d^3\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(e + f*x^2)^(3/2))/Sqrt[c + d*x^2],x]
```

```
[Out] ((10*a*d*f*(2*d*e - c*f) + b*(3*d^2*e^2 - 13*c*d*e*f + 8*c^2*f^2))*x*Sqrt[c
 + d*x^2])/(15*d^3*Sqrt[e + f*x^2]) + ((3*b*d*e - 4*b*c*f + 5*a*d*f)*x*Sqrt
[c + d*x^2]*Sqrt[e + f*x^2])/(15*d^2) + (b*x*Sqrt[c + d*x^2]*(e + f*x^2)^(3
/2))/(5*d) - (Sqrt[e]*(10*a*d*f*(2*d*e - c*f) + b*(3*d^2*e^2 - 13*c*d*e*f +
 8*c^2*f^2))*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*
e)/(c*f)])/(15*d^3*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f
*x^2]) + (e^(3/2)*(5*a*d*(3*d*e - c*f) - b*(6*c*d*e - 4*c^2*f))*Sqrt[c + d*
x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(15*c*d^2*Sqr
t[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
```

```
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)\left(e+fx^2\right)^{3/2}}{\sqrt{c+dx^2}}\,dx = \frac{bx\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}}{5d} + \frac{\int \frac{\sqrt{e+fx^2}\left(-(bc-5ad)e+(3bde-4bcf+5adf)x^2\right)}{\sqrt{c+dx^2}}\,dx}{5d}$$

$$= \frac{(3bde-4bcf+5adf)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{15d^2} + \frac{bx\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}}{5d} + \frac{\int \frac{-e(2bc(3de-2cf}{}}{}$$

$$= \frac{(3bde-4bcf+5adf)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{15d^2} + \frac{bx\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}}{5d} - \frac{(e(2bc(3de-2}{}$$

$$= \frac{\left(10adf(2de-cf)+b\left(3d^2e^2-13cdef+8c^2f^2\right)\right)x\sqrt{c+dx^2}}{15d^3\sqrt{e+fx^2}} + \frac{(3bde-4bcf+5adf)x}{15d^2}$$

$$= \frac{\left(10adf(2de-cf)+b\left(3d^2e^2-13cdef+8c^2f^2\right)\right)x\sqrt{c+dx^2}}{15d^3\sqrt{e+fx^2}} + \frac{(3bde-4bcf+5adf)x}{15d^2}$$

Mathematica [C] time = 0.84414, size = 275, normalized size = 0.69

$$\frac{ie\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}(cf-de)(-5adf+4bcf-3bde)\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{\frac{d}{c}}\right),\frac{cf}{de}\right)-ie\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\left(10adf(2d\right.}{15c^2f\left(\frac{d}{c}\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(e + f*x^2)^(3/2))/Sqrt[c + d*x^2],x]
```

```
[Out] (-(Sqrt[d/c]*f*x*(c + d*x^2)*(e + f*x^2)*(4*b*c*f - 5*a*d*f - 3*b*d*(2*e +
f*x^2))) - I*e*(10*a*d*f*(2*d*e - c*f) + b*(3*d^2*e^2 - 13*c*d*e*f + 8*c^2*
f^2))*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]
*x], (c*f)/(d*e)] + I*e*(-(d*e) + c*f)*(-3*b*d*e + 4*b*c*f - 5*a*d*f)*Sqrt[
1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/
(d*e)])/(15*c^2*(d/c)^(5/2)*f*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Maple [B] time = 0.018, size = 870, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(f*x^2+e)^(3/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/15*(f*x^2+e)^(1/2)*(d*x^2+c)^(1/2)*(3*(-d/c)^(1/2)*x^7*b*d^2*f^3+5*(-d/c)
^(1/2)*x^5*a*d^2*f^3-(-d/c)^(1/2)*x^5*b*c*d*f^3+9*(-d/c)^(1/2)*x^5*b*d^2*e*
f^2+5*(-d/c)^(1/2)*x^3*a*c*d*f^3+5*(-d/c)^(1/2)*x^3*a*d^2*e*f^2-4*(-d/c)^(1
/2)*x^3*b*c^2*f^3+5*(-d/c)^(1/2)*x^3*b*c*d*e*f^2+6*(-d/c)^(1/2)*x^3*b*d^2*e
^2*f+5*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*
f/d/e)^(1/2))*a*c*d*e*f^2-5*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Ellipti
cF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d^2*e^2*f-4*((d*x^2+c)/c)^(1/2)*((f*x^
2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^2*e*f^2+7*((d*x
^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2)
)*b*c*d*e^2*f-3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^
(1/2),(c*f/d/e)^(1/2))*b*d^2*e^3-10*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)
*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c*d*e*f^2+20*((d*x^2+c)/c)^(1/
2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d^2*e^2*
f+8*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d
/e)^(1/2))*b*c^2*e*f^2-13*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE
(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c*d*e^2*f+3*((d*x^2+c)/c)^(1/2)*((f*x^2+
e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*d^2*e^3+5*(-d/c)^(1
/2)*x*a*c*d*e*f^2-4*(-d/c)^(1/2)*x*b*c^2*e*f^2+6*(-d/c)^(1/2)*x*b*c*d*e^2*f
)/d^2/f/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)/(-d/c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)\left(fx^2+e\right)^{\frac{3}{2}}}{\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(f*x^2+e)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)*(f*x^2 + e)^(3/2)/sqrt(d*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bfx^4+\left(be+af\right)x^2+ae\right)\sqrt{fx^2+e}}{\sqrt{dx^2+c}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(f*x^2+e)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*f*x^4 + (b*e + a*f)*x^2 + a*e)*sqrt(f*x^2 + e)/sqrt(d*x^2 + c),
 x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + bx^2\right)\left(e + fx^2\right)^{\frac{3}{2}}}{\sqrt{c + dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(f*x**2+e)**(3/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)*(e + f*x**2)**(3/2)/sqrt(c + d*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2 + a\right)\left(fx^2 + e\right)^{\frac{3}{2}}}{\sqrt{dx^2 + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(f*x^2+e)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)*(f*x^2 + e)^(3/2)/sqrt(d*x^2 + c), x)
```

3.31 $\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=369

$$\frac{e^{3/2}\sqrt{c+dx^2}(3adf-4bcf+3bde)\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{3cd^2\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{fx\sqrt{c+dx^2}\sqrt{e+fx^2}(4bc-3ad)}{3cd^2}+\frac{fx\sqrt{c+dx^2}(bc}{3}$$

```
[Out] (f*(b*c*(7*d*e - 8*c*f) - 3*a*d*(d*e - 2*c*f))*x*Sqrt[c + d*x^2])/(3*c*d^3*
Sqrt[e + f*x^2]) + ((4*b*c - 3*a*d)*f*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(3
*c*d^2) - ((b*c - a*d)*x*(e + f*x^2)^(3/2))/(c*d*Sqrt[c + d*x^2]) - (Sqrt[e
]*Sqrt[f]*(b*c*(7*d*e - 8*c*f) - 3*a*d*(d*e - 2*c*f))*Sqrt[c + d*x^2]*Ellip
ticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(3*c*d^3*Sqrt[(e*(c + d
*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (e^(3/2)*(3*b*d*e - 4*b*c*f + 3*
a*d*f)*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*
f)])/(3*c*d^2*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]
)
```

Rubi [A] time = 0.399909, antiderivative size = 369, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {526, 528, 531, 418, 492, 411}

$$\frac{e^{3/2}\sqrt{c+dx^2}(3adf-4bcf+3bde)F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3cd^2\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{fx\sqrt{c+dx^2}\sqrt{e+fx^2}(4bc-3ad)}{3cd^2}+\frac{fx\sqrt{c+dx^2}(bc(7de-8c}{3cd^3\sqrt{e+}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(3/2),x]
```



```
[Out] (f*(b*c*(7*d*e - 8*c*f) - 3*a*d*(d*e - 2*c*f))*x*Sqrt[c + d*x^2])/(3*c*d^3*
Sqrt[e + f*x^2]) + ((4*b*c - 3*a*d)*f*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(3
*c*d^2) - ((b*c - a*d)*x*(e + f*x^2)^(3/2))/(c*d*Sqrt[c + d*x^2]) - (Sqrt[e
]*Sqrt[f]*(b*c*(7*d*e - 8*c*f) - 3*a*d*(d*e - 2*c*f))*Sqrt[c + d*x^2]*Ellip
ticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(3*c*d^3*Sqrt[(e*(c + d
*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (e^(3/2)*(3*b*d*e - 4*b*c*f + 3*
a*d*f)*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*
f)])/(3*c*d^2*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]
)
```


Rule 526

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^q)/(a*b*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p +
 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
```

```
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)\left(e+fx^2\right)^{3/2}}{\left(c+dx^2\right)^{3/2}}\,dx = -\frac{(bc-ad)x\left(e+fx^2\right)^{3/2}}{cd\sqrt{c+dx^2}} - \frac{\int \frac{\sqrt{e+fx^2}\left(-bce-(4bc-3ad)fx^2\right)}{\sqrt{c+dx^2}}\,dx}{cd}$$

$$= \frac{(4bc-3ad)fx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3cd^2} - \frac{(bc-ad)x\left(e+fx^2\right)^{3/2}}{cd\sqrt{c+dx^2}} - \frac{\int \frac{-ce(3bde-4bcf+3adf)-f(bc(7}{\sqrt{c+dx^2}\sqrt{e}}}{3cd^2}$$

$$= \frac{(4bc-3ad)fx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3cd^2} - \frac{(bc-ad)x\left(e+fx^2\right)^{3/2}}{cd\sqrt{c+dx^2}} + \frac{(e(3bde-4bcf+3adf))}{3d^2}$$

$$= \frac{f(bc(7de-8cf)-3ad(de-2cf))x\sqrt{c+dx^2}}{3cd^3\sqrt{e+fx^2}} + \frac{(4bc-3ad)fx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3cd^2} - \frac{(bc}{}$$

$$= \frac{f(bc(7de-8cf)-3ad(de-2cf))x\sqrt{c+dx^2}}{3cd^3\sqrt{e+fx^2}} + \frac{(4bc-3ad)fx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3cd^2} - \frac{(bc}{}$$

Mathematica [C] time = 0.706238, size = 248, normalized size = 0.67

$$\frac{\sqrt{\frac{d}{c}}\left(-ie\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}(4bc-3ad)(cf-de)\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{\frac{d}{c}}\right),\frac{cf}{de}\right)+x\sqrt{\frac{d}{c}}\left(e+fx^2\right)\left(3ad(de-cf)+bc\left(4cf\right.\right.\right.}{3d^3\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(3/2),x]
```

```
[Out] (Sqrt[d/c]*(Sqrt[d/c]*x*(e + f*x^2)*(3*a*d*(d*e - c*f) + b*c*(-3*d*e + 4*c*
f + d*f*x^2)) + I*e*(3*a*d*(d*e - 2*c*f) + b*c*(-7*d*e + 8*c*f))*Sqrt[1 + (
d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)
] - I*(4*b*c - 3*a*d)*e*(-(d*e) + c*f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)
/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]))/(3*d^3*Sqrt[c + d*x^2]
*Sqrt[e + f*x^2])
```

Maple [A] time = 0.025, size = 671, normalized size = 1.8

$$-\frac{1}{3\,d^2\left(dfx^4+cfx^2+dex^2+ce\right)c}\sqrt{fx^2+e}\sqrt{dx^2+c}\left(-x^5bcdf^2\sqrt{-\frac{d}{c}}+3\,x^3acdf^2\sqrt{-\frac{d}{c}}-3\,x^3ad^2ef\sqrt{-\frac{d}{c}}-4\,x^3bc^2f^2\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(f*x^2+e)^(3/2)/(d*x^2+c)^(3/2),x)
```

```
[Out] -1/3*(f*x^2+e)^(1/2)*(d*x^2+c)^(1/2)*(-x^5*b*c*d*f^2*(-d/c)^(1/2)+3*x^3*a*c
*d*f^2*(-d/c)^(1/2)-3*x^3*a*d^2*e*f*(-d/c)^(1/2)-4*x^3*b*c^2*f^2*(-d/c)^(1/
2)+2*x^3*b*c*d*e*f*(-d/c)^(1/2)+3*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))
*a*c*d*e*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-3*EllipticF(x*(-d/c)^(1/
2),(c*f/d/e)^(1/2))*a*d^2*e^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-4*Ell
ipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^2*e*f*((d*x^2+c)/c)^(1/2)*((f*x^
2+e)/e)^(1/2)+4*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c*d*e^2*((d*x^2
+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-6*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2)
)*a*c*d*e*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+3*EllipticE(x*(-d/c)^(1
/2),(c*f/d/e)^(1/2))*a*d^2*e^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+8*El
lipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^2*e*f*((d*x^2+c)/c)^(1/2)*((f*x
^2+e)/e)^(1/2)-7*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c*d*e^2*((d*x^
2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+3*x*a*c*d*e*f*(-d/c)^(1/2)-3*x*a*d^2*e^2*
(-d/c)^(1/2)-4*x*b*c^2*e*f*(-d/c)^(1/2)+3*x*b*c*d*e^2*(-d/c)^(1/2))/d^2/(d*
f*x^4+c*f*x^2+d*e*x^2+c*e)/(-d/c)^(1/2)/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)\left(fx^2+e\right)^{\frac{3}{2}}}{\left(dx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(f*x^2+e)^(3/2)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)*(f*x^2 + e)^(3/2)/(d*x^2 + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bfx^4 + \left(be + af\right)x^2 + ae\right)\sqrt{dx^2 + c}\sqrt{fx^2 + e}}{d^2x^4 + 2\,cdx^2 + c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(f*x^2+e)^(3/2)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*f*x^4 + (b*e + a*f)*x^2 + a*e)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/
(d^2*x^4 + 2*c*d*x^2 + c^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + bx^2\right)\left(e + fx^2\right)^{\frac{3}{2}}}{\left(c + dx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(f*x**2+e)**(3/2)/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral((a + b*x**2)*(e + f*x**2)**(3/2)/(c + d*x**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2 + a\right)\left(fx^2 + e\right)^{\frac{3}{2}}}{\left(dx^2 + c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(f*x^2+e)^(3/2)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)*(f*x^2 + e)^(3/2)/(d*x^2 + c)^(3/2), x)
```

3.32 $\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=373

$$\frac{e^{3/2}\sqrt{f}\sqrt{c+dx^2}(4bc-ad)\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{3c^2d^2\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{x\sqrt{e+fx^2}(ad(cf+2de)+bc(de-4cf))}{3c^2d^2\sqrt{c+dx^2}}-\frac{fx\sqrt{c+dx^2}(2ad}{3c^2}$$

```
[Out] -(f*(b*c*(d*e - 8*c*f) + 2*a*d*(d*e + c*f))*x*Sqrt[c + d*x^2])/(3*c^2*d^3*S
qrt[e + f*x^2]) + ((b*c*(d*e - 4*c*f) + a*d*(2*d*e + c*f))*x*Sqrt[e + f*x^2
])/(3*c^2*d^2*Sqrt[c + d*x^2]) - ((b*c - a*d)*x*(e + f*x^2)^(3/2))/(3*c*d*(
c + d*x^2)^(3/2)) + (Sqrt[e]*Sqrt[f]*(b*c*(d*e - 8*c*f) + 2*a*d*(d*e + c*f)
)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/
(3*c^2*d^3*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + ((4*b*c
 - a*d)*e^(3/2)*Sqrt[f]*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e
]], 1 - (d*e)/(c*f)])/(3*c^2*d^2*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt
[e + f*x^2])
```

Rubi [A] time = 0.393749, antiderivative size = 373, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {526, 531, 418, 492, 411}

$$\frac{e^{3/2}\sqrt{f}\sqrt{c+dx^2}(4bc-ad)F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3c^2d^2\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{x\sqrt{e+fx^2}(ad(cf+2de)+bc(de-4cf))}{3c^2d^2\sqrt{c+dx^2}}-\frac{fx\sqrt{c+dx^2}(2ad(cf+de}{3c^2d^3\sqrt{e+}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(5/2),x]
```


```
[Out] -(f*(b*c*(d*e - 8*c*f) + 2*a*d*(d*e + c*f))*x*Sqrt[c + d*x^2])/(3*c^2*d^3*S
qrt[e + f*x^2]) + ((b*c*(d*e - 4*c*f) + a*d*(2*d*e + c*f))*x*Sqrt[e + f*x^2
])/(3*c^2*d^2*Sqrt[c + d*x^2]) - ((b*c - a*d)*x*(e + f*x^2)^(3/2))/(3*c*d*(
c + d*x^2)^(3/2)) + (Sqrt[e]*Sqrt[f]*(b*c*(d*e - 8*c*f) + 2*a*d*(d*e + c*f)
)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/
(3*c^2*d^3*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + ((4*b*c
 - a*d)*e^(3/2)*Sqrt[f]*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e
]], 1 - (d*e)/(c*f)])/(3*c^2*d^2*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt
[e + f*x^2])
```


Rule 526

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^q)/(a*b*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p +
 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
```

```
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{(c+dx^2)^{5/2}}\,dx = -\frac{(bc-ad)x(e+fx^2)^{3/2}}{3cd(c+dx^2)^{3/2}} - \frac{\int \frac{\sqrt{e+fx^2}\left(-(bc+2ad)e-(4bc-ad)fx^2\right)}{(c+dx^2)^{3/2}}\,dx}{3cd}$$

$$= \frac{(bc(de-4cf)+ad(2de+cf))x\sqrt{e+fx^2}}{3c^2d^2\sqrt{c+dx^2}} - \frac{(bc-ad)x(e+fx^2)^{3/2}}{3cd(c+dx^2)^{3/2}} + \frac{\int \frac{c(4bc-ad)ef-f(bc(d}{\sqrt{c+dx}}}{3}$$

$$= \frac{(bc(de-4cf)+ad(2de+cf))x\sqrt{e+fx^2}}{3c^2d^2\sqrt{c+dx^2}} - \frac{(bc-ad)x(e+fx^2)^{3/2}}{3cd(c+dx^2)^{3/2}} + \frac{((4bc-ad)ef)\int}{3cd}$$

$$= -\frac{f(bc(de-8cf)+2ad(de+cf))x\sqrt{c+dx^2}}{3c^2d^3\sqrt{e+fx^2}} + \frac{(bc(de-4cf)+ad(2de+cf))x\sqrt{e+fx^2}}{3c^2d^2\sqrt{c+dx^2}}$$

$$= -\frac{f(bc(de-8cf)+2ad(de+cf))x\sqrt{c+dx^2}}{3c^2d^3\sqrt{e+fx^2}} + \frac{(bc(de-4cf)+ad(2de+cf))x\sqrt{e+fx^2}}{3c^2d^2\sqrt{c+dx^2}}$$

Mathematica [C] time = 1.0019, size = 296, normalized size = 0.79

$$\frac{\left(\frac{d}{c}\right)^{3/2}\left(ie(c+dx^2)\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}(bc(4cf-de)-ad(cf+2de))\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{\frac{d}{c}}\right),\frac{cf}{de}\right)+x\sqrt{\frac{d}{c}}(e+fx^2)\right(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(5/2),x]
```

```
[Out] ((d/c)^(3/2)*(Sqrt[d/c]*x*(e + f*x^2)*(b*c*(-4*c^2*f + d^2*e*x^2 - 5*c*d*f*
x^2) + a*d*(c^2*f + 2*d^2*e*x^2 + c*d*(3*e + 2*f*x^2))) - I*e*(-2*a*d*(d*e
+ c*f) + b*c*(-(d*e) + 8*c*f))*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*
x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + I*e*(-(a*d*(2*d*e
+ c*f)) + b*c*(-(d*e) + 4*c*f))*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f
*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]))/(3*d^4*(c + d*x^2
)^(3/2)*Sqrt[e + f*x^2])
```

Maple [B] time = 0.028, size = 1231, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(f*x^2+e)^(3/2)/(d*x^2+c)^(5/2),x)
```

```
[Out] -1/3*(-x*a*c^2*d*e*f*(-d/c)^(1/2)+4*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2
))*b*c^3*e*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+2*EllipticE(x*(-d/c)^(
1/2),(c*f/d/e)^(1/2))*a*c^2*d*e*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-5
*x^3*a*c*d^2*e*f*(-d/c)^(1/2)+5*x^3*b*c^2*d*e*f*(-d/c)^(1/2)-3*x*a*c*d^2*e^
2*(-d/c)^(1/2)+4*x*b*c^3*e*f*(-d/c)^(1/2)-2*x^5*a*c*d^2*f^2*(-d/c)^(1/2)-2*
x^5*a*d^3*e*f*(-d/c)^(1/2)+5*x^5*b*c^2*d*f^2*(-d/c)^(1/2)-x^3*a*c^2*d*f^2*(
-d/c)^(1/2)-2*x^3*a*d^3*e^2*(-d/c)^(1/2)-x^3*b*c*d^2*e^2*(-d/c)^(1/2)+4*Ell
ipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*b*c^2*d*e*f*((d*x^2+c)/c)^(1/2)*
((f*x^2+e)/e)^(1/2)+2*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a*c*d^2
*e*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-x^5*b*c*d^2*e*f*(-d/c)^(1/2)+4
*x^3*b*c^3*f^2*(-d/c)^(1/2)-2*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2
*a*d^3*e^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-2*EllipticF(x*(-d/c)^(1/
2),(c*f/d/e)^(1/2))*a*c*d^2*e^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+2*E
llipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a*d^3*e^2*((d*x^2+c)/c)^(1/2)*
((f*x^2+e)/e)^(1/2)-EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^2*d*e^2*(
(d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+2*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)
^(1/2))*a*c*d^2*e^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-8*EllipticE(x*(
-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^3*e*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1
/2)+EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^2*d*e^2*((d*x^2+c)/c)^(1/
2)*((f*x^2+e)/e)^(1/2)-EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*b*c*d^
2*e^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+EllipticE(x*(-d/c)^(1/2),(c*f
/d/e)^(1/2))*x^2*b*c*d^2*e^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-Ellipt
icF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c^2*d*e*f*((d*x^2+c)/c)^(1/2)*((f*x^2
+e)/e)^(1/2)-8*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*b*c^2*d*e*f*((
d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1
/2))*x^2*a*c*d^2*e*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2))/(f*x^2+e)^(1/
2)/c^2/(-d/c)^(1/2)/(d*x^2+c)^(3/2)/d^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)\left(fx^2+e\right)^{\frac{3}{2}}}{\left(dx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(f*x^2+e)^(3/2)/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)*(f*x^2 + e)^(3/2)/(d*x^2 + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bfx^4+\left(be+af\right)x^2+ae\right)\sqrt{dx^2+c}\sqrt{fx^2+e}}{d^3x^6+3\,cd^2x^4+3\,c^2dx^2+c^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(f*x^2+e)^(3/2)/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b*f*x^4 + (b*e + a*f)*x^2 + a*e)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/
(d^3*x^6 + 3*c*d^2*x^4 + 3*c^2*d*x^2 + c^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(f*x**2+e)**(3/2)/(d*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)\left(fx^2+e\right)^{\frac{3}{2}}}{\left(dx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(f*x^2+e)^(3/2)/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)*(f*x^2 + e)^(3/2)/(d*x^2 + c)^(5/2), x)
```

3.33 $\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{(c+dx^2)^{7/2}}\,dx$

Optimal. Leaf size=376

$$-\frac{e^{3/2}\sqrt{f}\sqrt{c+dx^2}(ad(4de-cf)+bc(de-4cf))\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{15c^3d^2\sqrt{e+fx^2}(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{\sqrt{e+fx^2}\left(ad\left(-2c^2f^2-3cdef+8d^2e^2\right.\right.}{15c^{5/2}d^{5/2}}$$

```
[Out] ((d*(b*c + 4*a*d)*e - c*(4*b*c + a*d)*f)*x*Sqrt[e + f*x^2])/(15*c^2*d^2*(c
+ d*x^2)^(3/2)) - ((b*c - a*d)*x*(e + f*x^2)^(3/2))/(5*c*d*(c + d*x^2)^(5/2
)) + ((b*c*(2*d^2*e^2 + 3*c*d*e*f - 8*c^2*f^2) + a*d*(8*d^2*e^2 - 3*c*d*e*f
 - 2*c^2*f^2))*Sqrt[e + f*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (
c*f)/(d*e)])/(15*c^(5/2)*d^(5/2)*(d*e - c*f)*Sqrt[c + d*x^2]*Sqrt[(c*(e + f
*x^2))/(e*(c + d*x^2))]) - (e^(3/2)*Sqrt[f]*(b*c*(d*e - 4*c*f) + a*d*(4*d*e
 - c*f))*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(
c*f)])/(15*c^3*d^2*(d*e - c*f)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e
 + f*x^2])
```

Rubi [A] time = 0.423039, antiderivative size = 376, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {526, 525, 418, 411}

$$\frac{\sqrt{e+fx^2}\left(ad\left(-2c^2f^2-3cdef+8d^2e^2\right)+bc\left(-8c^2f^2+3cdef+2d^2e^2\right)\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{15c^{5/2}d^{5/2}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}-\frac{e^{3/2}\sqrt{f}\sqrt{c+dx^2}(ad(4\ldots}{15c^3\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(7/2),x]
```


```
[Out] ((d*(b*c + 4*a*d)*e - c*(4*b*c + a*d)*f)*x*Sqrt[e + f*x^2])/(15*c^2*d^2*(c
+ d*x^2)^(3/2)) - ((b*c - a*d)*x*(e + f*x^2)^(3/2))/(5*c*d*(c + d*x^2)^(5/2
)) + ((b*c*(2*d^2*e^2 + 3*c*d*e*f - 8*c^2*f^2) + a*d*(8*d^2*e^2 - 3*c*d*e*f
 - 2*c^2*f^2))*Sqrt[e + f*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (
c*f)/(d*e)])/(15*c^(5/2)*d^(5/2)*(d*e - c*f)*Sqrt[c + d*x^2]*Sqrt[(c*(e + f
*x^2))/(e*(c + d*x^2))]) - (e^(3/2)*Sqrt[f]*(b*c*(d*e - 4*c*f) + a*d*(4*d*e
 - c*f))*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(
c*f)])/(15*c^3*d^2*(d*e - c*f)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e
 + f*x^2])
```


Rule 526

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^q)/(a*b*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p +
 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 525

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
```

```
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)\left(e+fx^2\right)^{3/2}}{\left(c+dx^2\right)^{7/2}}\,dx = -\frac{(bc-ad)x\left(e+fx^2\right)^{3/2}}{5cd\left(c+dx^2\right)^{5/2}} - \frac{\int \frac{\sqrt{e+fx^2}\left(-(bc+4ad)e-(4bc+ad)fx^2\right)}{\left(c+dx^2\right)^{5/2}}\,dx}{5cd}$$

$$= \frac{(d(bc+4ad)e-c(4bc+ad)f)x\sqrt{e+fx^2}}{15c^2d^2\left(c+dx^2\right)^{3/2}} - \frac{(bc-ad)x\left(e+fx^2\right)^{3/2}}{5cd\left(c+dx^2\right)^{5/2}} + \frac{\int \frac{e(ad(8de+cf)+2bc(d}{}}{}$$

$$= \frac{(d(bc+4ad)e-c(4bc+ad)f)x\sqrt{e+fx^2}}{15c^2d^2\left(c+dx^2\right)^{3/2}} - \frac{(bc-ad)x\left(e+fx^2\right)^{3/2}}{5cd\left(c+dx^2\right)^{5/2}} - \frac{(ef(bc(de-4cf)}{}$$

$$= \frac{(d(bc+4ad)e-c(4bc+ad)f)x\sqrt{e+fx^2}}{15c^2d^2\left(c+dx^2\right)^{3/2}} - \frac{(bc-ad)x\left(e+fx^2\right)^{3/2}}{5cd\left(c+dx^2\right)^{5/2}} + \frac{\left(bc\left(2d^2e^2+3cde\right.\right.}{}$$

Mathematica [C] time = 1.30004, size = 382, normalized size = 1.02

$$\frac{\sqrt{\frac{d}{c}}\left(-x\sqrt{\frac{d}{c}}\left(e+fx^2\right)\left(\left(c+dx^2\right)^2\left(ad\left(2c^2f^2+3cdef-8d^2e^2\right)+bc\left(8c^2f^2-3cdef-2d^2e^2\right)\right)+3c^2(bc-ad)(de-cf)^2\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(7/2),x]
```

```
[Out] (Sqrt[d/c]*(-(Sqrt[d/c]*x*(e + f*x^2)*(3*c^2*(b*c - a*d)*(d*e - c*f)^2 - c*
(d*e - c*f)*(b*c*(d*e - 7*c*f) + 2*a*d*(2*d*e + c*f))*(c + d*x^2) + (a*d*(-
8*d^2*e^2 + 3*c*d*e*f + 2*c^2*f^2) + b*c*(-2*d^2*e^2 - 3*c*d*e*f + 8*c^2*f^
2))*(c + d*x^2)^2)) - I*e*(c + d*x^2)^2*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2
)/e]*((a*d*(-8*d^2*e^2 + 3*c*d*e*f + 2*c^2*f^2) + b*c*(-2*d^2*e^2 - 3*c*d*e
*f + 8*c^2*f^2))*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + (d*e - c*
f)*(a*d*(8*d*e + c*f) + 2*b*c*(d*e + 2*c*f))*EllipticF[I*ArcSinh[Sqrt[d/c]*
x], (c*f)/(d*e)])))/(15*c^2*d^3*(d*e - c*f)*(c + d*x^2)^(5/2)*Sqrt[e + f*x^
```

```
2])
```

Maple [B] time = 0.037, size = 2860, normalized size = 7.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(f*x^2+e)^(3/2)/(d*x^2+c)^(7/2),x)
```

```
[Out] -1/15*(-3*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^4*d*e^2*f*((d*x^2+c
)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+16*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))
*x^2*a*c*d^4*e^3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+4*EllipticF(x*(-d/
c)^(1/2),(c*f/d/e)^(1/2))*x^2*b*c^2*d^3*e^3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/
e)^(1/2)-16*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a*c*d^4*e^3*((d*x
^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+2*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/
2))*x^4*b*c*d^4*e^3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-2*EllipticE(x*(
-d/c)^(1/2),(c*f/d/e)^(1/2))*x^4*b*c*d^4*e^3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)
/e)^(1/2)+2*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^4*d*e^2*f*((d*x^2
+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+2*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2)
)*a*c^4*d*e*f^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+3*EllipticE(x*(-d/c
)^(1/2),(c*f/d/e)^(1/2))*a*c^3*d^2*e^2*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^
(1/2)-EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c^4*d*e*f^2*((d*x^2+c)/c)
^(1/2)*((f*x^2+e)/e)^(1/2)-7*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c^
3*d^2*e^2*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+2*EllipticF(x*(-d/c)^(1
/2),(c*f/d/e)^(1/2))*x^4*b*c^2*d^3*e^2*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^
(1/2)+2*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^4*a*c^2*d^3*e*f^2*((d*x
^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+3*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/
2))*x^4*a*c*d^4*e^2*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+8*x^5*a*d^5*e
^3*(-d/c)^(1/2)-6*x^5*a*c^3*d^2*f^3*(-d/c)^(1/2)-8*x^7*b*c^3*d^2*f^3*(-d/c)
^(1/2)-4*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*b*c^2*d^3*e^3*((d*x^
2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-9*x^5*b*c^4*d*f^3*(-d/c)^(1/2)+2*x^5*b*c*
d^4*e^3*(-d/c)^(1/2)-x^3*a*c^4*d*f^3*(-d/c)^(1/2)+20*x^3*a*c*d^4*e^3*(-d/c)
^(1/2)+5*x^3*b*c^2*d^3*e^3*(-d/c)^(1/2)+15*x*a*c^2*d^3*e^3*(-d/c)^(1/2)-4*x
*b*c^5*e*f^2*(-d/c)^(1/2)-2*x^7*a*c^2*d^3*f^3*(-d/c)^(1/2)+8*x^7*a*d^5*e^2*
f*(-d/c)^(1/2)+2*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^3*d^2*e^3*((
d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-8*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^
(1/2))*a*c^2*d^3*e^3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+8*EllipticE(x*
(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^5*e*f^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)
^(1/2)-2*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^3*d^2*e^3*((d*x^2+c)
/c)^(1/2)*((f*x^2+e)/e)^(1/2)-8*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x
^2*b*c^4*d*e*f^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+4*EllipticE(x*(-d/
c)^(1/2),(c*f/d/e)^(1/2))*x^2*a*c^3*d^2*e*f^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e
)/e)^(1/2)+6*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a*c^2*d^3*e^2*f*
((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-4*EllipticF(x*(-d/c)^(1/2),(c*f/d/e
)^(1/2))*b*c^5*e*f^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-3*x^7*a*c*d^4*
e*f^2*(-d/c)^(1/2)+3*x^7*b*c^2*d^3*e*f^2*(-d/c)^(1/2)+2*x^7*b*c*d^4*e^2*f*(
-d/c)^(1/2)-10*x^5*a*c^2*d^3*e*f^2*(-d/c)^(1/2)+17*x^5*a*c*d^4*e^2*f*(-d/c)
^(1/2)-10*x^5*b*c^3*d^2*e*f^2*(-d/c)^(1/2)+8*x^5*b*c^2*d^3*e^2*f*(-d/c)^(1/
2)-17*x^3*a*c^3*d^2*e*f^2*(-d/c)^(1/2)-x*a*c^4*d*e*f^2*(-d/c)^(1/2)-11*x*a*
c^3*d^2*e^2*f*(-d/c)^(1/2)+x*b*c^4*d*e^2*f*(-d/c)^(1/2)+8*EllipticF(x*(-d/c
)^(1/2),(c*f/d/e)^(1/2))*x^4*a*d^5*e^3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1
/2)+7*x^3*a*c^2*d^3*e^2*f*(-d/c)^(1/2)-8*x^3*b*c^4*d*e*f^2*(-d/c)^(1/2)-2*x
^3*b*c^3*d^2*e^2*f*(-d/c)^(1/2)-4*x^3*b*c^5*f^3*(-d/c)^(1/2)-8*EllipticE(x*
(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^4*a*d^5*e^3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/
e)^(1/2)+8*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c^2*d^3*e^3*((d*x^2+
c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+8*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))
```

```
*x^4*b*c^3*d^2*e*f^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-3*EllipticE(x*
(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^4*b*c^2*d^3*e^2*f*((d*x^2+c)/c)^(1/2)*((f*x
^2+e)/e)^(1/2)-2*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a*c^3*d^2*e*
f^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-14*EllipticF(x*(-d/c)^(1/2),(c*
f/d/e)^(1/2))*x^2*a*c^2*d^3*e^2*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+1
6*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*b*c^4*d*e*f^2*((d*x^2+c)/c)
^(1/2)*((f*x^2+e)/e)^(1/2)-6*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*
b*c^3*d^2*e^2*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-EllipticF(x*(-d/c)^
(1/2),(c*f/d/e)^(1/2))*x^4*a*c^2*d^3*e*f^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e
)^(1/2)-7*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^4*a*c*d^4*e^2*f*((d*x
^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-4*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/
2))*x^4*b*c^3*d^2*e*f^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+4*EllipticF
(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*b*c^3*d^2*e^2*f*((d*x^2+c)/c)^(1/2)*((
f*x^2+e)/e)^(1/2))/(f*x^2+e)^(1/2)/c^3/(c*f-d*e)/(-d/c)^(1/2)/(d*x^2+c)^(5/
2)/d^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)(fx^2+e)^{\frac{3}{2}}}{(dx^2+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(f*x^2+e)^(3/2)/(d*x^2+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)*(f*x^2 + e)^(3/2)/(d*x^2 + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bfx^4+(be+af)x^2+ae)\sqrt{dx^2+c}\sqrt{fx^2+e}}{d^4x^8+4\,cd^3x^6+6\,c^2d^2x^4+4\,c^3dx^2+c^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(f*x^2+e)^(3/2)/(d*x^2+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((b*f*x^4 + (b*e + a*f)*x^2 + a*e)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/
(d^4*x^8 + 4*c*d^3*x^6 + 6*c^2*d^2*x^4 + 4*c^3*d*x^2 + c^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(f*x**2+e)**(3/2)/(d*x**2+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2 + a\right)\left(fx^2 + e\right)^{\frac{3}{2}}}{\left(dx^2 + c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(f*x^2+e)^(3/2)/(d*x^2+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)*(f*x^2 + e)^(3/2)/(d*x^2 + c)^(7/2), x)
```

3.34 $\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{(c+dx^2)^{9/2}}\,dx$

Optimal. Leaf size=531

$$-\frac{e^{3/2}\sqrt{f}\sqrt{c+dx^2}\left(3ad\left(c^2f^2-11cdef+8d^2e^2\right)+2bc\left(2c^2f^2-cdef+2d^2e^2\right)\right)\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{105c^4d^2\sqrt{e+fx^2}(de-cf)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{x\sqrt{e\cdots}}{\cdots}$$

```
[Out] ((d*(b*c + 6*a*d)*e - c*(4*b*c + 3*a*d)*f)*x*Sqrt[e + f*x^2])/(35*c^2*d^2*(
c + d*x^2)^(5/2)) + ((b*c*(4*d^2*e^2 + c*d*e*f - 8*c^2*f^2) + 3*a*d*(8*d^2*
e^2 - 5*c*d*e*f - 2*c^2*f^2))*x*Sqrt[e + f*x^2])/(105*c^3*d^2*(d*e - c*f)*(
c + d*x^2)^(3/2)) - ((b*c - a*d)*x*(e + f*x^2)^(3/2))/(7*c*d*(c + d*x^2)^(7
/2)) + ((6*a*d*(8*d^3*e^3 - 12*c*d^2*e^2*f + 2*c^2*d*e*f^2 + c^3*f^3) + b*c
*(8*d^3*e^3 - 5*c*d^2*e^2*f - 5*c^2*d*e*f^2 + 8*c^3*f^3))*Sqrt[e + f*x^2]*E
llipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(105*c^(7/2)*d^(5/2
)*(d*e - c*f)^2*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))]) - (e
^(3/2)*Sqrt[f]*(3*a*d*(8*d^2*e^2 - 11*c*d*e*f + c^2*f^2) + 2*b*c*(2*d^2*e^2
 - c*d*e*f + 2*c^2*f^2))*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[
e]], 1 - (d*e)/(c*f)])/(105*c^4*d^2*(d*e - c*f)^2*Sqrt[(e*(c + d*x^2))/(c*(
e + f*x^2))]*Sqrt[e + f*x^2])
```

Rubi [A] time = 0.614756, antiderivative size = 531, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {526, 527, 525, 418, 411}

$$\frac{x\sqrt{e+fx^2}\left(3ad\left(-2c^2f^2-5cdef+8d^2e^2\right)+bc\left(-8c^2f^2+cdef+4d^2e^2\right)\right)}{105c^3d^2\left(c+dx^2\right)^{3/2}(de-cf)}-\frac{e^{3/2}\sqrt{f}\sqrt{c+dx^2}\left(3ad\left(c^2f^2-11cdef+8d\cdots\right.\right.}{105c^4d^2\sqrt{\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(9/2),x]
```

```
[Out] ((d*(b*c + 6*a*d)*e - c*(4*b*c + 3*a*d)*f)*x*Sqrt[e + f*x^2])/(35*c^2*d^2*(
c + d*x^2)^(5/2)) + ((b*c*(4*d^2*e^2 + c*d*e*f - 8*c^2*f^2) + 3*a*d*(8*d^2*
e^2 - 5*c*d*e*f - 2*c^2*f^2))*x*Sqrt[e + f*x^2])/(105*c^3*d^2*(d*e - c*f)*(
c + d*x^2)^(3/2)) - ((b*c - a*d)*x*(e + f*x^2)^(3/2))/(7*c*d*(c + d*x^2)^(7
/2)) + ((6*a*d*(8*d^3*e^3 - 12*c*d^2*e^2*f + 2*c^2*d*e*f^2 + c^3*f^3) + b*c
*(8*d^3*e^3 - 5*c*d^2*e^2*f - 5*c^2*d*e*f^2 + 8*c^3*f^3))*Sqrt[e + f*x^2]*E
llipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(105*c^(7/2)*d^(5/2
)*(d*e - c*f)^2*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))]) - (e
^(3/2)*Sqrt[f]*(3*a*d*(8*d^2*e^2 - 11*c*d*e*f + c^2*f^2) + 2*b*c*(2*d^2*e^2
 - c*d*e*f + 2*c^2*f^2))*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[
e]], 1 - (d*e)/(c*f)])/(105*c^4*d^2*(d*e - c*f)^2*Sqrt[(e*(c + d*x^2))/(c*(
e + f*x^2))]*Sqrt[e + f*x^2])
```

Rule 526

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^q)/(a*b*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p +
```

```
 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 525

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)\left(e+fx^2\right)^{3/2}}{\left(c+dx^2\right)^{9/2}}\,dx = -\frac{(bc-ad)x\left(e+fx^2\right)^{3/2}}{7cd\left(c+dx^2\right)^{7/2}} - \frac{\int \frac{\sqrt{e+fx^2}\left(-(bc+6ad)e-(4bc+3ad)fx^2\right)}{\left(c+dx^2\right)^{7/2}}\,dx}{7cd}$$

$$= \frac{(d(bc+6ad)e-c(4bc+3ad)f)x\sqrt{e+fx^2}}{35c^2d^2\left(c+dx^2\right)^{5/2}} - \frac{(bc-ad)x\left(e+fx^2\right)^{3/2}}{7cd\left(c+dx^2\right)^{7/2}} + \frac{\int \frac{e(4bc(de+cf)+3ad(8de\cdots}{(\cdots}}{\cdots}$$

$$= \frac{(d(bc+6ad)e-c(4bc+3ad)f)x\sqrt{e+fx^2}}{35c^2d^2\left(c+dx^2\right)^{5/2}} + \frac{\left(bc\left(4d^2e^2+cdef-8c^2f^2\right)+3ad\left(8d^2e^2-5c\cdots\right.\right.}{105c^3d^2(de-cf)\left(c+dx\cdots\right.}$$

$$= \frac{(d(bc+6ad)e-c(4bc+3ad)f)x\sqrt{e+fx^2}}{35c^2d^2\left(c+dx^2\right)^{5/2}} + \frac{\left(bc\left(4d^2e^2+cdef-8c^2f^2\right)+3ad\left(8d^2e^2-5c\cdots\right.\right.}{105c^3d^2(de-cf)\left(c+dx\cdots\right.}$$

$$= \frac{(d(bc+6ad)e-c(4bc+3ad)f)x\sqrt{e+fx^2}}{35c^2d^2\left(c+dx^2\right)^{5/2}} + \frac{\left(bc\left(4d^2e^2+cdef-8c^2f^2\right)+3ad\left(8d^2e^2-5c\cdots\right.\right.}{105c^3d^2(de-cf)\left(c+dx\cdots\right.}$$

Mathematica [C] time = 1.83476, size = 545, normalized size = 1.03

$$\sqrt{\frac{d}{c}}\left(-x\sqrt{\frac{d}{c}}\left(e+fx^2\right)\left(-c\left(c+dx^2\right)^2(de-cf)\left(3ad\left(-2c^2f^2-5cdef+8d^2e^2\right)+bc\left(-8c^2f^2+cdef+4d^2e^2\right)\right)-\left(c+dx\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(9/2),x]
```

```
[Out] (Sqrt[d/c]*(-(Sqrt[d/c]*x*(e + f*x^2)*(15*c^3*(b*c - a*d)*(d*e - c*f)^3 - 3
*c^2*(d*e - c*f)^2*(b*c*(d*e - 9*c*f) + 2*a*d*(3*d*e + c*f))*(c + d*x^2) -
c*(d*e - c*f)*(b*c*(4*d^2*e^2 + c*d*e*f - 8*c^2*f^2) + 3*a*d*(8*d^2*e^2 - 5
*c*d*e*f - 2*c^2*f^2))*(c + d*x^2)^2 - (6*a*d*(8*d^3*e^3 - 12*c*d^2*e^2*f +
 2*c^2*d*e*f^2 + c^3*f^3) + b*c*(8*d^3*e^3 - 5*c*d^2*e^2*f - 5*c^2*d*e*f^2
+ 8*c^3*f^3))*(c + d*x^2)^3)) + I*e*(c + d*x^2)^3*Sqrt[1 + (d*x^2)/c]*Sqrt[
1 + (f*x^2)/e]*((6*a*d*(8*d^3*e^3 - 12*c*d^2*e^2*f + 2*c^2*d*e*f^2 + c^3*f^
3) + b*c*(8*d^3*e^3 - 5*c*d^2*e^2*f - 5*c^2*d*e*f^2 + 8*c^3*f^3))*EllipticE
[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - (-(d*e) + c*f)*(3*a*d*(-16*d^2*e^2
+ 16*c*d*e*f + c^2*f^2) + b*c*(-8*d^2*e^2 + c*d*e*f + 4*c^2*f^2))*EllipticF
[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])))/(105*c^3*d^3*(d*e - c*f)^2*(c + d*
x^2)^(7/2)*Sqrt[e + f*x^2])
```

Maple [B] time = 0.069, size = 5113, normalized size = 9.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(f*x^2+e)^(3/2)/(d*x^2+c)^(9/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)\left(fx^2+e\right)^{\frac{3}{2}}}{\left(dx^2+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(f*x^2+e)^(3/2)/(d*x^2+c)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)*(f*x^2 + e)^(3/2)/(d*x^2 + c)^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bfx^4+\left(be+af\right)x^2+ae\right)\sqrt{dx^2+c}\sqrt{fx^2+e}}{d^5x^{10}+5\,cd^4x^8+10\,c^2d^3x^6+10\,c^3d^2x^4+5\,c^4dx^2+c^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(f*x^2+e)^(3/2)/(d*x^2+c)^(9/2),x, algorithm="fricas")
```

```
[Out] integral((b*f*x^4 + (b*e + a*f)*x^2 + a*e)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/
(d^5*x^10 + 5*c*d^4*x^8 + 10*c^2*d^3*x^6 + 10*c^3*d^2*x^4 + 5*c^4*d*x^2 + c
^5), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(f*x**2+e)**(3/2)/(d*x**2+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)(fx^2 + e)^{\frac{3}{2}}}{(dx^2 + c)^{\frac{9}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(f*x^2+e)^(3/2)/(d*x^2+c)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)*(f*x^2 + e)^(3/2)/(d*x^2 + c)^(9/2), x)
```

3.35 $\int \frac{(a+bx^2)(c+dx^2)^{5/2}}{\sqrt{e+fx^2}} \, dx$

Optimal. Leaf size=551

$$\frac{\sqrt{e}\sqrt{c+dx^2}\left(7af\left(15c^2f^2-11cdef+4d^2e^2\right)-be\left(45c^2f^2-61cdef+24d^2e^2\right)\right)\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{105f^{7/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-x\sqrt{\cdots}$$

```
[Out] ((7*a*d*f*(8*d^2*e^2 - 23*c*d*e*f + 23*c^2*f^2) - b*(48*d^3*e^3 - 128*c*d^2
*e^2*f + 103*c^2*d*e*f^2 - 15*c^3*f^3))*x*Sqrt[c + d*x^2])/(105*d*f^3*Sqrt[
e + f*x^2]) - ((28*a*d*f*(d*e - 2*c*f) - b*(24*d^2*e^2 - 43*c*d*e*f + 15*c^
2*f^2))*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(105*f^3) - ((6*b*d*e - 5*b*c*f
- 7*a*d*f)*x*(c + d*x^2)^(3/2)*Sqrt[e + f*x^2])/(35*f^2) + (b*x*(c + d*x^2)
^(5/2)*Sqrt[e + f*x^2])/(7*f) - (Sqrt[e]*(7*a*d*f*(8*d^2*e^2 - 23*c*d*e*f +
 23*c^2*f^2) - b*(48*d^3*e^3 - 128*c*d^2*e^2*f + 103*c^2*d*e*f^2 - 15*c^3*f
^3))*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)
])/(105*d*f^(7/2)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) +
(Sqrt[e]*(7*a*f*(4*d^2*e^2 - 11*c*d*e*f + 15*c^2*f^2) - b*e*(24*d^2*e^2 - 6
1*c*d*e*f + 45*c^2*f^2))*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[
e]], 1 - (d*e)/(c*f)])/(105*f^(7/2)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*S
qrt[e + f*x^2])
```

Rubi [A] time = 0.6285, antiderivative size = 551, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {528, 531, 418, 492, 411}

$$-\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}\left(28adf(de-2cf)-b\left(15c^2f^2-43cdef+24d^2e^2\right)\right)}{105f^3}+\frac{x\sqrt{c+dx^2}\left(7adf\left(23c^2f^2-23cdef+8d^2\cdots\right.\right.}{105\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(c + d*x^2)^(5/2))/Sqrt[e + f*x^2],x]
```


```
[Out] ((7*a*d*f*(8*d^2*e^2 - 23*c*d*e*f + 23*c^2*f^2) - b*(48*d^3*e^3 - 128*c*d^2
*e^2*f + 103*c^2*d*e*f^2 - 15*c^3*f^3))*x*Sqrt[c + d*x^2])/(105*d*f^3*Sqrt[
e + f*x^2]) - ((28*a*d*f*(d*e - 2*c*f) - b*(24*d^2*e^2 - 43*c*d*e*f + 15*c^
2*f^2))*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(105*f^3) - ((6*b*d*e - 5*b*c*f
- 7*a*d*f)*x*(c + d*x^2)^(3/2)*Sqrt[e + f*x^2])/(35*f^2) + (b*x*(c + d*x^2)
^(5/2)*Sqrt[e + f*x^2])/(7*f) - (Sqrt[e]*(7*a*d*f*(8*d^2*e^2 - 23*c*d*e*f +
 23*c^2*f^2) - b*(48*d^3*e^3 - 128*c*d^2*e^2*f + 103*c^2*d*e*f^2 - 15*c^3*f
^3))*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)
])/(105*d*f^(7/2)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) +
(Sqrt[e]*(7*a*f*(4*d^2*e^2 - 11*c*d*e*f + 15*c^2*f^2) - b*e*(24*d^2*e^2 - 6
1*c*d*e*f + 45*c^2*f^2))*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[
e]], 1 - (d*e)/(c*f)])/(105*f^(7/2)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*S
qrt[e + f*x^2])
```


Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
```

```
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)\left(c+dx^2\right)^{5/2}}{\sqrt{e+fx^2}}\,dx = \frac{bx\left(c+dx^2\right)^{5/2}\sqrt{e+fx^2}}{7f} + \frac{\int \frac{\left(c+dx^2\right)^{3/2}\left(-c(be-7af)+(-6bde+5bcf+7adf)x^2\right)}{\sqrt{e+fx^2}}\,dx}{7f}$$

$$= -\frac{(6bde-5bcf-7adf)x\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}}{35f^2} + \frac{bx\left(c+dx^2\right)^{5/2}\sqrt{e+fx^2}}{7f} + \frac{\int \frac{\sqrt{c+dx^2}(-c(7\ldots}{\ldots}}{\ldots}$$

$$= -\frac{\left(28adf(de-2cf)-b\left(24d^2e^2-43cdef+15c^2f^2\right)\right)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{105f^3} - \frac{(6bde-5bcf\ldots}{\ldots}$$

$$= -\frac{\left(28adf(de-2cf)-b\left(24d^2e^2-43cdef+15c^2f^2\right)\right)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{105f^3} - \frac{(6bde-5bcf\ldots}{\ldots}$$

$$= \frac{\left(7adf\left(8d^2e^2-23cdef+23c^2f^2\right)-b\left(48d^3e^3-128cd^2e^2f+103c^2def^2-15c^3f^3\right)\right)x\sqrt{c+\ldots}}{105df^3\sqrt{e+fx^2}}$$

$$= \frac{\left(7adf\left(8d^2e^2-23cdef+23c^2f^2\right)-b\left(48d^3e^3-128cd^2e^2f+103c^2def^2-15c^3f^3\right)\right)x\sqrt{c+\ldots}}{105df^3\sqrt{e+fx^2}}$$

Mathematica [C] time = 1.18045, size = 386, normalized size = 0.7

$$i\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}(cf-de)\left(4be\left(15c^2f^2-26cdef+12d^2e^2\right)-7af\left(15c^2f^2-19cdef+8d^2e^2\right)\right)\text{EllipticF}\left(i\sinh^{-1}\left(x\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(c + d*x^2)^(5/2))/Sqrt[e + f*x^2],x]
```

```
[Out] (Sqrt[d/c]*f*x*(c + d*x^2)*(e + f*x^2)*(7*a*d*f*(-4*d*e + 11*c*f + 3*d*f*x^
2) + b*(45*c^2*f^2 + c*d*f*(-61*e + 45*f*x^2) + 3*d^2*(8*e^2 - 6*e*f*x^2 +
5*f^2*x^4))) - I*e*(7*a*d*f*(8*d^2*e^2 - 23*c*d*e*f + 23*c^2*f^2) + b*(-48*
d^3*e^3 + 128*c*d^2*e^2*f - 103*c^2*d*e*f^2 + 15*c^3*f^3))*Sqrt[1 + (d*x^2)
/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + I*
(-(d*e) + c*f)*(4*b*e*(12*d^2*e^2 - 26*c*d*e*f + 15*c^2*f^2) - 7*a*f*(8*d^2
*e^2 - 19*c*d*e*f + 15*c^2*f^2))*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*El
lipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/(105*Sqrt[d/c]*f^4*Sqrt[c + d
*x^2]*Sqrt[e + f*x^2])
```

Maple [B] time = 0.025, size = 1386, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x)
```

```
[Out] 1/105*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)*(90*(-d/c)^(1/2)*x^5*b*c^2*d*f^4+105*
((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^
(1/2))*a*c^3*f^4+15*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d
/c)^(1/2),(c*f/d/e)^(1/2))*b*c^3*e*f^3-56*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)
^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d^3*e^3*f+24*(-d/c)^(1/2
)*x*b*c*d^2*e^3*f-103*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(
-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^2*d*e^2*f^2+15*(-d/c)^(1/2)*x^9*b*d^3*f^4+
164*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d
/e)^(1/2))*b*c^2*d*e^2*f^2-161*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Elli
pticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c*d^2*e^2*f^2-152*((d*x^2+c)/c)^(1/
2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c*d^2*e^
3*f+161*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c
*f/d/e)^(1/2))*a*c^2*d*e*f^3-238*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*El
lipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c^2*d*e*f^3-60*((d*x^2+c)/c)^(1/2
)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^3*e*f^3
+56*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d
/e)^(1/2))*a*d^3*e^3*f+98*(-d/c)^(1/2)*x^5*a*c*d^2*f^4-7*(-d/c)^(1/2)*x^5*a
*d^3*e*f^3+48*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1
/2),(c*f/d/e)^(1/2))*b*d^3*e^4-48*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*E
llipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*d^3*e^4-19*(-d/c)^(1/2)*x^5*b*c*
d^2*e*f^3+70*(-d/c)^(1/2)*x^3*a*c*d^2*e*f^3+29*(-d/c)^(1/2)*x^3*b*c^2*d*e*f
^3+128*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*
f/d/e)^(1/2))*b*c*d^2*e^3*f+21*(-d/c)^(1/2)*x^7*a*d^3*f^4+45*(-d/c)^(1/2)*x
^3*b*c^3*f^4-61*(-d/c)^(1/2)*x*b*c^2*d*e^2*f^2-55*(-d/c)^(1/2)*x^3*b*c*d^2*
e^2*f^2+77*(-d/c)^(1/2)*x*a*c^2*d*e*f^3-28*(-d/c)^(1/2)*x*a*c*d^2*e^2*f^2+6
*(-d/c)^(1/2)*x^5*b*d^3*e^2*f^2+77*(-d/c)^(1/2)*x^3*a*c^2*d*f^4-28*(-d/c)^(
1/2)*x^3*a*d^3*e^2*f^2+24*(-d/c)^(1/2)*x^3*b*d^3*e^3*f+45*(-d/c)^(1/2)*x*b*
c^3*e*f^3+60*(-d/c)^(1/2)*x^7*b*c*d^2*f^4-3*(-d/c)^(1/2)*x^7*b*d^3*e*f^3+18
```

```
9*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e
)^(1/2))*a*c*d^2*e^2*f^2)/f^4/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)/(-d/c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)(dx^2+c)^{\frac{5}{2}}}{\sqrt{fx^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)*(d*x^2 + c)^(5/2)/sqrt(f*x^2 + e), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bd^2x^6+\left(2\,bcd+ad^2\right)x^4+ac^2+\left(bc^2+2\,acd\right)x^2\right)\sqrt{dx^2+c}}{\sqrt{fx^2+e}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*d^2*x^6 + (2*b*c*d + a*d^2)*x^4 + a*c^2 + (b*c^2 + 2*a*c*d)*x^2
)*sqrt(d*x^2 + c)/sqrt(f*x^2 + e), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^2\right)\left(c+dx^2\right)^{\frac{5}{2}}}{\sqrt{e+fx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+c)**(5/2)/(f*x**2+e)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)*(c + d*x**2)**(5/2)/sqrt(e + f*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)(dx^2+c)^{\frac{5}{2}}}{\sqrt{fx^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)*(d*x^2 + c)^(5/2)/sqrt(f*x^2 + e), x)
```

3.36 $\int \frac{(a+bx^2)(c+dx^2)^{3/2}}{\sqrt{e+fx^2}}\,dx$

Optimal. Leaf size=396

$$-\frac{\sqrt{e}\sqrt{c+dx^2}\left(5af(de-3cf)-b\left(4de^2-6cef\right)\right)\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{15f^{5/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{x\sqrt{c+dx^2}\left(10adf(de-2cf)-b\left(3\right.\right.}{15df^2\sqrt{e+}}$$

```
[Out] -((10*a*d*f*(d*e - 2*c*f) - b*(8*d^2*e^2 - 13*c*d*e*f + 3*c^2*f^2))*x*Sqrt[
c + d*x^2])/(15*d*f^2*Sqrt[e + f*x^2]) - ((4*b*d*e - 3*b*c*f - 5*a*d*f)*x*S
qrt[c + d*x^2]*Sqrt[e + f*x^2])/(15*f^2) + (b*x*(c + d*x^2)^(3/2)*Sqrt[e +
f*x^2])/(5*f) + (Sqrt[e]*(10*a*d*f*(d*e - 2*c*f) - b*(8*d^2*e^2 - 13*c*d*e*
f + 3*c^2*f^2))*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 -
(d*e)/(c*f)])/(15*d*f^(5/2)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e +
f*x^2]) - (Sqrt[e]*(5*a*f*(d*e - 3*c*f) - b*(4*d*e^2 - 6*c*e*f))*Sqrt[c + d
*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(15*f^(5/2)*
Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rubi [A] time = 0.447957, antiderivative size = 396, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {528, 531, 418, 492, 411}

$$-\frac{x\sqrt{c+dx^2}\left(10adf(de-2cf)-b\left(3c^2f^2-13cdef+8d^2e^2\right)\right)}{15df^2\sqrt{e+fx^2}}+\frac{\sqrt{e}\sqrt{c+dx^2}\left(10adf(de-2cf)-b\left(3c^2f^2-13cdef+8\right.\right.}{15df^{5/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx}{c(e+fx}}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(c + d*x^2)^(3/2))/Sqrt[e + f*x^2],x]
```

```
[Out] -((10*a*d*f*(d*e - 2*c*f) - b*(8*d^2*e^2 - 13*c*d*e*f + 3*c^2*f^2))*x*Sqrt[
c + d*x^2])/(15*d*f^2*Sqrt[e + f*x^2]) - ((4*b*d*e - 3*b*c*f - 5*a*d*f)*x*S
qrt[c + d*x^2]*Sqrt[e + f*x^2])/(15*f^2) + (b*x*(c + d*x^2)^(3/2)*Sqrt[e +
f*x^2])/(5*f) + (Sqrt[e]*(10*a*d*f*(d*e - 2*c*f) - b*(8*d^2*e^2 - 13*c*d*e*
f + 3*c^2*f^2))*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 -
(d*e)/(c*f)])/(15*d*f^(5/2)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e +
f*x^2]) - (Sqrt[e]*(5*a*f*(d*e - 3*c*f) - b*(4*d*e^2 - 6*c*e*f))*Sqrt[c + d
*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(15*f^(5/2)*
Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
```

```
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}{\sqrt{e+fx^2}}\,dx = \frac{bx\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}}{5f} + \frac{\int \frac{\sqrt{c+dx^2}\left(-c(be-5af)+(-4bde+3bcf+5adf)x^2\right)}{\sqrt{e+fx^2}}\,dx}{5f}$$

$$= -\frac{(4bde-3bcf-5adf)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{15f^2} + \frac{bx\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}}{5f} + \frac{\int \frac{-c(5af(de-3cf)-2}{}}{}$$

$$= -\frac{(4bde-3bcf-5adf)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{15f^2} + \frac{bx\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}}{5f} - \frac{\left(c\left(5af(de-3cf\right.\right.}{}$$

$$= -\frac{\left(10adf(de-2cf)-b\left(8d^2e^2-13cdef+3c^2f^2\right)\right)x\sqrt{c+dx^2}}{15df^2\sqrt{e+fx^2}} - \frac{(4bde-3bcf-5adf)x\sqrt{c}}{15f^2}$$

$$= -\frac{\left(10adf(de-2cf)-b\left(8d^2e^2-13cdef+3c^2f^2\right)\right)x\sqrt{c+dx^2}}{15df^2\sqrt{e+fx^2}} - \frac{(4bde-3bcf-5adf)x\sqrt{c}}{15f^2}$$

Mathematica [C] time = 0.82325, size = 279, normalized size = 0.7

$$\frac{i\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}(cf-de)(5af(2de-3cf)+be(9cf-8de))\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{\frac{d}{c}}\right),\frac{cf}{de}\right)-ie\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\left(b\left(3\right.\right.}{15f^3\sqrt{\frac{d}{c}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(c + d*x^2)^(3/2))/Sqrt[e + f*x^2],x]
```

```
[Out] (Sqrt[d/c]*f*x*(c + d*x^2)*(e + f*x^2)*(5*a*d*f + b*(-4*d*e + 6*c*f + 3*d*f
*x^2)) - I*e*(-10*a*d*f*(d*e - 2*c*f) + b*(8*d^2*e^2 - 13*c*d*e*f + 3*c^2*f
^2))*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*
x], (c*f)/(d*e)] + I*(-(d*e) + c*f)*(5*a*f*(2*d*e - 3*c*f) + b*e*(-8*d*e +
9*c*f))*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/
c]*x], (c*f)/(d*e)])/(15*Sqrt[d/c]*f^3*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Maple [B] time = 0.02, size = 924, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x)
```

```
[Out] 1/15*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)*(3*(-d/c)^(1/2)*x^7*b*d^2*f^3+5*(-d/c)
^(1/2)*x^5*a*d^2*f^3+9*(-d/c)^(1/2)*x^5*b*c*d*f^3-(-d/c)^(1/2)*x^5*b*d^2*e*
f^2+5*(-d/c)^(1/2)*x^3*a*c*d*f^3+5*(-d/c)^(1/2)*x^3*a*d^2*e*f^2+6*(-d/c)^(1
/2)*x^3*b*c^2*f^3+5*(-d/c)^(1/2)*x^3*b*c*d*e*f^2-4*(-d/c)^(1/2)*x^3*b*d^2*e
^2*f+15*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c
*f/d/e)^(1/2))*a*c^2*f^3-25*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Ellipti
cF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c*d*e*f^2+10*((d*x^2+c)/c)^(1/2)*((f*x
^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d^2*e^2*f-9*((d*
x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2
))*b*c^2*e*f^2+17*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c
)^(1/2),(c*f/d/e)^(1/2))*b*c*d*e^2*f-8*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1
/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*d^2*e^3+20*((d*x^2+c)/c)^(1
/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c*d*e*f
^2-10*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f
/d/e)^(1/2))*a*d^2*e^2*f+3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Elliptic
E(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^2*e*f^2-13*((d*x^2+c)/c)^(1/2)*((f*x^
2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c*d*e^2*f+8*((d*x
^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2)
)*b*d^2*e^3+5*(-d/c)^(1/2)*x*a*c*d*e*f^2+6*(-d/c)^(1/2)*x*b*c^2*e*f^2-4*(-d
/c)^(1/2)*x*b*c*d*e^2*f)/f^3/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)/(-d/c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)\left(dx^2+c\right)^{\frac{3}{2}}}{\sqrt{fx^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)*(d*x^2 + c)^(3/2)/sqrt(f*x^2 + e), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bdx^4+(bc+ad)x^2+ac\right)\sqrt{dx^2+c}}{\sqrt{fx^2+e}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*d*x^4 + (b*c + a*d)*x^2 + a*c)*sqrt(d*x^2 + c)/sqrt(f*x^2 + e),
 x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + bx^2\right)\left(c + dx^2\right)^{\frac{3}{2}}}{\sqrt{e + fx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+c)**(3/2)/(f*x**2+e)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)*(c + d*x**2)**(3/2)/sqrt(e + f*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2 + a\right)\left(dx^2 + c\right)^{\frac{3}{2}}}{\sqrt{fx^2 + e}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)*(d*x^2 + c)^(3/2)/sqrt(f*x^2 + e), x)
```

3.37 $\int \frac{(a+bx^2)\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx$

Optimal. Leaf size=282

$$-\frac{\sqrt{e}\sqrt{c+dx^2}(be-3af)\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{3f^{3/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{\sqrt{e}\sqrt{c+dx^2}(-3adf-bcf+2bde)E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3df^{3/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

```
[Out] -((2*b*d*e - b*c*f - 3*a*d*f)*x*Sqrt[c + d*x^2])/(3*d*f*Sqrt[e + f*x^2]) +
(b*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(3*f) + (Sqrt[e]*(2*b*d*e - b*c*f - 3
*a*d*f)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c
*f)])/(3*d*f^(3/2)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) -
 (Sqrt[e]*(b*e - 3*a*f)*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e
]], 1 - (d*e)/(c*f)])/(3*f^(3/2)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt
[e + f*x^2])
```

Rubi [A] time = 0.18439, antiderivative size = 282, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {528, 531, 418, 492, 411}

$$-\frac{\sqrt{e}\sqrt{c+dx^2}(be-3af)F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3f^{3/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{\sqrt{e}\sqrt{c+dx^2}(-3adf-bcf+2bde)E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3df^{3/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{x\sqrt{c+}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*Sqrt[c + d*x^2])/Sqrt[e + f*x^2],x]
```


```
[Out] -((2*b*d*e - b*c*f - 3*a*d*f)*x*Sqrt[c + d*x^2])/(3*d*f*Sqrt[e + f*x^2]) +
(b*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(3*f) + (Sqrt[e]*(2*b*d*e - b*c*f - 3
*a*d*f)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c
*f)])/(3*d*f^(3/2)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) -
 (Sqrt[e]*(b*e - 3*a*f)*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e
]], 1 - (d*e)/(c*f)])/(3*f^(3/2)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt
[e + f*x^2])
```


Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx=\frac{bx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3f}+\frac{\int \frac{-c(be-3af)+(-2bde+bcf+3adf)x^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{3f}$$

$$=\frac{bx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3f}-\frac{(c(be-3af))\int \frac{1}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{3f}+\frac{(-2bde+bcf+3adf)\int \frac{x^2}{\sqrt{c+dx^2}\sqrt{e+}}}{3f}$$

$$=-\frac{(2bde-bcf-3adf)x\sqrt{c+dx^2}}{3df\sqrt{e+fx^2}}+\frac{bx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3f}-\frac{\sqrt{e}(be-3af)\sqrt{c+dx^2}F\left(\tan^{-1}\right.}{3f^{3/2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}\sqrt{e+f}}$$

$$=-\frac{(2bde-bcf-3adf)x\sqrt{c+dx^2}}{3df\sqrt{e+fx^2}}+\frac{bx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3f}+\frac{\sqrt{e}(2bde-bcf-3adf)\sqrt{c+dx^2}}{3df^{3/2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}$$

Mathematica [C] time = 0.424289, size = 215, normalized size = 0.76

$$\frac{i\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}(2be-3af)(cf-de)\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{\frac{d}{c}}\right),\frac{cf}{de}\right)-ie\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}(3adf+bcf-2bde)E\left(i\sin\right.}{3f^2\sqrt{\frac{d}{c}}\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*Sqrt[c + d*x^2])/Sqrt[e + f*x^2],x]
```

```
[Out] (b*Sqrt[d/c]*f*x*(c + d*x^2)*(e + f*x^2) - I*e*(-2*b*d*e + b*c*f + 3*a*d*f)
*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x],
(c*f)/(d*e)] + I*(2*b*e - 3*a*f)*(-(d*e) + c*f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1
+ (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/(3*Sqrt[d/c]*f
^2*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Maple [A] time = 0.016, size = 501, normalized size = 1.8

$$\frac{1}{\left(3dfx^4+3cfx^2+3dex^2+3ce\right)f^2}\sqrt{dx^2+c}\sqrt{fx^2+e}\left(\sqrt{-\frac{d}{c}}x^5bdf^2+\sqrt{-\frac{d}{c}}x^3bcf^2+\sqrt{-\frac{d}{c}}x^3bdef+3\sqrt{\frac{dx^2+c}{c}}\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

```
[Out] 1/3*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)*((-d/c)^(1/2)*x^5*b*d*f^2+(-d/c)^(1/2)*
x^3*b*c*f^2+(-d/c)^(1/2)*x^3*b*d*e*f+3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1
/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c*f^2-3*((d*x^2+c)/c)^(1/2)
*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d*e*f-2*((
d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1
/2))*b*c*e*f+2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(
1/2),(c*f/d/e)^(1/2))*b*d*e^2+3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Ell
ipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d*e*f+((d*x^2+c)/c)^(1/2)*((f*x^2+
e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c*e*f-2*((d*x^2+c)/
c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*d*
e^2+(-d/c)^(1/2)*x*b*c*e*f)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)/f^2/(-d/c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)\sqrt{dx^2+c}}{\sqrt{fx^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)*sqrt(d*x^2 + c)/sqrt(f*x^2 + e), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(bx^2+a\right)\sqrt{dx^2+c}}{\sqrt{fx^2+e}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)*sqrt(d*x^2 + c)/sqrt(f*x^2 + e), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^2\right)\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)*sqrt(c + d*x**2)/sqrt(e + f*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)\sqrt{dx^2 + c}}{\sqrt{fx^2 + e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)*sqrt(d*x^2 + c)/sqrt(f*x^2 + e), x)
```

3.38 $\int \frac{a+bx^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$

Optimal. Leaf size=206

$$\frac{a\sqrt{e}\sqrt{c+dx^2}\,\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{bx\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{b\sqrt{e}\sqrt{c+dx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{d\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

```
[Out] (b*x*Sqrt[c + d*x^2])/(d*Sqrt[e + f*x^2]) - (b*Sqrt[e]*Sqrt[c + d*x^2]*Elli
pticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(d*Sqrt[f]*Sqrt[(e*(c
+ d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (a*Sqrt[e]*Sqrt[c + d*x^2]*El
lipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(c*Sqrt[f]*Sqrt[(e*(
c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rubi [A] time = 0.101872, antiderivative size = 206, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {531, 418, 492, 411}

$$\frac{a\sqrt{e}\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{bx\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{b\sqrt{e}\sqrt{c+dx^2}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{d\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

```
[Out] (b*x*Sqrt[c + d*x^2])/(d*Sqrt[e + f*x^2]) - (b*Sqrt[e]*Sqrt[c + d*x^2]*Elli
pticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(d*Sqrt[f]*Sqrt[(e*(c
+ d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (a*Sqrt[e]*Sqrt[c + d*x^2]*El
lipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(c*Sqrt[f]*Sqrt[(e*(
c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{a+bx^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = a\int \frac{1}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx + b\int \frac{x^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

$$= \frac{bx\sqrt{c+dx^2}}{d\sqrt{e+fx^2}} + \frac{a\sqrt{e}\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} - \frac{(be)\int \frac{\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx}{d}$$

$$= \frac{bx\sqrt{c+dx^2}}{d\sqrt{e+fx^2}} - \frac{b\sqrt{e}\sqrt{c+dx^2}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{d\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} + \frac{a\sqrt{e}\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

Mathematica [C] time = 0.161497, size = 131, normalized size = 0.64

$$-\frac{i\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\left((af-be)\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{\frac{d}{c}}\right),\frac{cf}{de}\right)+beE\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)\right)}{f\sqrt{\frac{d}{c}}\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

```
[Out] ((-I)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*(b*e*EllipticE[I*ArcSinh[Sqrt
[d/c]*x], (c*f)/(d*e)] + (-(b*e) + a*f)*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (
c*f)/(d*e)]))/(Sqrt[d/c]*f*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Maple [A] time = 0.018, size = 158, normalized size = 0.8

$$\frac{1}{f\left(dfx^4+cfx^2+dex^2+ce\right)}\left(\text{EllipticF}\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)af-\text{EllipticF}\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)be+\text{EllipticE}\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

```
[Out] (EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*f-EllipticF(x*(-d/c)^(1/2),(c*
f/d/e)^(1/2))*b*e+EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*e)*((f*x^2+e)
/e)^(1/2)*((d*x^2+c)/c)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/f/(-d/c)^(1/2
)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx^2 + a}{\sqrt{dx^2 + c}\sqrt{fx^2 + e}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)/(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2 + a)\sqrt{dx^2 + c}\sqrt{fx^2 + e}}{dfx^4 + (de + cf)x^2 + ce}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(d*f*x^4 + (d*e + c*f)
*x^2 + c*e), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + bx^2}{\sqrt{c + dx^2}\sqrt{e + fx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/(d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)/(sqrt(c + d*x**2)*sqrt(e + f*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx^2 + a}{\sqrt{dx^2 + c}\sqrt{fx^2 + e}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)/(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

3.39 $\int \frac{a+bx^2}{\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}}\,dx$

Optimal. Leaf size=209

$$\frac{\sqrt{e}\sqrt{c+dx^2}(be-af)\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{e+fx^2}(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{\sqrt{e+fx^2}(bc-ad)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$

```
[Out] -(((b*c - a*d)*Sqrt[e + f*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (
c*f)/(d*e)])/(Sqrt[c]*Sqrt[d]*(d*e - c*f)*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*x^
2))/(e*(c + d*x^2))])) + (Sqrt[e]*(b*e - a*f)*Sqrt[c + d*x^2]*EllipticF[Arc
Tan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(c*Sqrt[f]*(d*e - c*f)*Sqrt[(e*
(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rubi [A] time = 0.0849165, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {525, 418, 411}

$$\frac{\sqrt{e}\sqrt{c+dx^2}(be-af)F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{e+fx^2}(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{\sqrt{e+fx^2}(bc-ad)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/((c + d*x^2)^(3/2)*Sqrt[e + f*x^2]),x]
```


```
[Out] -(((b*c - a*d)*Sqrt[e + f*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (
c*f)/(d*e)])/(Sqrt[c]*Sqrt[d]*(d*e - c*f)*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*x^
2))/(e*(c + d*x^2))])) + (Sqrt[e]*(b*e - a*f)*Sqrt[c + d*x^2]*EllipticF[Arc
Tan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(c*Sqrt[f]*(d*e - c*f)*Sqrt[(e*
(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```


Rule 525

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
```

```
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{a+bx^2}{\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}}\,dx = -\frac{(bc-ad)\int \frac{\sqrt{e+fx^2}}{\left(c+dx^2\right)^{3/2}}\,dx}{de-cf} + \frac{(be-af)\int \frac{1}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{de-cf}$$

$$= -\frac{(bc-ad)\sqrt{e+fx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\sqrt{d}(de-cf)\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}} + \frac{\sqrt{e}(be-af)\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\right.}{c\sqrt{f}(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

Mathematica [C] time = 0.636782, size = 206, normalized size = 0.99

$$\frac{\sqrt{\frac{d}{c}}\left(-ia\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}(cf-de)\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{\frac{d}{c}}\right),\frac{cf}{de}\right)+x\sqrt{\frac{d}{c}}\left(e+fx^2\right)(bc-ad)+ie\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1(\right.}{d\sqrt{c+dx^2}\sqrt{e+fx^2}(cf-de)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/((c + d*x^2)^(3/2)*Sqrt[e + f*x^2]),x]
```

```
[Out] (Sqrt[d/c]*(Sqrt[d/c]*(b*c - a*d)*x*(e + f*x^2) + I*(b*c - a*d)*e*Sqrt[1 +
(d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e
)] - I*a*(-(d*e) + c*f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I
*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]))/(d*(-(d*e) + c*f)*Sqrt[c + d*x^2]*Sqr
t[e + f*x^2])
```

Maple [A] time = 0.024, size = 334, normalized size = 1.6

$$\frac{1}{c\left(cf-de\right)\left(dfx^4+cfx^2+dex^2+ce\right)}\left(-x^3adf\sqrt{-\frac{d}{c}}+x^3bcf\sqrt{-\frac{d}{c}}+\text{EllipticF}\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)acf\sqrt{\frac{fx^2+e}{e}}\sqrt{\frac{dx^2}{c}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x)
```

```
[Out] (-x^3*a*d*f*(-d/c)^(1/2)+x^3*b*c*f*(-d/c)^(1/2)+EllipticF(x*(-d/c)^(1/2),(c
*f/d/e)^(1/2))*a*c*f*((f*x^2+e)/e)^(1/2)*((d*x^2+c)/c)^(1/2)-EllipticF(x*(-
d/c)^(1/2),(c*f/d/e)^(1/2))*a*d*e*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+E
llipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d*e*((d*x^2+c)/c)^(1/2)*((f*x^2+
e)/e)^(1/2)-EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c*e*((d*x^2+c)/c)^(
1/2)*((f*x^2+e)/e)^(1/2)-x*a*d*e*(-d/c)^(1/2)+x*b*c*e*(-d/c)^(1/2))*(f*x^2+
e)^(1/2)*(d*x^2+c)^(1/2)/c/(-d/c)^(1/2)/(c*f-d*e)/(d*f*x^4+c*f*x^2+d*e*x^2+
c*e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx^2+a}{\left(dx^2+c\right)^{\frac{3}{2}}\sqrt{fx^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)/((d*x^2 + c)^(3/2)*sqrt(f*x^2 + e)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(bx^2+a\right)\sqrt{dx^2+c}\sqrt{fx^2+e}}{d^2fx^6+\left(d^2e+2\,cdf\right)x^4+c^2e+\left(2\,cde+c^2f\right)x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(d^2*f*x^6 + (d^2*e +
2*c*d*f)*x^4 + c^2*e + (2*c*d*e + c^2*f)*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a+bx^2}{\left(c+dx^2\right)^{\frac{3}{2}}\sqrt{e+fx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/(d*x**2+c)**(3/2)/(f*x**2+e)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)/((c + d*x**2)**(3/2)*sqrt(e + f*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx^2+a}{\left(dx^2+c\right)^{\frac{3}{2}}\sqrt{fx^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)/((d*x^2 + c)^(3/2)*sqrt(f*x^2 + e)), x)
```

3.40 $\int \frac{a+bx^2}{(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx$

Optimal. Leaf size=284

$$-\frac{\sqrt{e}\sqrt{f}\sqrt{c+dx^2}(-3acf+ade+2bce)\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{3c^2\sqrt{e+fx^2}(de-cf)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{\sqrt{e+fx^2}(2ad(de-2cf)+bc(cf+de))E\left(\text{ta}\right.}{3c^{3/2}\sqrt{d}\sqrt{c+dx^2}(de-cf)^2\sqrt{\frac{c(}{e(}}}$$

```
[Out] -((b*c - a*d)*x*Sqrt[e + f*x^2])/(3*c*(d*e - c*f)*(c + d*x^2)^(3/2)) + ((2*
a*d*(d*e - 2*c*f) + b*c*(d*e + c*f))*Sqrt[e + f*x^2]*EllipticE[ArcTan[(Sqrt
[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(3*c^(3/2)*Sqrt[d]*(d*e - c*f)^2*Sqrt[c
+ d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))]) - (Sqrt[e]*Sqrt[f]*(2*b*c*e
 + a*d*e - 3*a*c*f)*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]],
1 - (d*e)/(c*f)])/(3*c^2*(d*e - c*f)^2*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))
]*Sqrt[e + f*x^2])
```

Rubi [A] time = 0.210812, antiderivative size = 284, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {527, 525, 418, 411}

$$-\frac{\sqrt{e}\sqrt{f}\sqrt{c+dx^2}(-3acf+ade+2bce)F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3c^2\sqrt{e+fx^2}(de-cf)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{\sqrt{e+fx^2}(2ad(de-2cf)+bc(cf+de))E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right.\right.}{3c^{3/2}\sqrt{d}\sqrt{c+dx^2}(de-cf)^2\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/((c + d*x^2)^(5/2)*Sqrt[e + f*x^2]),x]
```


```
[Out] -((b*c - a*d)*x*Sqrt[e + f*x^2])/(3*c*(d*e - c*f)*(c + d*x^2)^(3/2)) + ((2*
a*d*(d*e - 2*c*f) + b*c*(d*e + c*f))*Sqrt[e + f*x^2]*EllipticE[ArcTan[(Sqrt
[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(3*c^(3/2)*Sqrt[d]*(d*e - c*f)^2*Sqrt[c
+ d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))]) - (Sqrt[e]*Sqrt[f]*(2*b*c*e
 + a*d*e - 3*a*c*f)*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]],
1 - (d*e)/(c*f)])/(3*c^2*(d*e - c*f)^2*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))
]*Sqrt[e + f*x^2])
```


Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 525

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{a+bx^2}{\left(c+dx^2\right)^{5/2}\sqrt{e+fx^2}}\,dx = -\frac{(bc-ad)x\sqrt{e+fx^2}}{3c(de-cf)\left(c+dx^2\right)^{3/2}} - \frac{\int \frac{-bce-2ade+3acf+(bc-ad)fx^2}{\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}}\,dx}{3c(de-cf)}$$

$$= -\frac{(bc-ad)x\sqrt{e+fx^2}}{3c(de-cf)\left(c+dx^2\right)^{3/2}} - \frac{(f(2bce+ade-3acf))\int \frac{1}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{3c(de-cf)^2} + \frac{(2ad(de-2cf)+ \ldots}{3c \ldots}$$

$$= -\frac{(bc-ad)x\sqrt{e+fx^2}}{3c(de-cf)\left(c+dx^2\right)^{3/2}} + \frac{(2ad(de-2cf)+bc(de+cf))\sqrt{e+fx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de} \ldots\right.}{3c^{3/2}\sqrt{d}(de-cf)^2\sqrt{c+dx^2}\sqrt{\frac{c\left(e+fx^2\right)}{e\left(c+dx^2\right)}}}$$

Mathematica [C] time = 1.08201, size = 302, normalized size = 1.06

$$\frac{i\left(c+dx^2\right)\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}(cf-de)(-3acf+2ade+bce)\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{\frac{d}{c}}\right),\frac{cf}{de}\right)+x\sqrt{\frac{d}{c}}\left(e+fx^2\right)\left(ad\left(-5c^2f \ldots\right.\right.}{3c^2\sqrt{\frac{d}{c}}\left(\ldots\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/((c + d*x^2)^(5/2)*Sqrt[e + f*x^2]),x]
```

```
[Out] (Sqrt[d/c]*x*(e + f*x^2)*(b*c*(2*c^2*f + d^2*e*x^2 + c*d*f*x^2) + a*d*(-5*c
^2*f + 2*d^2*e*x^2 + c*d*(3*e - 4*f*x^2))) + I*e*(2*a*d*(d*e - 2*c*f) + b*c
*(d*e + c*f))*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE
[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + I*(-(d*e) + c*f)*(b*c*e + 2*a*d*e -
 3*a*c*f)*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*A
rcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/(3*c^2*Sqrt[d/c]*(d*e - c*f)^2*(c + d*x^
2)^(3/2)*Sqrt[e + f*x^2])
```

Maple [B] time = 0.031, size = 1352, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x)
```

```
[Out] 1/3*(-5*x*a*c^2*d*e*f*(-d/c)^(1/2)-EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2)
)*b*c^3*e*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+4*EllipticE(x*(-d/c)^(1
/2),(c*f/d/e)^(1/2))*a*c^2*d*e*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-x^
3*a*c*d^2*e*f*(-d/c)^(1/2)+x^3*b*c^2*d*e*f*(-d/c)^(1/2)+3*x*a*c*d^2*e^2*(-d
/c)^(1/2)+2*x*b*c^3*e*f*(-d/c)^(1/2)-4*x^5*a*c*d^2*f^2*(-d/c)^(1/2)+2*x^5*a
*d^3*e*f*(-d/c)^(1/2)+x^5*b*c^2*d*f^2*(-d/c)^(1/2)-5*x^3*a*c^2*d*f^2*(-d/c)
^(1/2)+2*x^3*a*d^3*e^2*(-d/c)^(1/2)+3*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1
/2))*a*c^3*f^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+x^3*b*c*d^2*e^2*(-d/
c)^(1/2)-EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*b*c^2*d*e*f*((d*x^2+
c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+4*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))
*x^2*a*c*d^2*e*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+x^5*b*c*d^2*e*f*(-
d/c)^(1/2)+2*x^3*b*c^3*f^2*(-d/c)^(1/2)+3*EllipticF(x*(-d/c)^(1/2),(c*f/d/e
)^(1/2))*x^2*a*c^2*d*f^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+2*Elliptic
F(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a*d^3*e^2*((d*x^2+c)/c)^(1/2)*((f*x^2
+e)/e)^(1/2)+2*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c*d^2*e^2*((d*x^
2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-2*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2
))*x^2*a*d^3*e^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+EllipticF(x*(-d/c)
^(1/2),(c*f/d/e)^(1/2))*b*c^2*d*e^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)
-2*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c*d^2*e^2*((d*x^2+c)/c)^(1/2
)*((f*x^2+e)/e)^(1/2)-EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^3*e*f*(
(d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(
1/2))*b*c^2*d*e^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+EllipticF(x*(-d/c
)^(1/2),(c*f/d/e)^(1/2))*x^2*b*c*d^2*e^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^
(1/2)-EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*b*c*d^2*e^2*((d*x^2+c)/
c)^(1/2)*((f*x^2+e)/e)^(1/2)-5*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*
c^2*d*e*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-EllipticE(x*(-d/c)^(1/2),
(c*f/d/e)^(1/2))*x^2*b*c^2*d*e*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-5*
EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a*c*d^2*e*f*((d*x^2+c)/c)^(1/
2)*((f*x^2+e)/e)^(1/2))/(f*x^2+e)^(1/2)/(c*f-d*e)^2/(-d/c)^(1/2)/c^2/(d*x^2
+c)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx^2 + a}{\left(dx^2 + c\right)^{\frac{5}{2}}\sqrt{fx^2 + e}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)/((d*x^2 + c)^(5/2)*sqrt(f*x^2 + e)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2 + a\right)\sqrt{dx^2 + c}\sqrt{fx^2 + e}}{d^3fx^8 + \left(d^3e + 3\,cd^2f\right)x^6 + 3\left(cd^2e + c^2df\right)x^4 + c^3e + \left(3\,c^2de + c^3f\right)x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(d^3*f*x^8 + (d^3*e +
3*c*d^2*f)*x^6 + 3*(c*d^2*e + c^2*d*f)*x^4 + c^3*e + (3*c^2*d*e + c^3*f)*x^
```

```
2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a+bx^2}{\left(c+dx^2\right)^{\frac{5}{2}}\sqrt{e+fx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/(d*x**2+c)**(5/2)/(f*x**2+e)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)/((c + d*x**2)**(5/2)*sqrt(e + f*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx^2+a}{\left(dx^2+c\right)^{\frac{5}{2}}\sqrt{fx^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)/((d*x^2 + c)^(5/2)*sqrt(f*x^2 + e)), x)
```

3.41 $\int \frac{a+bx^2}{\left(c+dx^2\right)^{7/2}\sqrt{e+fx^2}}\,dx$

Optimal. Leaf size=401

$$-\frac{\sqrt{e}\sqrt{f}\sqrt{c+dx^2}\left(a\left(15c^2f^2-11cdef+4d^2e^2\right)+bce(de-9cf)\right)\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{15c^3\sqrt{e+fx^2}(de-cf)^3\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}+\frac{\sqrt{e+fx^2}\left(ad\left(23c\right.\right.}{}$$

```
[Out] -((b*c - a*d)*x*Sqrt[e + f*x^2])/(5*c*(d*e - c*f)*(c + d*x^2)^(5/2)) + ((4*
a*d*(d*e - 2*c*f) + b*c*(d*e + 3*c*f))*x*Sqrt[e + f*x^2])/(15*c^2*(d*e - c*
f)^2*(c + d*x^2)^(3/2)) + ((b*c*(2*d^2*e^2 - 7*c*d*e*f - 3*c^2*f^2) + a*d*(
8*d^2*e^2 - 23*c*d*e*f + 23*c^2*f^2))*Sqrt[e + f*x^2]*EllipticE[ArcTan[(Sqr
t[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(15*c^(5/2)*Sqrt[d]*(d*e - c*f)^3*Sqrt[
c + d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))]) - (Sqrt[e]*Sqrt[f]*(b*c*e
*(d*e - 9*c*f) + a*(4*d^2*e^2 - 11*c*d*e*f + 15*c^2*f^2))*Sqrt[c + d*x^2]*E
llipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(15*c^3*(d*e - c*f)
^3*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rubi [A] time = 0.413409, antiderivative size = 401, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {527, 525, 418, 411}

$$-\frac{\sqrt{e}\sqrt{f}\sqrt{c+dx^2}\left(a\left(15c^2f^2-11cdef+4d^2e^2\right)+bce(de-9cf)\right)F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15c^3\sqrt{e+fx^2}(de-cf)^3\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}+\frac{\sqrt{e+fx^2}\left(ad\left(23c^2f^2-23\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/((c + d*x^2)^(7/2)*Sqrt[e + f*x^2]),x]
```


```
[Out] -((b*c - a*d)*x*Sqrt[e + f*x^2])/(5*c*(d*e - c*f)*(c + d*x^2)^(5/2)) + ((4*
a*d*(d*e - 2*c*f) + b*c*(d*e + 3*c*f))*x*Sqrt[e + f*x^2])/(15*c^2*(d*e - c*
f)^2*(c + d*x^2)^(3/2)) + ((b*c*(2*d^2*e^2 - 7*c*d*e*f - 3*c^2*f^2) + a*d*(
8*d^2*e^2 - 23*c*d*e*f + 23*c^2*f^2))*Sqrt[e + f*x^2]*EllipticE[ArcTan[(Sqr
t[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(15*c^(5/2)*Sqrt[d]*(d*e - c*f)^3*Sqrt[
c + d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))]) - (Sqrt[e]*Sqrt[f]*(b*c*e
*(d*e - 9*c*f) + a*(4*d^2*e^2 - 11*c*d*e*f + 15*c^2*f^2))*Sqrt[c + d*x^2]*E
llipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(15*c^3*(d*e - c*f)
^3*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```


Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 525

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
```

```
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{a+bx^2}{\left(c+dx^2\right)^{7/2}\sqrt{e+fx^2}}\,dx = -\frac{(bc-ad)x\sqrt{e+fx^2}}{5c(de-cf)\left(c+dx^2\right)^{5/2}} - \frac{\int \frac{-bce-4ade+5acf+3(bc-ad)fx^2}{\left(c+dx^2\right)^{5/2}\sqrt{e+fx^2}}\,dx}{5c(de-cf)}$$

$$= -\frac{(bc-ad)x\sqrt{e+fx^2}}{5c(de-cf)\left(c+dx^2\right)^{5/2}} + \frac{(4ad(de-2cf)+bc(de+3cf))x\sqrt{e+fx^2}}{15c^2(de-cf)^2\left(c+dx^2\right)^{3/2}} + \frac{\int \frac{2bce(de-3cf)+a\left(8d\right.}{}}{}$$

$$= -\frac{(bc-ad)x\sqrt{e+fx^2}}{5c(de-cf)\left(c+dx^2\right)^{5/2}} + \frac{(4ad(de-2cf)+bc(de+3cf))x\sqrt{e+fx^2}}{15c^2(de-cf)^2\left(c+dx^2\right)^{3/2}} - \frac{\left(f\left(bce(de-9cf)\right.\right.}{}$$

$$= -\frac{(bc-ad)x\sqrt{e+fx^2}}{5c(de-cf)\left(c+dx^2\right)^{5/2}} + \frac{(4ad(de-2cf)+bc(de+3cf))x\sqrt{e+fx^2}}{15c^2(de-cf)^2\left(c+dx^2\right)^{3/2}} + \frac{\left(bc\left(2d^2e^2-7cde\right.\right.}{}$$

Mathematica [C] time = 1.29057, size = 393, normalized size = 0.98

$$\frac{-x\sqrt{\frac{d}{c}}\left(e+fx^2\right)\left(\left(c+dx^2\right)^2\left(ad\left(-23c^2f^2+23cdef-8d^2e^2\right)+bc\left(3c^2f^2+7cdef-2d^2e^2\right)\right)+3c^2(bc-ad)(de-cf)^2+c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/((c + d*x^2)^(7/2)*Sqrt[e + f*x^2]),x]
```

```
[Out] (-(Sqrt[d/c]*x*(e + f*x^2)*(3*c^2*(b*c - a*d)*(d*e - c*f)^2 + c*(-(d*e) + c
*f)*(4*a*d*(d*e - 2*c*f) + b*c*(d*e + 3*c*f))*(c + d*x^2) + (a*d*(-8*d^2*e^
2 + 23*c*d*e*f - 23*c^2*f^2) + b*c*(-2*d^2*e^2 + 7*c*d*e*f + 3*c^2*f^2))*(c
 + d*x^2)^2)) - I*(c + d*x^2)^2*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*(e*
(a*d*(-8*d^2*e^2 + 23*c*d*e*f - 23*c^2*f^2) + b*c*(-2*d^2*e^2 + 7*c*d*e*f +
 3*c^2*f^2))*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + (d*e - c*f)*(
2*b*c*e*(d*e - 3*c*f) + a*(8*d^2*e^2 - 19*c*d*e*f + 15*c^2*f^2))*EllipticF[
I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]))/(15*c^3*Sqrt[d/c]*(d*e - c*f)^3*(c +
```

```
d*x^2)^(5/2)*Sqrt[e + f*x^2])
```

Maple [B] time = 0.042, size = 3039, normalized size = 7.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/(d*x^2+c)^(7/2)/(f*x^2+e)^(1/2),x)
```

```
[Out] 1/15*(30*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a*c^4*d*f^3*((d*x^2+
c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-7*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))
*b*c^4*d*e^2*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-16*EllipticF(x*(-d/c
)^(1/2),(c*f/d/e)^(1/2))*x^2*a*c*d^4*e^3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^
(1/2)-4*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*b*c^2*d^3*e^3*((d*x^2
+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+16*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2
))*x^2*a*c*d^4*e^3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-2*EllipticF(x*(-
d/c)^(1/2),(c*f/d/e)^(1/2))*x^4*b*c*d^4*e^3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/
e)^(1/2)+2*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^4*b*c*d^4*e^3*((d*x^
2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+8*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2
))*b*c^4*d*e^2*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+23*EllipticE(x*(-d
/c)^(1/2),(c*f/d/e)^(1/2))*a*c^4*d*e*f^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^
(1/2)-23*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c^3*d^2*e^2*f*((d*x^2+
c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-34*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2)
)*a*c^4*d*e*f^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+27*EllipticF(x*(-d/
c)^(1/2),(c*f/d/e)^(1/2))*a*c^3*d^2*e^2*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)
^(1/2)+8*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^4*b*c^2*d^3*e^2*f*((d*
x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+23*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(
1/2))*x^4*a*c^2*d^3*e*f^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-23*Ellipt
icE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^4*a*c*d^4*e^2*f*((d*x^2+c)/c)^(1/2)*(
(f*x^2+e)/e)^(1/2)-8*x^5*a*d^5*e^3*(-d/c)^(1/2)+15*EllipticF(x*(-d/c)^(1/2)
,(c*f/d/e)^(1/2))*a*c^5*f^3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-54*x^5*
a*c^3*d^2*f^3*(-d/c)^(1/2)+3*x^7*b*c^3*d^2*f^3*(-d/c)^(1/2)+4*EllipticE(x*(
-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*b*c^2*d^3*e^3*((d*x^2+c)/c)^(1/2)*((f*x^2+
e)/e)^(1/2)+9*x^5*b*c^4*d*f^3*(-d/c)^(1/2)-2*x^5*b*c*d^4*e^3*(-d/c)^(1/2)-3
4*x^3*a*c^4*d*f^3*(-d/c)^(1/2)-20*x^3*a*c*d^4*e^3*(-d/c)^(1/2)-5*x^3*b*c^2*
d^3*e^3*(-d/c)^(1/2)-15*x*a*c^2*d^3*e^3*(-d/c)^(1/2)+9*x*b*c^5*e*f^2*(-d/c)
^(1/2)-23*x^7*a*c^2*d^3*f^3*(-d/c)^(1/2)-8*x^7*a*d^5*e^2*f*(-d/c)^(1/2)-2*E
llipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^3*d^2*e^3*((d*x^2+c)/c)^(1/2)*
((f*x^2+e)/e)^(1/2)+8*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c^2*d^3*e
^3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-3*EllipticE(x*(-d/c)^(1/2),(c*f/
d/e)^(1/2))*b*c^5*e*f^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+2*EllipticE
(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^3*d^2*e^3*((d*x^2+c)/c)^(1/2)*((f*x^2+
e)/e)^(1/2)+15*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^4*a*c^3*d^2*f^3*
((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-12*EllipticF(x*(-d/c)^(1/2),(c*f/d/
e)^(1/2))*x^2*b*c^4*d*e*f^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+46*Elli
pticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a*c^3*d^2*e*f^2*((d*x^2+c)/c)^(1/
2)*((f*x^2+e)/e)^(1/2)-46*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a*c
^2*d^3*e^2*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-6*EllipticF(x*(-d/c)^(
1/2),(c*f/d/e)^(1/2))*b*c^5*e*f^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+2
3*x^7*a*c*d^4*e*f^2*(-d/c)^(1/2)+7*x^7*b*c^2*d^3*e*f^2*(-d/c)^(1/2)-2*x^7*b
*c*d^4*e^2*f*(-d/c)^(1/2)+35*x^5*a*c^2*d^3*e*f^2*(-d/c)^(1/2)+3*x^5*a*c*d^4
*e^2*f*(-d/c)^(1/2)+15*x^5*b*c^3*d^2*e*f^2*(-d/c)^(1/2)+2*x^5*b*c^2*d^3*e^2
*f*(-d/c)^(1/2)-13*x^3*a*c^3*d^2*e*f^2*(-d/c)^(1/2)-34*x*a*c^4*d*e*f^2*(-d/
c)^(1/2)+41*x*a*c^3*d^2*e^2*f*(-d/c)^(1/2)-x*b*c^4*d*e^2*f*(-d/c)^(1/2)-8*E
llipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^4*a*d^5*e^3*((d*x^2+c)/c)^(1/2)*
((f*x^2+e)/e)^(1/2)+43*x^3*a*c^2*d^3*e^2*f*(-d/c)^(1/2)+8*x^3*b*c^4*d*e*f^2
```

```
*(-d/c)^(1/2)+12*x^3*b*c^3*d^2*e^2*f*(-d/c)^(1/2)+9*x^3*b*c^5*f^3*(-d/c)^(1
/2)+8*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^4*a*d^5*e^3*((d*x^2+c)/c)
^(1/2)*((f*x^2+e)/e)^(1/2)-8*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c^
2*d^3*e^3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-3*EllipticE(x*(-d/c)^(1/2
),(c*f/d/e)^(1/2))*x^4*b*c^3*d^2*e*f^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1
/2)-7*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^4*b*c^2*d^3*e^2*f*((d*x^2
+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-68*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2
))*x^2*a*c^3*d^2*e*f^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+54*EllipticF
(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a*c^2*d^3*e^2*f*((d*x^2+c)/c)^(1/2)*((
f*x^2+e)/e)^(1/2)-6*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*b*c^4*d*e
*f^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-14*EllipticE(x*(-d/c)^(1/2),(c
*f/d/e)^(1/2))*x^2*b*c^3*d^2*e^2*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-
34*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^4*a*c^2*d^3*e*f^2*((d*x^2+c)
/c)^(1/2)*((f*x^2+e)/e)^(1/2)+27*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*
x^4*a*c*d^4*e^2*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-6*EllipticF(x*(-d
/c)^(1/2),(c*f/d/e)^(1/2))*x^4*b*c^3*d^2*e*f^2*((d*x^2+c)/c)^(1/2)*((f*x^2+
e)/e)^(1/2)+16*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*b*c^3*d^2*e^2*
f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2))/(f*x^2+e)^(1/2)/(c*f-d*e)^3/(-d/
c)^(1/2)/c^3/(d*x^2+c)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx^2 + a}{\left(dx^2 + c\right)^{\frac{7}{2}} \sqrt{fx^2 + e}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+c)^(7/2)/(f*x^2+e)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)/((d*x^2 + c)^(7/2)*sqrt(f*x^2 + e)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(bx^2 + a\right)\sqrt{dx^2 + c}\sqrt{fx^2 + e}}{d^4fx^{10} + \left(d^4e + 4\,cd^3f\right)x^8 + 2\left(2\,cd^3e + 3\,c^2d^2f\right)x^6 + c^4e + 2\left(3\,c^2d^2e + 2\,c^3df\right)x^4 + \left(4\,c^3de + c^4f\right)x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+c)^(7/2)/(f*x^2+e)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(d^4*f*x^10 + (d^4*e +
 4*c*d^3*f)*x^8 + 2*(2*c*d^3*e + 3*c^2*d^2*f)*x^6 + c^4*e + 2*(3*c^2*d^2*e
+ 2*c^3*d*f)*x^4 + (4*c^3*d*e + c^4*f)*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/(d*x**2+c)**(7/2)/(f*x**2+e)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx^2 + a}{\left(dx^2 + c\right)^{\frac{7}{2}}\sqrt{fx^2 + e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+c)^(7/2)/(f*x^2+e)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)/((d*x^2 + c)^(7/2)*sqrt(f*x^2 + e)), x)
```

3.42 $\int \frac{(a+bx^2)(c+dx^2)^{5/2}}{(e+fx^2)^{3/2}}\,dx$

Optimal. Leaf size=501

$$-\frac{\sqrt{e}\sqrt{c+dx^2}\left(10adf(2de-3cf)-b\left(15c^2f^2-41cdef+24d^2e^2\right)\right)\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{15f^{7/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{x\sqrt{c+dx^2}\left(5af\left(3c\right.\right.}{}$$

```
[Out] -((5*a*f*(8*d^2*e^2 - 13*c*d*e*f + 3*c^2*f^2) - 2*b*e*(24*d^2*e^2 - 44*c*d*
e*f + 19*c^2*f^2))*x*Sqrt[c + d*x^2])/(15*e*f^3*Sqrt[e + f*x^2]) - ((b*e -
a*f)*x*(c + d*x^2)^(5/2))/(e*f*Sqrt[e + f*x^2]) - (d*(b*e*(24*d*e - 23*c*f)
 - 5*a*f*(4*d*e - 3*c*f))*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(15*e*f^3) + (
d*(6*b*e - 5*a*f)*x*(c + d*x^2)^(3/2)*Sqrt[e + f*x^2])/(5*e*f^2) + ((5*a*f*
(8*d^2*e^2 - 13*c*d*e*f + 3*c^2*f^2) - 2*b*e*(24*d^2*e^2 - 44*c*d*e*f + 19*
c^2*f^2))*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/
(c*f)])/(15*Sqrt[e]*f^(7/2)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e +
f*x^2]) - (Sqrt[e]*(10*a*d*f*(2*d*e - 3*c*f) - b*(24*d^2*e^2 - 41*c*d*e*f +
 15*c^2*f^2))*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d
*e)/(c*f)])/(15*f^(7/2)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^
2])
```

Rubi [A] time = 0.598706, antiderivative size = 501, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {526, 528, 531, 418, 492, 411}

$$-\frac{x\sqrt{c+dx^2}\left(5af\left(3c^2f^2-13cdef+8d^2e^2\right)-2be\left(19c^2f^2-44cdef+24d^2e^2\right)\right)}{15ef^3\sqrt{e+fx^2}}-\frac{\sqrt{e}\sqrt{c+dx^2}\left(10adf(2de-3cf)-b\left(15\right.\right.}{15f^{7/2}\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(c + d*x^2)^(5/2))/(e + f*x^2)^(3/2),x]
```


```
[Out] -((5*a*f*(8*d^2*e^2 - 13*c*d*e*f + 3*c^2*f^2) - 2*b*e*(24*d^2*e^2 - 44*c*d*
e*f + 19*c^2*f^2))*x*Sqrt[c + d*x^2])/(15*e*f^3*Sqrt[e + f*x^2]) - ((b*e -
a*f)*x*(c + d*x^2)^(5/2))/(e*f*Sqrt[e + f*x^2]) - (d*(b*e*(24*d*e - 23*c*f)
 - 5*a*f*(4*d*e - 3*c*f))*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(15*e*f^3) + (
d*(6*b*e - 5*a*f)*x*(c + d*x^2)^(3/2)*Sqrt[e + f*x^2])/(5*e*f^2) + ((5*a*f*
(8*d^2*e^2 - 13*c*d*e*f + 3*c^2*f^2) - 2*b*e*(24*d^2*e^2 - 44*c*d*e*f + 19*
c^2*f^2))*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/
(c*f)])/(15*Sqrt[e]*f^(7/2)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e +
f*x^2]) - (Sqrt[e]*(10*a*d*f*(2*d*e - 3*c*f) - b*(24*d^2*e^2 - 41*c*d*e*f +
 15*c^2*f^2))*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d
*e)/(c*f)])/(15*f^(7/2)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^
2])
```


Rule 526

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^q)/(a*b*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p +
```

```
 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)\left(c+dx^2\right)^{5/2}}{\left(e+fx^2\right)^{3/2}}\,dx = -\frac{(be-af)x\left(c+dx^2\right)^{5/2}}{ef\sqrt{e+fx^2}} - \frac{\int \frac{\left(c+dx^2\right)^{3/2}\left(-bce-d(6be-5af)x^2\right)}{\sqrt{e+fx^2}}\,dx}{ef}$$

$$= -\frac{(be-af)x\left(c+dx^2\right)^{5/2}}{ef\sqrt{e+fx^2}} + \frac{d(6be-5af)x\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}}{5ef^2} - \int \sqrt{c+dx^2}\left(ce(6bde-5bcf-5ad\right.$$

$$= -\frac{(be-af)x\left(c+dx^2\right)^{5/2}}{ef\sqrt{e+fx^2}} - \frac{d(be(24de-23cf)-5af(4de-3cf))x\sqrt{c+dx^2}\sqrt{e+fx^2}}{15ef^3} + \frac{d(6}{}$$

$$= -\frac{(be-af)x\left(c+dx^2\right)^{5/2}}{ef\sqrt{e+fx^2}} - \frac{d(be(24de-23cf)-5af(4de-3cf))x\sqrt{c+dx^2}\sqrt{e+fx^2}}{15ef^3} + \frac{d(6}{}$$

$$= -\frac{\left(5af\left(8d^2e^2-13cdef+3c^2f^2\right)-2be\left(24d^2e^2-44cdef+19c^2f^2\right)\right)x\sqrt{c+dx^2}}{15ef^3\sqrt{e+fx^2}} - \frac{(be-af}{ef}$$

$$= -\frac{\left(5af\left(8d^2e^2-13cdef+3c^2f^2\right)-2be\left(24d^2e^2-44cdef+19c^2f^2\right)\right)x\sqrt{c+dx^2}}{15ef^3\sqrt{e+fx^2}} - \frac{(be-af}{ef}$$

Mathematica [C] time = 1.16102, size = 369, normalized size = 0.74

$$\frac{-ie\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}(cf-de)\left(5adf(9cf-8de)+b\left(15c^2f^2-64cdef+48d^2e^2\right)\right)\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{\frac{d}{c}}\right),\frac{cf}{de}\right)+fx\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(c + d*x^2)^(5/2))/(e + f*x^2)^(3/2),x]
```

```
[Out] (Sqrt[d/c]*f*x*(c + d*x^2)*(5*a*f*(-6*c*d*e*f + 3*c^2*f^2 + d^2*e*(4*e + f*
x^2)) + b*e*(-15*c^2*f^2 + c*d*f*(41*e + 11*f*x^2) - 3*d^2*(8*e^2 + 2*e*f*x
^2 - f^2*x^4))) - I*d*e*(-5*a*f*(8*d^2*e^2 - 13*c*d*e*f + 3*c^2*f^2) + 2*b*
e*(24*d^2*e^2 - 44*c*d*e*f + 19*c^2*f^2))*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x
^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - I*e*(-(d*e) + c*f)*
(5*a*d*f*(-8*d*e + 9*c*f) + b*(48*d^2*e^2 - 64*c*d*e*f + 15*c^2*f^2))*Sqrt[
1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/
(d*e)])/(15*Sqrt[d/c]*e*f^4*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Maple [B] time = 0.046, size = 1169, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x^2+c)^(5/2)/(f*x^2+e)^(3/2),x)
```

```
[Out] -1/15*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)*(-40*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e
)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d^3*e^3*f+24*(-d/c)^(1/
```

```
2)*x*b*c*d^2*e^3*f-38*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(
-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^2*d*e^2*f^2-15*x*a*c^3*f^4*(-d/c)^(1/2)+79
*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)
^(1/2))*b*c^2*d*e^2*f^2-65*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Elliptic
E(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c*d^2*e^2*f^2-112*((d*x^2+c)/c)^(1/2)*(
(f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c*d^2*e^3*f+
15*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/
e)^(1/2))*a*c^2*d*e*f^3-45*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Elliptic
F(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c^2*d*e*f^3-15*((d*x^2+c)/c)^(1/2)*((f*
x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^3*e*f^3+40*((
d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1
/2))*a*d^3*e^3*f-5*(-d/c)^(1/2)*x^5*a*d^3*e*f^3+48*((d*x^2+c)/c)^(1/2)*((f*
x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*d^3*e^4-48*((d*
x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2
))*b*d^3*e^4-14*(-d/c)^(1/2)*x^5*b*c*d^2*e*f^3+25*(-d/c)^(1/2)*x^3*a*c*d^2*
e*f^3+4*(-d/c)^(1/2)*x^3*b*c^2*d*e*f^3+88*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)
^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c*d^2*e^3*f-41*(-d/c)^(1
/2)*x*b*c^2*d*e^2*f^2-35*(-d/c)^(1/2)*x^3*b*c*d^2*e^2*f^2+30*(-d/c)^(1/2)*x
*a*c^2*d*e*f^3-20*(-d/c)^(1/2)*x*a*c*d^2*e^2*f^2+6*(-d/c)^(1/2)*x^5*b*d^3*e
^2*f^2-15*(-d/c)^(1/2)*x^3*a*c^2*d*f^4-20*(-d/c)^(1/2)*x^3*a*d^3*e^2*f^2+24
*(-d/c)^(1/2)*x^3*b*d^3*e^3*f+15*(-d/c)^(1/2)*x*b*c^3*e*f^3-3*(-d/c)^(1/2)*
x^7*b*d^3*e*f^3+85*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/
c)^(1/2),(c*f/d/e)^(1/2))*a*c*d^2*e^2*f^2)/f^4/(d*f*x^4+c*f*x^2+d*e*x^2+c*e
)/(-d/c)^(1/2)/e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)\left(dx^2+c\right)^{\frac{5}{2}}}{\left(fx^2+e\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^(5/2)/(f*x^2+e)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)*(d*x^2 + c)^(5/2)/(f*x^2 + e)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bd^2x^6+\left(2\,bcd+ad^2\right)x^4+ac^2+\left(bc^2+2\,acd\right)x^2\right)\sqrt{dx^2+c}\sqrt{fx^2+e}}{f^2x^4+2\,efx^2+e^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^(5/2)/(f*x^2+e)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*d^2*x^6 + (2*b*c*d + a*d^2)*x^4 + a*c^2 + (b*c^2 + 2*a*c*d)*x^2
)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(f^2*x^4 + 2*e*f*x^2 + e^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^2\right)\left(c+dx^2\right)^{\frac{5}{2}}}{\left(e+fx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+c)**(5/2)/(f*x**2+e)**(3/2),x)
```

```
[Out] Integral((a + b*x**2)*(c + d*x**2)**(5/2)/(e + f*x**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)\left(dx^2+c\right)^{\frac{5}{2}}}{\left(fx^2+e\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^(5/2)/(f*x^2+e)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)*(d*x^2 + c)^(5/2)/(f*x^2 + e)^(3/2), x)
```

3.43 $\int \frac{(a+bx^2)(c+dx^2)^{3/2}}{(e+fx^2)^{3/2}}\,dx$

Optimal. Leaf size=358

$$-\frac{\sqrt{e}\sqrt{c+dx^2}(-3adf-3bcf+4bde)\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{3f^{5/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{dx\sqrt{c+dx^2}\sqrt{e+fx^2}(4be-3af)}{3ef^2}-\frac{x\sqrt{c+dx^2}}{}$$

```
[Out] -((b*e*(8*d*e - 7*c*f) - 3*a*f*(2*d*e - c*f))*x*Sqrt[c + d*x^2])/(3*e*f^2*S
qrt[e + f*x^2]) - ((b*e - a*f)*x*(c + d*x^2)^(3/2))/(e*f*Sqrt[e + f*x^2]) +
 (d*(4*b*e - 3*a*f)*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(3*e*f^2) + ((b*e*(8
*d*e - 7*c*f) - 3*a*f*(2*d*e - c*f))*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt
[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(3*Sqrt[e]*f^(5/2)*Sqrt[(e*(c + d*x^2))/
(c*(e + f*x^2))]*Sqrt[e + f*x^2]) - (Sqrt[e]*(4*b*d*e - 3*b*c*f - 3*a*d*f)*
Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(3
*f^(5/2)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rubi [A] time = 0.36314, antiderivative size = 358, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {526, 528, 531, 418, 492, 411}

$$\frac{dx\sqrt{c+dx^2}\sqrt{e+fx^2}(4be-3af)}{3ef^2}-\frac{x\sqrt{c+dx^2}(be(8de-7cf)-3af(2de-cf))}{3ef^2\sqrt{e+fx^2}}-\frac{\sqrt{e}\sqrt{c+dx^2}(-3adf-3bcf+4bde)F}{3f^{5/2}\sqrt{e+fx^2}\sqrt{\frac{e(c}{c(e}}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(c + d*x^2)^(3/2))/(e + f*x^2)^(3/2),x]
```


```
[Out] -((b*e*(8*d*e - 7*c*f) - 3*a*f*(2*d*e - c*f))*x*Sqrt[c + d*x^2])/(3*e*f^2*S
qrt[e + f*x^2]) - ((b*e - a*f)*x*(c + d*x^2)^(3/2))/(e*f*Sqrt[e + f*x^2]) +
 (d*(4*b*e - 3*a*f)*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(3*e*f^2) + ((b*e*(8
*d*e - 7*c*f) - 3*a*f*(2*d*e - c*f))*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt
[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(3*Sqrt[e]*f^(5/2)*Sqrt[(e*(c + d*x^2))/
(c*(e + f*x^2))]*Sqrt[e + f*x^2]) - (Sqrt[e]*(4*b*d*e - 3*b*c*f - 3*a*d*f)*
Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(3
*f^(5/2)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```


Rule 526

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^q)/(a*b*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p +
 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
```

```
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}{\left(e+fx^2\right)^{3/2}}\,dx = -\frac{(be-af)x\left(c+dx^2\right)^{3/2}}{ef\sqrt{e+fx^2}} - \frac{\int \frac{\sqrt{c+dx^2}\left(-bce-d(4be-3af)x^2\right)}{\sqrt{e+fx^2}}\,dx}{ef}$$

$$= -\frac{(be-af)x\left(c+dx^2\right)^{3/2}}{ef\sqrt{e+fx^2}} + \frac{d(4be-3af)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{3ef^2} - \frac{\int \frac{ce(4bde-3bcf-3adf)+d(be(8de-\cdots}{\sqrt{c+dx^2}\sqrt{e+f\cdots}}}{3ef^2}$$

$$= -\frac{(be-af)x\left(c+dx^2\right)^{3/2}}{ef\sqrt{e+fx^2}} + \frac{d(4be-3af)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{3ef^2} - \frac{(c(4bde-3bcf-3adf))\int\cdots}{3f^2}$$

$$= -\frac{(be(8de-7cf)-3af(2de-cf))x\sqrt{c+dx^2}}{3ef^2\sqrt{e+fx^2}} - \frac{(be-af)x\left(c+dx^2\right)^{3/2}}{ef\sqrt{e+fx^2}} + \frac{d(4be-3af)x\sqrt{c\cdots}}{3e\cdots}$$

$$= -\frac{(be(8de-7cf)-3af(2de-cf))x\sqrt{c+dx^2}}{3ef^2\sqrt{e+fx^2}} - \frac{(be-af)x\left(c+dx^2\right)^{3/2}}{ef\sqrt{e+fx^2}} + \frac{d(4be-3af)x\sqrt{c\cdots}}{3e\cdots}$$

Mathematica [C] time = 0.765159, size = 260, normalized size = 0.73

$$\frac{-ie\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}(cf-de)(6adf+3bcf-8bde)\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{\frac{d}{c}}\right),\frac{cf}{de}\right)+fx\sqrt{\frac{d}{c}}\left(c+dx^2\right)\left(3af(cf-de)\right.}{3ef^3\sqrt{\frac{d}{c}}\sqrt{c+dx^2}\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(c + d*x^2)^(3/2))/(e + f*x^2)^(3/2),x]
```

```
[Out] (Sqrt[d/c]*f*x*(c + d*x^2)*(3*a*f*(-(d*e) + c*f) + b*e*(4*d*e - 3*c*f + d*f
*x^2)) - I*d*e*(-3*a*f*(-2*d*e + c*f) + b*e*(-8*d*e + 7*c*f))*Sqrt[1 + (d*x
^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] -
 I*e*(-(d*e) + c*f)*(-8*b*d*e + 3*b*c*f + 6*a*d*f)*Sqrt[1 + (d*x^2)/c]*Sqrt
[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/(3*Sqrt[d/c
]*e*f^3*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Maple [A] time = 0.025, size = 750, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x)
```

```
[Out] 1/3*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)*((-d/c)^(1/2)*x^5*b*d^2*e*f^2+3*(-d/c)^
(1/2)*x^3*a*c*d*f^3-3*(-d/c)^(1/2)*x^3*a*d^2*e*f^2-2*(-d/c)^(1/2)*x^3*b*c*d
*e*f^2+4*(-d/c)^(1/2)*x^3*b*d^2*e^2*f+6*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(
1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c*d*e*f^2-6*((d*x^2+c)/c)^
(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d^2*e
^2*f+3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*
f/d/e)^(1/2))*b*c^2*e*f^2-11*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Ellipt
icF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c*d*e^2*f+8*((d*x^2+c)/c)^(1/2)*((f*x
^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*d^2*e^3-3*((d*x^
2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))
*a*c*d*e*f^2+6*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(
1/2),(c*f/d/e)^(1/2))*a*d^2*e^2*f+7*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)
*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c*d*e^2*f-8*((d*x^2+c)/c)^(1/2
)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*d^2*e^3+3
*x*a*c^2*f^3*(-d/c)^(1/2)-3*(-d/c)^(1/2)*x*a*c*d*e*f^2-3*(-d/c)^(1/2)*x*b*c
^2*e*f^2+4*(-d/c)^(1/2)*x*b*c*d*e^2*f)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)/f^3/e/
(-d/c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)\left(dx^2+c\right)^{\frac{3}{2}}}{\left(fx^2+e\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)*(d*x^2 + c)^(3/2)/(f*x^2 + e)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bdx^4+(bc+ad)x^2+ac\right)\sqrt{dx^2+c}\sqrt{fx^2+e}}{f^2x^4+2\,efx^2+e^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*d*x^4 + (b*c + a*d)*x^2 + a*c)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/
(f^2*x^4 + 2*e*f*x^2 + e^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a+bx^2\right)\left(c+dx^2\right)^{\frac{3}{2}}}{\left(e+fx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+c)**(3/2)/(f*x**2+e)**(3/2),x)
```

```
[Out] Integral((a + b*x**2)*(c + d*x**2)**(3/2)/(e + f*x**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(bx^2+a\right)\left(dx^2+c\right)^{\frac{3}{2}}}{\left(fx^2+e\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)*(d*x^2 + c)^(3/2)/(f*x^2 + e)^(3/2), x)
```

3.44 $\int \frac{(a+bx^2)\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx$

Optimal. Leaf size=258

$$\frac{b\sqrt{e}\sqrt{c+dx^2}\,\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{f^{3/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{\sqrt{c+dx^2}(2be-af)E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{\sqrt{e}f^{3/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{x\sqrt{c+dx^2}(be-af)}{ef\sqrt{e+fx^2}}+\cdots$$

```
[Out] -(((b*e - a*f)*x*Sqrt[c + d*x^2])/(e*f*Sqrt[e + f*x^2])) + ((2*b*e - a*f)*x
*Sqrt[c + d*x^2])/(e*f*Sqrt[e + f*x^2]) - ((2*b*e - a*f)*Sqrt[c + d*x^2]*El
lipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(Sqrt[e]*f^(3/2)*Sqr
t[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (b*Sqrt[e]*Sqrt[c + d
*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(f^(3/2)*Sqr
t[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rubi [A] time = 0.162362, antiderivative size = 258, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {526, 531, 418, 492, 411}

$$-\frac{\sqrt{c+dx^2}(2be-af)E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{\sqrt{e}f^{3/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{x\sqrt{c+dx^2}(be-af)}{ef\sqrt{e+fx^2}}+\frac{x\sqrt{c+dx^2}(2be-af)}{ef\sqrt{e+fx^2}}+\frac{b\sqrt{e}\sqrt{c+dx^2}F\left(\tan^{-1}\left(\cdots\right.\right.}{f^{3/2}\sqrt{e+fx^2}\sqrt{\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*Sqrt[c + d*x^2])/(e + f*x^2)^(3/2),x]
```

```
[Out] -(((b*e - a*f)*x*Sqrt[c + d*x^2])/(e*f*Sqrt[e + f*x^2])) + ((2*b*e - a*f)*x
*Sqrt[c + d*x^2])/(e*f*Sqrt[e + f*x^2]) - ((2*b*e - a*f)*Sqrt[c + d*x^2]*El
lipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(Sqrt[e]*f^(3/2)*Sqr
t[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (b*Sqrt[e]*Sqrt[c + d
*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(f^(3/2)*Sqr
t[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rule 526

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^q)/(a*b*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p +
 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)\sqrt{c+dx^2}}{\left(e+fx^2\right)^{3/2}}\,dx = -\frac{(be-af)x\sqrt{c+dx^2}}{ef\sqrt{e+fx^2}} - \frac{\int \frac{-bce-d(2be-af)x^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{ef}$$

$$= -\frac{(be-af)x\sqrt{c+dx^2}}{ef\sqrt{e+fx^2}} + \frac{(bc)\int \frac{1}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{f} + \frac{(d(2be-af))\int \frac{x^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{ef}$$

$$= -\frac{(be-af)x\sqrt{c+dx^2}}{ef\sqrt{e+fx^2}} + \frac{(2be-af)x\sqrt{c+dx^2}}{ef\sqrt{e+fx^2}} + \frac{b\sqrt{e}\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{f^{3/2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}\sqrt{e+fx^2}} - \frac{(2be \cdots}{\cdots}$$

$$= -\frac{(be-af)x\sqrt{c+dx^2}}{ef\sqrt{e+fx^2}} + \frac{(2be-af)x\sqrt{c+dx^2}}{ef\sqrt{e+fx^2}} - \frac{(2be-af)\sqrt{c+dx^2}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{\sqrt{e}f^{3/2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}\sqrt{e+fx^2}} \cdots$$

Mathematica [C] time = 0.381981, size = 208, normalized size = 0.81

$$\frac{-ie\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}(adf+bcf-2bde)\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{\frac{d}{c}}\right),\frac{cf}{de}\right)+fx\sqrt{\frac{d}{c}}\left(c+dx^2\right)(af-be)-ide\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}} \cdots}{ef^2\sqrt{\frac{d}{c}}\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*Sqrt[c + d*x^2])/(e + f*x^2)^(3/2),x]
```

```
[Out] (Sqrt[d/c]*f*(-(b*e) + a*f)*x*(c + d*x^2) - I*d*e*(2*b*e - a*f)*Sqrt[1 + (d
*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]
 - I*e*(-2*b*d*e + b*c*f + a*d*f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*E
llipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/(Sqrt[d/c]*e*f^2*Sqrt[c + d*
x^2]*Sqrt[e + f*x^2])
```

Maple [A] time = 0.023, size = 393, normalized size = 1.5

$$\frac{1}{\left(dfx^4+cfx^2+dex^2+ce\right)f^2e}\sqrt{dx^2+c}\sqrt{fx^2+e}\left(x^3adf^2\sqrt{-\frac{d}{c}}-\sqrt{-\frac{d}{c}}x^3bdef+\sqrt{\frac{dx^2+c}{c}}\sqrt{\frac{fx^2+e}{e}}\mathrm{EllipticF}\left(x\sqrt{\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

```
[Out] (d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)*(x^3*a*d*f^2*(-d/c)^(1/2)-(-d/c)^(1/2)*x^3*
b*d*e*f+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c
*f/d/e)^(1/2))*a*d*e*f+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*
(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c*e*f-2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(
1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*d*e^2-((d*x^2+c)/c)^(1/2)*
((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d*e*f+2*((d
*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/
2))*b*d*e^2+x*a*c*f^2*(-d/c)^(1/2)-(-d/c)^(1/2)*x*b*c*e*f)/(d*f*x^4+c*f*x^2
+d*e*x^2+c*e)/f^2/e/(-d/c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)\sqrt{dx^2+c}}{\left(fx^2+e\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)*sqrt(d*x^2 + c)/(f*x^2 + e)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(bx^2+a\right)\sqrt{dx^2+c}\sqrt{fx^2+e}}{f^2x^4+2\,efx^2+e^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(f^2*x^4 + 2*e*f*x^2 +
 e^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^2\right)\sqrt{c+dx^2}}{\left(e+fx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+c)**(1/2)/(f*x**2+e)**(3/2),x)
```

```
[Out] Integral((a + b*x**2)*sqrt(c + d*x**2)/(e + f*x**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)\sqrt{dx^2+c}}{\left(fx^2+e\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)*sqrt(d*x^2 + c)/(f*x^2 + e)^(3/2), x)
```

3.45 $\int \frac{a+bx^2}{\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=209

$$\frac{\sqrt{c+dx^2}(be-af)E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{\sqrt{e}\sqrt{f}\sqrt{e+fx^2}(de-cf)\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}-\frac{\sqrt{e}\sqrt{c+dx^2}(bc-ad)\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{e+fx^2}(de-cf)\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}$$

```
[Out] ((b*e - a*f)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*
e)/(c*f)])/(Sqrt[e]*Sqrt[f]*(d*e - c*f)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2)
)]*Sqrt[e + f*x^2]) - ((b*c - a*d)*Sqrt[e]*Sqrt[c + d*x^2]*EllipticF[ArcTan
[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(c*Sqrt[f]*(d*e - c*f)*Sqrt[(e*(c
+ d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rubi [A] time = 0.0845786, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {525, 418, 411}

$$\frac{\sqrt{c+dx^2}(be-af)E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{\sqrt{e}\sqrt{f}\sqrt{e+fx^2}(de-cf)\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}-\frac{\sqrt{e}\sqrt{c+dx^2}(bc-ad)F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{e+fx^2}(de-cf)\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(Sqrt[c + d*x^2]*(e + f*x^2)^(3/2)),x]
```


```
[Out] ((b*e - a*f)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*
e)/(c*f)])/(Sqrt[e]*Sqrt[f]*(d*e - c*f)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2)
)]*Sqrt[e + f*x^2]) - ((b*c - a*d)*Sqrt[e]*Sqrt[c + d*x^2]*EllipticF[ArcTan
[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(c*Sqrt[f]*(d*e - c*f)*Sqrt[(e*(c
+ d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```


Rule 525

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
```

```
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{a+bx^2}{\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}}\,dx = -\frac{(bc-ad)\int \frac{1}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{de-cf}+\frac{(be-af)\int \frac{\sqrt{c+dx^2}}{\left(e+fx^2\right)^{3/2}}\,dx}{de-cf}$$

$$=\frac{(be-af)\sqrt{c+dx^2}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{\sqrt{e}\sqrt{f}(de-cf)\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}\sqrt{e+fx^2}}-\frac{(bc-ad)\sqrt{e}\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{c\sqrt{f}(de-cf)\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}\sqrt{e+fx^2}}$$

Mathematica [C] time = 0.373597, size = 212, normalized size = 1.01

$$\frac{-ibe\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}(cf-de)\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{\frac{d}{c}}\right),\frac{cf}{de}\right)+fx\sqrt{\frac{d}{c}}\left(c+dx^2\right)(af-be)-ide\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}(be-}{ef\sqrt{\frac{d}{c}}\sqrt{c+dx^2}\sqrt{e+fx^2}(cf-de)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(Sqrt[c + d*x^2]*(e + f*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[d/c]*f*(-(b*e) + a*f)*x*(c + d*x^2) - I*d*e*(b*e - a*f)*Sqrt[1 + (d*x
^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] -
 I*b*e*(-(d*e) + c*f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*A
rcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/(Sqrt[d/c]*e*f*(-(d*e) + c*f)*Sqrt[c + d
*x^2]*Sqrt[e + f*x^2])
```

Maple [A] time = 0.025, size = 349, normalized size = 1.7

$$\frac{1}{ef\left(cf-de\right)\left(dfx^4+cfx^2+dex^2+ce\right)}\left(x^3adf^2\sqrt{-\frac{d}{c}}-\sqrt{-\frac{d}{c}}x^3bdef+\sqrt{\frac{dx^2+c}{c}}\sqrt{\frac{fx^2+e}{e}}\text{EllipticF}\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)bc$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

```
[Out] (x^3*a*d*f^2*(-d/c)^(1/2)-(-d/c)^(1/2)*x^3*b*d*e*f+((d*x^2+c)/c)^(1/2)*((f*
x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c*e*f-((d*x^2+c
)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*
d*e^2-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f
/d/e)^(1/2))*a*d*e*f+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-
d/c)^(1/2),(c*f/d/e)^(1/2))*b*d*e^2+x*a*c*f^2*(-d/c)^(1/2)-(-d/c)^(1/2)*x*b
*c*e*f)*(f*x^2+e)^(1/2)*(d*x^2+c)^(1/2)/e/f/(-d/c)^(1/2)/(c*f-d*e)/(d*f*x^4
+c*f*x^2+d*e*x^2+c*e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx^2+a}{\sqrt{dx^2+c}\left(fx^2+e\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)/(sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)\sqrt{dx^2+c}\sqrt{fx^2+e}}{df^2x^6+\left(2\,def+cf^2\right)x^4+ce^2+\left(de^2+2\,cef\right)x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(d*f^2*x^6 + (2*d*e*f
+ c*f^2)*x^4 + c*e^2 + (d*e^2 + 2*c*e*f)*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a+bx^2}{\sqrt{c+dx^2}\left(e+fx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/(d*x**2+c)**(1/2)/(f*x**2+e)**(3/2),x)
```

```
[Out] Integral((a + b*x**2)/(sqrt(c + d*x**2)*(e + f*x**2)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx^2+a}{\sqrt{dx^2+c}\left(fx^2+e\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)/(sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)), x)
```

3.46 $\int \frac{a+bx^2}{\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=272

$$\frac{\sqrt{e}\sqrt{c+dx^2}(-2adf+bcf+bde)\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{e+fx^2}(de-cf)^2\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}-\frac{x(bc-ad)}{c\sqrt{c+dx^2}\sqrt{e+fx^2}(de-cf)}-\frac{\sqrt{f}\sqrt{c+dx^2}(-acf-}{c\sqrt{e}\sqrt{e+}$$

```
[Out] -(((b*c - a*d)*x)/(c*(d*e - c*f)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])) - (Sqrt[
f]*(2*b*c*e - a*d*e - a*c*f)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/S
qrt[e]], 1 - (d*e)/(c*f)])/(c*Sqrt[e]*(d*e - c*f)^2*Sqrt[(e*(c + d*x^2))/(c
*(e + f*x^2))]*Sqrt[e + f*x^2]) + (Sqrt[e]*(b*d*e + b*c*f - 2*a*d*f)*Sqrt[c
 + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(c*Sqrt[
f]*(d*e - c*f)^2*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rubi [A] time = 0.220669, antiderivative size = 272, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {527, 525, 418, 411}

$$-\frac{x(bc-ad)}{c\sqrt{c+dx^2}\sqrt{e+fx^2}(de-cf)}+\frac{\sqrt{e}\sqrt{c+dx^2}(-2adf+bcf+bde)F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{e+fx^2}(de-cf)^2\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}-\frac{\sqrt{f}\sqrt{c+dx^2}(-acf-ade+2}{c\sqrt{e}\sqrt{e+fx^2}(de}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/((c + d*x^2)^(3/2)*(e + f*x^2)^(3/2)),x]
```


```
[Out] -(((b*c - a*d)*x)/(c*(d*e - c*f)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])) - (Sqrt[
f]*(2*b*c*e - a*d*e - a*c*f)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/S
qrt[e]], 1 - (d*e)/(c*f)])/(c*Sqrt[e]*(d*e - c*f)^2*Sqrt[(e*(c + d*x^2))/(c
*(e + f*x^2))]*Sqrt[e + f*x^2]) + (Sqrt[e]*(b*d*e + b*c*f - 2*a*d*f)*Sqrt[c
 + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(c*Sqrt[
f]*(d*e - c*f)^2*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```


Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 525

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{a+bx^2}{\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^{3/2}}\,dx = -\frac{(bc-ad)x}{c(de-cf)\sqrt{c+dx^2}\sqrt{e+fx^2}} - \frac{\int \frac{-c(be-af)+(bc-ad)fx^2}{\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}}\,dx}{c(de-cf)}$$

$$= -\frac{(bc-ad)x}{c(de-cf)\sqrt{c+dx^2}\sqrt{e+fx^2}} - \frac{(f(2bce-ade-acf))\int \frac{\sqrt{c+dx^2}}{\left(e+fx^2\right)^{3/2}}\,dx}{c(de-cf)^2} + \frac{(bde+bcf-\cdots}{\cdots}$$

$$= -\frac{(bc-ad)x}{c(de-cf)\sqrt{c+dx^2}\sqrt{e+fx^2}} - \frac{\sqrt{f}(2bce-ade-acf)\sqrt{c+dx^2}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\cdots\right.}{c\sqrt{e}(de-cf)^2\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}\sqrt{e+fx^2}}$$

Mathematica [C] time = 0.71995, size = 262, normalized size = 0.96

$$\frac{\sqrt{\frac{d}{c}}\left(-ie\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}(bc-ad)(cf-de)\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{\frac{d}{c}}\right),\frac{cf}{de}\right)+x\sqrt{\frac{d}{c}}\left(a\left(c^2f^2+cdf^2x^2+d^2e\left(e+fx^2\right)\cdots\right.\right.\right.}{de\sqrt{c+dx^2}\sqrt{e+fx^2}(de-\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/((c + d*x^2)^(3/2)*(e + f*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[d/c]*(Sqrt[d/c]*x*(a*(c^2*f^2 + c*d*f^2*x^2 + d^2*e*(e + f*x^2)) - b*
c*e*(c*f + d*(e + 2*f*x^2))) - I*d*e*(2*b*c*e - a*(d*e + c*f))*Sqrt[1 + (d*
x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]
- I*(b*c - a*d)*e*(-(d*e) + c*f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*El
lipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]))/(d*e*(d*e - c*f)^2*Sqrt[c +
d*x^2]*Sqrt[e + f*x^2])
```

Maple [A] time = 0.029, size = 581, normalized size = 2.1

$$\frac{1}{ce\left(cf-de\right)^2\left(dfx^4+cfx^2+dex^2+ce\right)}\left(x^3acdf^2\sqrt{-\frac{d}{c}}+x^3ad^2ef\sqrt{-\frac{d}{c}}-2x^3bcdef\sqrt{-\frac{d}{c}}-\text{EllipticF}\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\cdots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x)
```

```
[Out] (x^3*a*c*d*f^2*(-d/c)^(1/2)+x^3*a*d^2*e*f*(-d/c)^(1/2)-2*x^3*b*c*d*e*f*(-d/
c)^(1/2)-EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c*d*e*f*((d*x^2+c)/c)^
(1/2)*((f*x^2+e)/e)^(1/2)+EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d^2*e
^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+EllipticF(x*(-d/c)^(1/2),(c*f/d/
e)^(1/2))*b*c^2*e*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-EllipticF(x*(-d
/c)^(1/2),(c*f/d/e)^(1/2))*b*c*d*e^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2
)-EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c*d*e*f*((d*x^2+c)/c)^(1/2)*(
(f*x^2+e)/e)^(1/2)-EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d^2*e^2*((d*
x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+2*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1
/2))*b*c*d*e^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+x*a*c^2*f^2*(-d/c)^(
1/2)+x*a*d^2*e^2*(-d/c)^(1/2)-x*b*c^2*e*f*(-d/c)^(1/2)-x*b*c*d*e^2*(-d/c)^(
1/2))*(f*x^2+e)^(1/2)*(d*x^2+c)^(1/2)/c/e/(-d/c)^(1/2)/(c*f-d*e)^2/(d*f*x^4
+c*f*x^2+d*e*x^2+c*e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx^2 + a}{\left(dx^2 + c\right)^{\frac{3}{2}} \left(fx^2 + e\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)/((d*x^2 + c)^(3/2)*(f*x^2 + e)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2 + a\right)\sqrt{dx^2 + c}\sqrt{fx^2 + e}}{d^2f^2x^8 + 2\left(d^2ef + cdf^2\right)x^6 + \left(d^2e^2 + 4\,cdef + c^2f^2\right)x^4 + c^2e^2 + 2\left(cde^2 + c^2ef\right)x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(d^2*f^2*x^8 + 2*(d^2*
e*f + c*d*f^2)*x^6 + (d^2*e^2 + 4*c*d*e*f + c^2*f^2)*x^4 + c^2*e^2 + 2*(c*d
*e^2 + c^2*e*f)*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/(d*x**2+c)**(3/2)/(f*x**2+e)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx^2 + a}{\left(dx^2 + c\right)^{\frac{3}{2}}\left(fx^2 + e\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)/((d*x^2 + c)^(3/2)*(f*x^2 + e)^(3/2)), x)
```

3.47 $\int \frac{a+bx^2}{(c+dx^2)^{5/2}(e+fx^2)^{3/2}}\,dx$

Optimal. Leaf size=375

$$-\frac{\sqrt{e}\sqrt{f}\sqrt{c+dx^2}(ad(de-9cf)+bc(3cf+5de))\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{3c^2\sqrt{e+fx^2}(de-cf)^3\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{\sqrt{f}\sqrt{c+dx^2}\left(a\left(-3c^2f^2-7cdef+2d\right.\right.}{3c^2\sqrt{e}\sqrt{e+fx}}$$

```
[Out] -((b*c - a*d)*x)/(3*c*(d*e - c*f)*(c + d*x^2)^(3/2)*Sqrt[e + f*x^2]) + ((2*
a*d*(d*e - 3*c*f) + b*c*(d*e + 3*c*f))*x)/(3*c^2*(d*e - c*f)^2*Sqrt[c + d*x
^2]*Sqrt[e + f*x^2]) + (Sqrt[f]*(b*c*e*(d*e + 7*c*f) + a*(2*d^2*e^2 - 7*c*d
*e*f - 3*c^2*f^2))*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1
 - (d*e)/(c*f)])/(3*c^2*Sqrt[e]*(d*e - c*f)^3*Sqrt[(e*(c + d*x^2))/(c*(e +
f*x^2))]*Sqrt[e + f*x^2]) - (Sqrt[e]*Sqrt[f]*(a*d*(d*e - 9*c*f) + b*c*(5*d*
e + 3*c*f))*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e
)/(c*f)])/(3*c^2*(d*e - c*f)^3*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e
 + f*x^2])
```

Rubi [A] time = 0.3925, antiderivative size = 375, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {527, 525, 418, 411}

$$\frac{\sqrt{f}\sqrt{c+dx^2}\left(a\left(-3c^2f^2-7cdef+2d^2e^2\right)+bce(7cf+de)\right)E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3c^2\sqrt{e}\sqrt{e+fx^2}(de-cf)^3\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{x(2ad(de-3cf)+bc(3cf+de))}{3c^2\sqrt{c+dx^2}\sqrt{e+fx^2}(de-cf)^2}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/((c + d*x^2)^(5/2)*(e + f*x^2)^(3/2)),x]
```

```
[Out] -((b*c - a*d)*x)/(3*c*(d*e - c*f)*(c + d*x^2)^(3/2)*Sqrt[e + f*x^2]) + ((2*
a*d*(d*e - 3*c*f) + b*c*(d*e + 3*c*f))*x)/(3*c^2*(d*e - c*f)^2*Sqrt[c + d*x
^2]*Sqrt[e + f*x^2]) + (Sqrt[f]*(b*c*e*(d*e + 7*c*f) + a*(2*d^2*e^2 - 7*c*d
*e*f - 3*c^2*f^2))*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1
 - (d*e)/(c*f)])/(3*c^2*Sqrt[e]*(d*e - c*f)^3*Sqrt[(e*(c + d*x^2))/(c*(e +
f*x^2))]*Sqrt[e + f*x^2]) - (Sqrt[e]*Sqrt[f]*(a*d*(d*e - 9*c*f) + b*c*(5*d*
e + 3*c*f))*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e
)/(c*f)])/(3*c^2*(d*e - c*f)^3*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e
 + f*x^2])
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 525

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
```

```
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{a+bx^2}{\left(c+dx^2\right)^{5/2}\left(e+fx^2\right)^{3/2}}\,dx = -\frac{(bc-ad)x}{3c(de-cf)\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}} - \frac{\int \frac{-bce-2ade+3acf+3(bc-ad)fx^2}{\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^{3/2}}\,dx}{3c(de-cf)}$$

$$= -\frac{(bc-ad)x}{3c(de-cf)\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}} + \frac{(2ad(de-3cf)+bc(de+3cf))x}{3c^2(de-cf)^2\sqrt{c+dx^2}\sqrt{e+fx^2}} + \frac{\int \frac{-cf(4bce-\cdots}{\cdots}}{\cdots}$$

$$= -\frac{(bc-ad)x}{3c(de-cf)\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}} + \frac{(2ad(de-3cf)+bc(de+3cf))x}{3c^2(de-cf)^2\sqrt{c+dx^2}\sqrt{e+fx^2}} - \frac{(f(ad(de-\cdots}{\cdots}$$

$$= -\frac{(bc-ad)x}{3c(de-cf)\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}} + \frac{(2ad(de-3cf)+bc(de+3cf))x}{3c^2(de-cf)^2\sqrt{c+dx^2}\sqrt{e+fx^2}} + \frac{\sqrt{f}\left(bce(de\cdots\right.}{\cdots}$$

Mathematica [C] time = 2.06465, size = 428, normalized size = 1.14

$$\frac{-ie\left(c+dx^2\right)\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}(cf-de)(2ad(de-3cf)+bc(3cf+de))\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{\frac{d}{c}}\right),\frac{cf}{de}\right)+x\sqrt{\frac{d}{c}}\left(a\left(c^2\cdots\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/((c + d*x^2)^(5/2)*(e + f*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[d/c]*x*(-(b*c*e*(3*c^3*f^2 + d^3*e*x^2*(e + f*x^2) + c*d^2*f*x^2*(4*e
 + 7*f*x^2) + c^2*d*f*(5*e + 11*f*x^2))) + a*(3*c^4*f^3 + 6*c^3*d*f^3*x^2 -
 2*d^4*e^2*x^2*(e + f*x^2) + c^2*d^2*f*(8*e^2 + 8*e*f*x^2 + 3*f^2*x^4) + c*
d^3*e*(-3*e^2 + 4*e*f*x^2 + 7*f^2*x^4))) - I*d*e*(b*c*e*(d*e + 7*c*f) + a*(
2*d^2*e^2 - 7*c*d*e*f - 3*c^2*f^2))*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[1
+ (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - I*e*(-(d*e) +
 c*f)*(2*a*d*(d*e - 3*c*f) + b*c*(d*e + 3*c*f))*(c + d*x^2)*Sqrt[1 + (d*x^2
)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/(3
```

```
*c^2*Sqrt[d/c]*e*(-(d*e) + c*f)^3*(c + d*x^2)^(3/2)*Sqrt[e + f*x^2])
```

Maple [B] time = 0.038, size = 1742, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/(d*x^2+c)^(5/2)/(f*x^2+e)^(3/2),x)
```

```
[Out] -1/3*(3*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c^3*d*e*f^2*((d*x^2+c)/
c)^(1/2)*((f*x^2+e)/e)^(1/2)+7*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*
c^2*d^2*e^2*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+EllipticF(x*(-d/c)^(1
/2),(c*f/d/e)^(1/2))*x^2*b*c*d^3*e^3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2
)-EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*b*c*d^3*e^3*((d*x^2+c)/c)^(
1/2)*((f*x^2+e)/e)^(1/2)+6*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c^3*
d*e*f^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-8*EllipticF(x*(-d/c)^(1/2),
(c*f/d/e)^(1/2))*a*c^2*d^2*e^2*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+2*
EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^3*d*e^2*f*((d*x^2+c)/c)^(1/2)
*((f*x^2+e)/e)^(1/2)-3*x*a*c^4*f^3*(-d/c)^(1/2)-7*EllipticE(x*(-d/c)^(1/2),
(c*f/d/e)^(1/2))*b*c^3*d*e^2*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+2*x^
3*a*d^4*e^3*(-d/c)^(1/2)-7*x^5*a*c*d^3*e*f^2*(-d/c)^(1/2)-2*EllipticE(x*(-d
/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a*d^4*e^3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^
(1/2)+2*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c*d^3*e^3*((d*x^2+c)/c)
^(1/2)*((f*x^2+e)/e)^(1/2)-3*x^5*a*c^2*d^2*f^3*(-d/c)^(1/2)+5*x*b*c^3*d*e^2
*f*(-d/c)^(1/2)+2*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a*d^4*e^3*(
(d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-8*x^3*a*c^2*d^2*e*f^2*(-d/c)^(1/2)-4
*x^3*a*c*d^3*e^2*f*(-d/c)^(1/2)+11*x^3*b*c^3*d*e*f^2*(-d/c)^(1/2)+4*x^3*b*c
^2*d^2*e^2*f*(-d/c)^(1/2)+7*x^5*b*c^2*d^2*e*f^2*(-d/c)^(1/2)+x^5*b*c*d^3*e^
2*f*(-d/c)^(1/2)-3*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^4*e*f^2*((
d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1
/2))*b*c^2*d^2*e^3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-2*EllipticE(x*(-
d/c)^(1/2),(c*f/d/e)^(1/2))*a*c*d^3*e^3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(
1/2)-EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^2*d^2*e^3*((d*x^2+c)/c)^
(1/2)*((f*x^2+e)/e)^(1/2)-8*x*a*c^2*d^2*e^2*f*(-d/c)^(1/2)+2*x^5*a*d^4*e^2*
f*(-d/c)^(1/2)-6*x^3*a*c^3*d*f^3*(-d/c)^(1/2)+x^3*b*c*d^3*e^3*(-d/c)^(1/2)+
3*x*a*c*d^3*e^3*(-d/c)^(1/2)+3*x*b*c^4*e*f^2*(-d/c)^(1/2)-7*EllipticE(x*(-d
/c)^(1/2),(c*f/d/e)^(1/2))*x^2*b*c^2*d^2*e^2*f*((d*x^2+c)/c)^(1/2)*((f*x^2+
e)/e)^(1/2)+6*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a*c^2*d^2*e*f^2
*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-8*EllipticF(x*(-d/c)^(1/2),(c*f/d/
e)^(1/2))*x^2*a*c*d^3*e^2*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-3*Ellip
ticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*b*c^3*d*e*f^2*((d*x^2+c)/c)^(1/2)*
((f*x^2+e)/e)^(1/2)+2*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*b*c^2*d
^2*e^2*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+3*EllipticE(x*(-d/c)^(1/2)
,(c*f/d/e)^(1/2))*x^2*a*c^2*d^2*e*f^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/
2)+7*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a*c*d^3*e^2*f*((d*x^2+c)
/c)^(1/2)*((f*x^2+e)/e)^(1/2))/(f*x^2+e)^(1/2)/(c*f-d*e)^3/(-d/c)^(1/2)/e/c
^2/(d*x^2+c)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx^2 + a}{\left(dx^2 + c\right)^{\frac{5}{2}} \left(fx^2 + e\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+c)^(5/2)/(f*x^2+e)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)/((d*x^2 + c)^(5/2)*(f*x^2 + e)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)\sqrt{dx^2+c}\sqrt{fx^2+e}}{d^3f^2x^{10}+\left(2\,d^3ef+3\,cd^2f^2\right)x^8+\left(d^3e^2+6\,cd^2ef+3\,c^2df^2\right)x^6+c^3e^2+\left(3\,cd^2e^2+6\,c^2def+c^3f^2\right)x^4+\left(3\,c\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+c)^(5/2)/(f*x^2+e)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(d^3*f^2*x^10 + (2*d^3
*e*f + 3*c*d^2*f^2)*x^8 + (d^3*e^2 + 6*c*d^2*e*f + 3*c^2*d*f^2)*x^6 + c^3*e
^2 + (3*c*d^2*e^2 + 6*c^2*d*e*f + c^3*f^2)*x^4 + (3*c^2*d*e^2 + 2*c^3*e*f)*
x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/(d*x**2+c)**(5/2)/(f*x**2+e)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx^2+a}{\left(dx^2+c\right)^{\frac{5}{2}}\left(fx^2+e\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+c)^(5/2)/(f*x^2+e)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)/((d*x^2 + c)^(5/2)*(f*x^2 + e)^(3/2)), x)
```

3.48 $\int \frac{e+fx^2}{\sqrt{a+bx^2}(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=209

$$\frac{\sqrt{c}\sqrt{a+bx^2}(be-af)\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{a+bx^2}(de-cf)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

```
[Out] -(((d*e - c*f)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (
b*c)/(a*d)])/(Sqrt[c]*Sqrt[d]*(b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^
2))]*Sqrt[c + d*x^2])) + (Sqrt[c]*(b*e - a*f)*Sqrt[a + b*x^2]*EllipticF[Arc
Tan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*(b*c - a*d)*Sqrt[(c*
(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.0863239, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {525, 418, 411}

$$\frac{\sqrt{c}\sqrt{a+bx^2}(be-af)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{a+bx^2}(de-cf)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x^2)/(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)),x]
```

```
[Out] -(((d*e - c*f)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (
b*c)/(a*d)])/(Sqrt[c]*Sqrt[d]*(b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^
2))]*Sqrt[c + d*x^2])) + (Sqrt[c]*(b*e - a*f)*Sqrt[a + b*x^2]*EllipticF[Arc
Tan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*(b*c - a*d)*Sqrt[(c*
(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rule 525

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
```

```
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}\,dx = \frac{(be-af)\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{bc-ad} - \frac{(de-cf)\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^{3/2}}\,dx}{bc-ad}$$

$$= -\frac{(de-cf)\sqrt{a+bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}\sqrt{c+dx^2}} + \frac{\sqrt{c}(be-af)\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\right.}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}\sqrt{c+dx^2}}$$

Mathematica [C] time = 0.409333, size = 212, normalized size = 1.01

$$\frac{-icf\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(ad-bc)\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{\frac{b}{a}}\right),\frac{ad}{bc}\right)+dx\sqrt{\frac{b}{a}}\left(a+bx^2\right)(de-cf)-ibc\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(c}{cd\sqrt{\frac{b}{a}}\sqrt{a+bx^2}\sqrt{c+dx^2}(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x^2)/(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[b/a]*d*(d*e - c*f)*x*(a + b*x^2) - I*b*c*(-(d*e) + c*f)*Sqrt[1 + (b*x
^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] -
 I*c*(-(b*c) + a*d)*f*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*A
rcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(Sqrt[b/a]*c*d*(-(b*c) + a*d)*Sqrt[a + b
*x^2]*Sqrt[c + d*x^2])
```

Maple [A] time = 0.036, size = 349, normalized size = 1.7

$$\frac{1}{cd\left(ad-bc\right)\left(bdx^4+adx^2+bcx^2+ac\right)}\left(-x^3bcdf\sqrt{-\frac{b}{a}}+x^3bd^2e\sqrt{-\frac{b}{a}}+\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)acdf\sqrt{\frac{bx^2+a}{a}}\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e)/(d*x^2+c)^(3/2)/(b*x^2+a)^(1/2),x)
```

```
[Out] (-x^3*b*c*d*f*(-b/a)^(1/2)+x^3*b*d^2*e*(-b/a)^(1/2)+EllipticF(x*(-b/a)^(1/2
),(a*d/b/c)^(1/2))*a*c*d*f*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)-Elliptic
F(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b*c^2*f*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)
^(1/2)+EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b*c^2*f*((b*x^2+a)/a)^(1/2
)*((d*x^2+c)/c)^(1/2)-EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b*c*d*e*((b
*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)-x*a*c*d*f*(-b/a)^(1/2)+x*a*d^2*e*(-b/a
)^(1/2))*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/c/d/(-b/a)^(1/2)/(a*d-b*c)/(b*d*x^
4+a*d*x^2+b*c*x^2+a*c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx^2+e}{\sqrt{bx^2+a}\left(dx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e)/(d*x^2+c)^(3/2)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x^2 + e)/(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}{bd^2x^6+\left(2\,bcd+ad^2\right)x^4+ac^2+\left(bc^2+2\,acd\right)x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e)/(d*x^2+c)^(3/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)/(b*d^2*x^6 + (2*b*c*d
+ a*d^2)*x^4 + a*c^2 + (b*c^2 + 2*a*c*d)*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{e+fx^2}{\sqrt{a+bx^2}\left(c+dx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e)/(d*x**2+c)**(3/2)/(b*x**2+a)**(1/2),x)
```

```
[Out] Integral((e + f*x**2)/(sqrt(a + b*x**2)*(c + d*x**2)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{fx^2+e}{\sqrt{bx^2+a}\left(dx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e)/(d*x^2+c)^(3/2)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((f*x^2 + e)/(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)), x)
```

3.49 $\int \frac{e+fx^2}{\sqrt{a-bx^2}\left(c+dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=247

$$\frac{\sqrt{a}f\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}+\frac{x\sqrt{a-bx^2}(de-cf)}{c\sqrt{c+dx^2}(ad+bc)}+\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(de-cf)E\left(\sin\right.}{cd\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}(ad+b}$$

```
[Out] ((d*e - c*f)*x*Sqrt[a - b*x^2])/(c*(b*c + a*d)*Sqrt[c + d*x^2]) + (Sqrt[a]*
Sqrt[b]*(d*e - c*f)*Sqrt[1 - (b*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin[(S
qrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(c*d*(b*c + a*d)*Sqrt[a - b*x^2]*Sqrt[
1 + (d*x^2)/c]) + (Sqrt[a]*f*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Ellipt
icF[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*d*Sqrt[a - b*x^2
]*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.239852, antiderivative size = 247, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {527, 524, 427, 426, 424, 421, 419}

$$\frac{x\sqrt{a-bx^2}(de-cf)}{c\sqrt{c+dx^2}(ad+bc)}+\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(de-cf)E\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{cd\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}(ad+bc)}+\frac{\sqrt{a}f\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}F\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x^2)/(Sqrt[a - b*x^2]*(c + d*x^2)^(3/2)),x]
```

```
[Out] ((d*e - c*f)*x*Sqrt[a - b*x^2])/(c*(b*c + a*d)*Sqrt[c + d*x^2]) + (Sqrt[a]*
Sqrt[b]*(d*e - c*f)*Sqrt[1 - (b*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin[(S
qrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(c*d*(b*c + a*d)*Sqrt[a - b*x^2]*Sqrt[
1 + (d*x^2)/c]) + (Sqrt[a]*f*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Ellipt
icF[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*d*Sqrt[a - b*x^2
]*Sqrt[c + d*x^2])
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 427

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d*x^2)/c
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{e+fx^2}{\sqrt{a-bx^2}\left(c+dx^2\right)^{3/2}}\,dx = \frac{(de-cf)x\sqrt{a-bx^2}}{c(bc+ad)\sqrt{c+dx^2}} - \frac{\int \frac{-c(be+af)-b(de-cf)x^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx}{c(bc+ad)}$$

$$= \frac{(de-cf)x\sqrt{a-bx^2}}{c(bc+ad)\sqrt{c+dx^2}} + \frac{f\int \frac{1}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx}{d} + \frac{(b(de-cf))\int \frac{\sqrt{c+dx^2}}{\sqrt{a-bx^2}}\,dx}{cd(bc+ad)}$$

$$= \frac{(de-cf)x\sqrt{a-bx^2}}{c(bc+ad)\sqrt{c+dx^2}} + \frac{\left(b(de-cf)\sqrt{1-\frac{bx^2}{a}}\right)\int \frac{\sqrt{c+dx^2}}{\sqrt{1-\frac{bx^2}{a}}}\,dx}{cd(bc+ad)\sqrt{a-bx^2}} + \frac{\left(f\sqrt{1+\frac{dx^2}{c}}\right)\int \frac{1}{\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}}\,d}{d\sqrt{c+dx^2}}$$

$$= \frac{(de-cf)x\sqrt{a-bx^2}}{c(bc+ad)\sqrt{c+dx^2}} + \frac{\left(b(de-cf)\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\right)\int \frac{\sqrt{1+\frac{dx^2}{c}}}{\sqrt{1-\frac{bx^2}{a}}}\,dx}{cd(bc+ad)\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}} + \frac{\left(f\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\right.}{d\sqrt{a-bx^2}}$$

$$= \frac{(de-cf)x\sqrt{a-bx^2}}{c(bc+ad)\sqrt{c+dx^2}} + \frac{\sqrt{a}\sqrt{b}(de-cf)\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{cd(bc+ad)\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}} + \frac{\sqrt{a}f\sqrt{1-}}{}$$

Mathematica [C] time = 0.69438, size = 220, normalized size = 0.89

$$\frac{-icf\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{-\frac{b}{a}}\right),-\frac{ad}{bc}\right)+dx\sqrt{-\frac{b}{a}}\left(a-bx^2\right)(de-cf)+ibc\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(}{cd\sqrt{-\frac{b}{a}}\sqrt{a-bx^2}\sqrt{c+dx^2}(ad+bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x^2)/(Sqrt[a - b*x^2]*(c + d*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[-(b/a)]*d*(d*e - c*f)*x*(a - b*x^2) + I*b*c*(-(d*e) + c*f)*Sqrt[1 - (
b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/
(b*c))] - I*c*(b*c + a*d)*f*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Ellipti
cF[I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))])/(Sqrt[-(b/a)]*c*d*(b*c + a*d
)*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])
```

Maple [A] time = 0.048, size = 359, normalized size = 1.5

$$\frac{1}{cd\,(ad+bc)\left(bdx^4-adx^2+bcx^2-ac\right)}\left(-x^3bcdf\sqrt{\frac{b}{a}}+x^3bd^2e\sqrt{\frac{b}{a}}-\mathrm{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)acdf\sqrt{-\frac{bx^2-a}{a}}\sqrt{dx^2\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e)/(d*x^2+c)^(3/2)/(-b*x^2+a)^(1/2),x)
```

```
[Out] (-x^3*b*c*d*f*(b/a)^(1/2)+x^3*b*d^2*e*(b/a)^(1/2)-EllipticF(x*(b/a)^(1/2),(
-a*d/b/c)^(1/2))*a*c*d*f*(-(b*x^2-a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)-EllipticF
(x*(b/a)^(1/2),(-a*d/b/c)^(1/2))*b*c^2*f*(-(b*x^2-a)/a)^(1/2)*((d*x^2+c)/c)
^(1/2)+EllipticE(x*(b/a)^(1/2),(-a*d/b/c)^(1/2))*b*c^2*f*(-(b*x^2-a)/a)^(1/
2)*((d*x^2+c)/c)^(1/2)-EllipticE(x*(b/a)^(1/2),(-a*d/b/c)^(1/2))*b*c*d*e*(-
(b*x^2-a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)+x*a*c*d*f*(b/a)^(1/2)-x*a*d^2*e*(b/a
)^(1/2))*(-b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/c/d/(b/a)^(1/2)/(a*d+b*c)/(b*d*x^
4-a*d*x^2+b*c*x^2-a*c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx^2+e}{\sqrt{-bx^2+a}\left(dx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e)/(d*x^2+c)^(3/2)/(-b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x^2 + e)/(sqrt(-b*x^2 + a)*(d*x^2 + c)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}{bd^2x^6+\left(2\,bcd-ad^2\right)x^4-ac^2+\left(bc^2-2\,acd\right)x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e)/(d*x^2+c)^(3/2)/(-b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)/(b*d^2*x^6 + (2*b*c*
d - a*d^2)*x^4 - a*c^2 + (b*c^2 - 2*a*c*d)*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e+fx^2}{\sqrt{a-bx^2}\left(c+dx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e)/(d*x**2+c)**(3/2)/(-b*x**2+a)**(1/2),x)
```

```
[Out] Integral((e + f*x**2)/(sqrt(a - b*x**2)*(c + d*x**2)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx^2+e}{\sqrt{-bx^2+a}\left(dx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e)/(d*x^2+c)^(3/2)/(-b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((f*x^2 + e)/(sqrt(-b*x^2 + a)*(d*x^2 + c)^(3/2)), x)
```

3.50 $\int \frac{e+fx^2}{\sqrt{a+bx^2}\left(c-dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=237

$$\frac{e\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}\sqrt{c-dx^2}}+\frac{x\sqrt{a+bx^2}(cf+de)}{c\sqrt{c-dx^2}(ad+bc)}-\frac{\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}(cf+de)E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\right.}{\sqrt{c}\sqrt{d}\sqrt{\frac{bx^2}{a}+1}\sqrt{c-dx^2}(ad+bc)}$$

```
[Out] ((d*e + c*f)*x*Sqrt[a + b*x^2])/(c*(b*c + a*d)*Sqrt[c - d*x^2]) - ((d*e + c
*f)*Sqrt[a + b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(Sqrt[d]*x)/Sqrt[c
]], -((b*c)/(a*d))])/(Sqrt[c]*Sqrt[d]*(b*c + a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[
c - d*x^2]) + (e*Sqrt[1 + (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(
Sqrt[d]*x)/Sqrt[c]], -((b*c)/(a*d))])/(Sqrt[c]*Sqrt[d]*Sqrt[a + b*x^2]*Sqrt
[c - d*x^2])
```

Rubi [A] time = 0.218324, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {527, 524, 427, 426, 424, 421, 419}

$$\frac{x\sqrt{a+bx^2}(cf+de)}{c\sqrt{c-dx^2}(ad+bc)}-\frac{\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}(cf+de)E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{\frac{bx^2}{a}+1}\sqrt{c-dx^2}(ad+bc)}+\frac{e\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x^2)/(Sqrt[a + b*x^2]*(c - d*x^2)^(3/2)),x]
```

```
[Out] ((d*e + c*f)*x*Sqrt[a + b*x^2])/(c*(b*c + a*d)*Sqrt[c - d*x^2]) - ((d*e + c
*f)*Sqrt[a + b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(Sqrt[d]*x)/Sqrt[c
]], -((b*c)/(a*d))])/(Sqrt[c]*Sqrt[d]*(b*c + a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[
c - d*x^2]) + (e*Sqrt[1 + (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(
Sqrt[d]*x)/Sqrt[c]], -((b*c)/(a*d))])/(Sqrt[c]*Sqrt[d]*Sqrt[a + b*x^2]*Sqrt
[c - d*x^2])
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 427

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d*x^2)/c
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\left(c-dx^2\right)^{3/2}}\,dx = \frac{(de+cf)x\sqrt{a+bx^2}}{c(bc+ad)\sqrt{c-dx^2}} + \frac{\int \frac{c(be-af)-b(de+cf)x^2}{\sqrt{a+bx^2}\sqrt{c-dx^2}}\,dx}{c(bc+ad)}$$

$$= \frac{(de+cf)x\sqrt{a+bx^2}}{c(bc+ad)\sqrt{c-dx^2}} + \frac{e\int \frac{1}{\sqrt{a+bx^2}\sqrt{c-dx^2}}\,dx}{c} - \frac{(de+cf)\int \frac{\sqrt{a+bx^2}}{\sqrt{c-dx^2}}\,dx}{c(bc+ad)}$$

$$= \frac{(de+cf)x\sqrt{a+bx^2}}{c(bc+ad)\sqrt{c-dx^2}} + \frac{\left(e\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{1}{\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}}\,dx}{c\sqrt{c-dx^2}} - \frac{\left((de+cf)\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{\sqrt{a+bx^2}}{\sqrt{1-\frac{dx^2}{c}}}\,dx}{c(bc+ad)\sqrt{c-dx^2}}$$

$$= \frac{(de+cf)x\sqrt{a+bx^2}}{c(bc+ad)\sqrt{c-dx^2}} - \frac{\left((de+cf)\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{\sqrt{1+\frac{bx^2}{a}}}{\sqrt{1-\frac{dx^2}{c}}}\,dx}{c(bc+ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{c-dx^2}} + \frac{\left(e\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\right) \cdots}{c\sqrt{a+bx^2} \cdots}$$

$$= \frac{(de+cf)x\sqrt{a+bx^2}}{c(bc+ad)\sqrt{c-dx^2}} - \frac{(de+cf)\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}(bc+ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{c-dx^2}} + \frac{e\sqrt{1+\frac{bx^2}{a}}\sqrt{1- \cdots}}{\sqrt{c}\sqrt{d} \cdots}$$

Mathematica [C] time = 0.432011, size = 213, normalized size = 0.9

$$\frac{icf\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}(ad+bc)\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{\frac{b}{a}}\right),-\frac{ad}{bc}\right)+dx\sqrt{\frac{b}{a}}\left(a+bx^2\right)(cf+de)-ibc\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}(cf+ \cdots}{cd\sqrt{\frac{b}{a}}\sqrt{a+bx^2}\sqrt{c-dx^2}(ad+bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x^2)/(Sqrt[a + b*x^2]*(c - d*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[b/a]*d*(d*e + c*f)*x*(a + b*x^2) - I*b*c*(d*e + c*f)*Sqrt[1 + (b*x^2)
/a]*Sqrt[1 - (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], -((a*d)/(b*c))] +
 I*c*(b*c + a*d)*f*Sqrt[1 + (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*EllipticF[I*ArcS
inh[Sqrt[b/a]*x], -((a*d)/(b*c))])/(Sqrt[b/a]*c*d*(b*c + a*d)*Sqrt[a + b*x^
2]*Sqrt[c - d*x^2])
```

Maple [A] time = 0.042, size = 345, normalized size = 1.5

$$\frac{1}{c\,(ad+bc)\left(bdx^4+adx^2-bcx^2-ac\right)}\left(-x^3bcf\sqrt{\frac{d}{c}}-x^3bde\sqrt{\frac{d}{c}}-\mathrm{EllipticF}\left(x\sqrt{\frac{d}{c}},\sqrt{-\frac{bc}{ad}}\right)ade\sqrt{-\frac{dx^2-c}{c}}\sqrt{\frac{bx^2+a}{a}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e)/(-d*x^2+c)^(3/2)/(b*x^2+a)^(1/2),x)
```

```
[Out] (-x^3*b*c*f*(d/c)^(1/2)-x^3*b*d*e*(d/c)^(1/2)-EllipticF(x*(d/c)^(1/2),(-b*c
/a/d)^(1/2))*a*d*e*(-(d*x^2-c)/c)^(1/2)*((b*x^2+a)/a)^(1/2)-EllipticF(x*(d/
c)^(1/2),(-b*c/a/d)^(1/2))*b*c*e*(-(d*x^2-c)/c)^(1/2)*((b*x^2+a)/a)^(1/2)+E
llipticE(x*(d/c)^(1/2),(-b*c/a/d)^(1/2))*a*c*f*(-(d*x^2-c)/c)^(1/2)*((b*x^2
+a)/a)^(1/2)+EllipticE(x*(d/c)^(1/2),(-b*c/a/d)^(1/2))*a*d*e*(-(d*x^2-c)/c)
^(1/2)*((b*x^2+a)/a)^(1/2)-x*a*c*f*(d/c)^(1/2)-x*a*d*e*(d/c)^(1/2))*(b*x^2+
a)^(1/2)*(-d*x^2+c)^(1/2)/c/(d/c)^(1/2)/(a*d+b*c)/(b*d*x^4+a*d*x^2-b*c*x^2-
a*c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx^2+e}{\sqrt{bx^2+a}\left(-dx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e)/(-d*x^2+c)^(3/2)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x^2 + e)/(sqrt(b*x^2 + a)*(-d*x^2 + c)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{bx^2+a}\sqrt{-dx^2+c}\left(fx^2+e\right)}{bd^2x^6-\left(2\,bcd-ad^2\right)x^4+ac^2+\left(bc^2-2\,acd\right)x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e)/(-d*x^2+c)^(3/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(-d*x^2 + c)*(f*x^2 + e)/(b*d^2*x^6 - (2*b*c*d
 - a*d^2)*x^4 + a*c^2 + (b*c^2 - 2*a*c*d)*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e + fx^2}{\sqrt{a + bx^2} \left(c - dx^2\right)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e)/(-d*x**2+c)**(3/2)/(b*x**2+a)**(1/2),x)
```

```
[Out] Integral((e + f*x**2)/(sqrt(a + b*x**2)*(c - d*x**2)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx^2 + e}{\sqrt{bx^2 + a}\left(-dx^2 + c\right)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e)/(-d*x^2+c)^(3/2)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((f*x^2 + e)/(sqrt(b*x^2 + a)*(-d*x^2 + c)^(3/2)), x)
```

3.51 $\int \frac{e+fx^2}{\sqrt{a-bx^2}\left(c-dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=242

$$\frac{e\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{a-bx^2}\sqrt{c-dx^2}}-\frac{x\sqrt{a-bx^2}(cf+de)}{c\sqrt{c-dx^2}(bc-ad)}+\frac{\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}(cf+de)E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}(bc-ad)}$$

```
[Out] -(((d*e + c*f)*x*Sqrt[a - b*x^2])/(c*(b*c - a*d)*Sqrt[c - d*x^2])) + ((d*e
+ c*f)*Sqrt[a - b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(Sqrt[d]*x)/Sqr
t[c]], (b*c)/(a*d)])/(Sqrt[c]*Sqrt[d]*(b*c - a*d)*Sqrt[1 - (b*x^2)/a]*Sqrt[
c - d*x^2]) + (e*Sqrt[1 - (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(
Sqrt[d]*x)/Sqrt[c]], (b*c)/(a*d)])/(Sqrt[c]*Sqrt[d]*Sqrt[a - b*x^2]*Sqrt[c
- d*x^2])
```

Rubi [A] time = 0.237454, antiderivative size = 242, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {527, 524, 427, 426, 424, 421, 419}

$$-\frac{x\sqrt{a-bx^2}(cf+de)}{c\sqrt{c-dx^2}(bc-ad)}+\frac{\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}(cf+de)E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}(bc-ad)}+\frac{e\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{a-bx^2}\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x^2)/(Sqrt[a - b*x^2]*(c - d*x^2)^(3/2)),x]
```



```
[Out] -(((d*e + c*f)*x*Sqrt[a - b*x^2])/(c*(b*c - a*d)*Sqrt[c - d*x^2])) + ((d*e
+ c*f)*Sqrt[a - b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(Sqrt[d]*x)/Sqr
t[c]], (b*c)/(a*d)])/(Sqrt[c]*Sqrt[d]*(b*c - a*d)*Sqrt[1 - (b*x^2)/a]*Sqrt[
c - d*x^2]) + (e*Sqrt[1 - (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(
Sqrt[d]*x)/Sqrt[c]], (b*c)/(a*d)])/(Sqrt[c]*Sqrt[d]*Sqrt[a - b*x^2]*Sqrt[c
- d*x^2])
```


Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 427

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d*x^2)/c
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{e+fx^2}{\sqrt{a-bx^2}\left(c-dx^2\right)^{3/2}}\,dx = -\frac{(de+cf)x\sqrt{a-bx^2}}{c(bc-ad)\sqrt{c-dx^2}} - \frac{\int \frac{-c(be+af)+b(de+cf)x^2}{\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx}{c(bc-ad)}$$

$$= -\frac{(de+cf)x\sqrt{a-bx^2}}{c(bc-ad)\sqrt{c-dx^2}} + \frac{e\int \frac{1}{\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx}{c} + \frac{(de+cf)\int \frac{\sqrt{a-bx^2}}{\sqrt{c-dx^2}}\,dx}{c(bc-ad)}$$

$$= -\frac{(de+cf)x\sqrt{a-bx^2}}{c(bc-ad)\sqrt{c-dx^2}} + \frac{\left(e\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{1}{\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}}\,dx}{c\sqrt{c-dx^2}} + \frac{\left((de+cf)\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{\sqrt{a-bx^2}}{\sqrt{1-\frac{dx^2}{c}}}\,d}{c(bc-ad)\sqrt{c-dx^2}}$$

$$= -\frac{(de+cf)x\sqrt{a-bx^2}}{c(bc-ad)\sqrt{c-dx^2}} + \frac{\left((de+cf)\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{\sqrt{1-\frac{bx^2}{a}}}{\sqrt{1-\frac{dx^2}{c}}}\,dx}{c(bc-ad)\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}} + \frac{\left(e\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\right.}{c\sqrt{a-bx^2}}$$

$$= -\frac{(de+cf)x\sqrt{a-bx^2}}{c(bc-ad)\sqrt{c-dx^2}} + \frac{(de+cf)\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}(bc-ad)\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}} + \frac{e\sqrt{1-\frac{bx^2}{a}}\sqrt{1-}}{\sqrt{c}\sqrt{d}\sqrt{a}}$$

Mathematica [C] time = 0.465846, size = 221, normalized size = 0.91

$$\frac{icf\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}(ad-bc)\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{-\frac{b}{a}}\right),\frac{ad}{bc}\right)+dx\sqrt{-\frac{b}{a}}\left(a-bx^2\right)(cf+de)+ibc\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}(cf+}{cd\sqrt{-\frac{b}{a}}\sqrt{a-bx^2}\sqrt{c-dx^2}(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x^2)/(Sqrt[a - b*x^2]*(c - d*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[-(b/a)]*d*(d*e + c*f)*x*(a - b*x^2) + I*b*c*(d*e + c*f)*Sqrt[1 - (b*x
^2)/a]*Sqrt[1 - (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[-(b/a)]*x], (a*d)/(b*c)
] + I*c*(-(b*c) + a*d)*f*Sqrt[1 - (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*EllipticF[
I*ArcSinh[Sqrt[-(b/a)]*x], (a*d)/(b*c)])/(Sqrt[-(b/a)]*c*d*(-(b*c) + a*d)*S
qrt[a - b*x^2]*Sqrt[c - d*x^2])
```

Maple [A] time = 0.048, size = 354, normalized size = 1.5

$$\frac{1}{c\left(ad-bc\right)\left(bdx^{4}-adx^{2}-bcx^{2}+ac\right)}\left(-x^{3}bcf\sqrt{\frac{d}{c}}-x^{3}bde\sqrt{\frac{d}{c}}+\mathrm{EllipticF}\left(x\sqrt{\frac{d}{c}},\sqrt{\frac{bc}{ad}}\right)ade\sqrt{-\frac{dx^{2}-c}{c}}\sqrt{-\frac{bx^{2}-a}{a}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e)/(-d*x^2+c)^(3/2)/(-b*x^2+a)^(1/2),x)
```

```
[Out] (-x^3*b*c*f*(d/c)^(1/2)-x^3*b*d*e*(d/c)^(1/2)+EllipticF(x*(d/c)^(1/2),(b*c/
a/d)^(1/2))*a*d*e*(-(d*x^2-c)/c)^(1/2)*(-(b*x^2-a)/a)^(1/2)-EllipticF(x*(d/
c)^(1/2),(b*c/a/d)^(1/2))*b*c*e*(-(d*x^2-c)/c)^(1/2)*(-(b*x^2-a)/a)^(1/2)-E
llipticE(x*(d/c)^(1/2),(b*c/a/d)^(1/2))*a*c*f*(-(d*x^2-c)/c)^(1/2)*(-(b*x^2
-a)/a)^(1/2)-EllipticE(x*(d/c)^(1/2),(b*c/a/d)^(1/2))*a*d*e*(-(d*x^2-c)/c)^
(1/2)*(-(b*x^2-a)/a)^(1/2)+x*a*c*f*(d/c)^(1/2)+x*a*d*e*(d/c)^(1/2))*(-b*x^2
+a)^(1/2)*(-d*x^2+c)^(1/2)/c/(d/c)^(1/2)/(a*d-b*c)/(b*d*x^4-a*d*x^2-b*c*x^2
+a*c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx^{2}+e}{\sqrt{-bx^{2}+a}\left(-dx^{2}+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e)/(-d*x^2+c)^(3/2)/(-b*x^2+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((f*x^2 + e)/(sqrt(-b*x^2 + a)*(-d*x^2 + c)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-bx^{2}+a}\sqrt{-dx^{2}+c}\left(fx^{2}+e\right)}{bd^{2}x^{6}-\left(2\,bcd+ad^{2}\right)x^{4}-ac^{2}+\left(bc^{2}+2\,acd\right)x^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e)/(-d*x^2+c)^(3/2)/(-b*x^2+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(-sqrt(-b*x^2 + a)*sqrt(-d*x^2 + c)*(f*x^2 + e)/(b*d^2*x^6 - (2*b*c
*d + a*d^2)*x^4 - a*c^2 + (b*c^2 + 2*a*c*d)*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e + fx^2}{\sqrt{a - bx^2}\left(c - dx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e)/(-d*x**2+c)**(3/2)/(-b*x**2+a)**(1/2),x)
```

```
[Out] Integral((e + f*x**2)/(sqrt(a - b*x**2)*(c - d*x**2)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx^2 + e}{\sqrt{-bx^2 + a}\left(-dx^2 + c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e)/(-d*x^2+c)^(3/2)/(-b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((f*x^2 + e)/(sqrt(-b*x^2 + a)*(-d*x^2 + c)^(3/2)), x)
```

3.52 $\int \frac{a+bx^2}{\sqrt{2+dx^2}\sqrt{3+fx^2}}\,dx$

Optimal. Leaf size=191

$$\frac{a\sqrt{dx^2+2}\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{3}}\right),1-\frac{3d}{2f}\right)}{\sqrt{2}\sqrt{f}\sqrt{fx^2+3}\sqrt{\frac{dx^2+2}{fx^2+3}}}+\frac{bx\sqrt{dx^2+2}}{d\sqrt{fx^2+3}}-\frac{\sqrt{2}b\sqrt{dx^2+2}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{3}}\right)|1-\frac{3d}{2f}\right)}{d\sqrt{f}\sqrt{fx^2+3}\sqrt{\frac{dx^2+2}{fx^2+3}}}$$

```
[Out] (b*x*Sqrt[2 + d*x^2])/(d*Sqrt[3 + f*x^2]) - (Sqrt[2]*b*Sqrt[2 + d*x^2]*Elli
pticE[ArcTan[(Sqrt[f]*x)/Sqrt[3]], 1 - (3*d)/(2*f)])/(d*Sqrt[f]*Sqrt[(2 + d
*x^2)/(3 + f*x^2)]*Sqrt[3 + f*x^2]) + (a*Sqrt[2 + d*x^2]*EllipticF[ArcTan[(
Sqrt[f]*x)/Sqrt[3]], 1 - (3*d)/(2*f)])/(Sqrt[2]*Sqrt[f]*Sqrt[(2 + d*x^2)/(3
 + f*x^2)]*Sqrt[3 + f*x^2])
```

Rubi [A] time = 0.0960592, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {531, 418, 492, 411}

$$\frac{a\sqrt{dx^2+2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{3}}\right)|1-\frac{3d}{2f}\right)}{\sqrt{2}\sqrt{f}\sqrt{fx^2+3}\sqrt{\frac{dx^2+2}{fx^2+3}}}+\frac{bx\sqrt{dx^2+2}}{d\sqrt{fx^2+3}}-\frac{\sqrt{2}b\sqrt{dx^2+2}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{3}}\right)|1-\frac{3d}{2f}\right)}{d\sqrt{f}\sqrt{fx^2+3}\sqrt{\frac{dx^2+2}{fx^2+3}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(Sqrt[2 + d*x^2]*Sqrt[3 + f*x^2]),x]
```

```
[Out] (b*x*Sqrt[2 + d*x^2])/(d*Sqrt[3 + f*x^2]) - (Sqrt[2]*b*Sqrt[2 + d*x^2]*Elli
pticE[ArcTan[(Sqrt[f]*x)/Sqrt[3]], 1 - (3*d)/(2*f)])/(d*Sqrt[f]*Sqrt[(2 + d
*x^2)/(3 + f*x^2)]*Sqrt[3 + f*x^2]) + (a*Sqrt[2 + d*x^2]*EllipticF[ArcTan[(
Sqrt[f]*x)/Sqrt[3]], 1 - (3*d)/(2*f)])/(Sqrt[2]*Sqrt[f]*Sqrt[(2 + d*x^2)/(3
 + f*x^2)]*Sqrt[3 + f*x^2])
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{a+bx^2}{\sqrt{2+dx^2}\sqrt{3+fx^2}}\,dx = a\int \frac{1}{\sqrt{2+dx^2}\sqrt{3+fx^2}}\,dx + b\int \frac{x^2}{\sqrt{2+dx^2}\sqrt{3+fx^2}}\,dx$$

$$= \frac{bx\sqrt{2+dx^2}}{d\sqrt{3+fx^2}} + \frac{a\sqrt{2+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{3}}\right)|1-\frac{3d}{2f}\right)}{\sqrt{2}\sqrt{f}\sqrt{\frac{2+dx^2}{3+fx^2}}\sqrt{3+fx^2}} - \frac{(3b)\int \frac{\sqrt{2+dx^2}}{\left(3+fx^2\right)^{3/2}}\,dx}{d}$$

$$= \frac{bx\sqrt{2+dx^2}}{d\sqrt{3+fx^2}} - \frac{\sqrt{2}b\sqrt{2+dx^2}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{3}}\right)|1-\frac{3d}{2f}\right)}{d\sqrt{f}\sqrt{\frac{2+dx^2}{3+fx^2}}\sqrt{3+fx^2}} + \frac{a\sqrt{2+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{3}}\right)|1-\frac{3d}{2f}\right)}{\sqrt{2}\sqrt{f}\sqrt{\frac{2+dx^2}{3+fx^2}}\sqrt{3+fx^2}}$$

Mathematica [C] time = 0.141978, size = 81, normalized size = 0.42

$$-\frac{i\left((af-3b)\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{2}}\right),\frac{2f}{3d}\right)+3bE\left(i\sinh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{2}}\right)|\frac{2f}{3d}\right)\right)}{\sqrt{3}\sqrt{d}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(Sqrt[2 + d*x^2]*Sqrt[3 + f*x^2]),x]
```

```
[Out] ((-I)*(3*b*EllipticE[I*ArcSinh[(Sqrt[d]*x)/Sqrt[2]], (2*f)/(3*d)] + (-3*b +
 a*f)*EllipticF[I*ArcSinh[(Sqrt[d]*x)/Sqrt[2]], (2*f)/(3*d)]))/(Sqrt[3]*Sqr
t[d]*f)
```

Maple [A] time = 0.037, size = 105, normalized size = 0.6

$$\frac{\sqrt{2}}{2d}\left(\text{EllipticF}\left(\frac{x\sqrt{3}}{3}\sqrt{-f},\frac{\sqrt{2}\sqrt{3}}{2}\sqrt{\frac{d}{f}}\right)ad-2\,\text{EllipticF}\left(1/3\,x\sqrt{3}\sqrt{-f},1/2\,\sqrt{2}\sqrt{3}\sqrt{\frac{d}{f}}\right)b+2\,\text{EllipticE}\left(1/3\,x\sqrt{3}\sqrt{-f},1/2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/(d*x^2+2)^(1/2)/(f*x^2+3)^(1/2),x)
```

```
[Out] 1/2*2^(1/2)*(EllipticF(1/3*x*3^(1/2)*(-f)^(1/2),1/2*2^(1/2)*3^(1/2)*(1/f*d)
^(1/2))*a*d-2*EllipticF(1/3*x*3^(1/2)*(-f)^(1/2),1/2*2^(1/2)*3^(1/2)*(1/f*d
)^(1/2))*b+2*EllipticE(1/3*x*3^(1/2)*(-f)^(1/2),1/2*2^(1/2)*3^(1/2)*(1/f*d)
^(1/2))*b)/(-f)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx^2+a}{\sqrt{dx^2+2}\sqrt{fx^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+2)^(1/2)/(f*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)/(sqrt(d*x^2 + 2)*sqrt(f*x^2 + 3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)\sqrt{dx^2+2}\sqrt{fx^2+3}}{dfx^4+\left(3\,d+2\,f\right)x^2+6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+2)^(1/2)/(f*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)*sqrt(d*x^2 + 2)*sqrt(f*x^2 + 3)/(d*f*x^4 + (3*d + 2*f)
*x^2 + 6), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a+bx^2}{\sqrt{dx^2+2}\sqrt{fx^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/(d*x**2+2)**(1/2)/(f*x**2+3)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)/(sqrt(d*x**2 + 2)*sqrt(f*x**2 + 3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx^2+a}{\sqrt{dx^2+2}\sqrt{fx^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+2)^(1/2)/(f*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)/(sqrt(d*x^2 + 2)*sqrt(f*x^2 + 3)), x)
```

3.53 $\int \frac{(a+bx^2)\sqrt{2+dx^2}}{\sqrt{3+fx^2}}\,dx$

Optimal. Leaf size=262

$$-\frac{\sqrt{2}\sqrt{dx^2+2}(b-af)\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{3}}\right),1-\frac{3d}{2f}\right)}{f^{3/2}\sqrt{fx^2+3}\sqrt{\frac{dx^2+2}{fx^2+3}}}+\frac{\sqrt{2}\sqrt{dx^2+2}(-3adf+6bd-2bf)E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{3}}\right)|1-\frac{3d}{2f}\right)}{3df^{3/2}\sqrt{fx^2+3}\sqrt{\frac{dx^2+2}{fx^2+3}}}-x\sqrt{\ldots}$$

```
[Out] -((6*b*d - 2*b*f - 3*a*d*f)*x*Sqrt[2 + d*x^2])/(3*d*f*Sqrt[3 + f*x^2]) + (b
*x*Sqrt[2 + d*x^2]*Sqrt[3 + f*x^2])/(3*f) + (Sqrt[2]*(6*b*d - 2*b*f - 3*a*d
*f)*Sqrt[2 + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[3]], 1 - (3*d)/(2*f)]
)/(3*d*f^(3/2)*Sqrt[(2 + d*x^2)/(3 + f*x^2)]*Sqrt[3 + f*x^2]) - (Sqrt[2]*(b
 - a*f)*Sqrt[2 + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[3]], 1 - (3*d)/(2
*f)])/(f^(3/2)*Sqrt[(2 + d*x^2)/(3 + f*x^2)]*Sqrt[3 + f*x^2])
```

Rubi [A] time = 0.167557, antiderivative size = 262, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {528, 531, 418, 492, 411}

$$-\frac{\sqrt{2}\sqrt{dx^2+2}(b-af)F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{3}}\right)|1-\frac{3d}{2f}\right)}{f^{3/2}\sqrt{fx^2+3}\sqrt{\frac{dx^2+2}{fx^2+3}}}+\frac{\sqrt{2}\sqrt{dx^2+2}(-3adf+6bd-2bf)E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{3}}\right)|1-\frac{3d}{2f}\right)}{3df^{3/2}\sqrt{fx^2+3}\sqrt{\frac{dx^2+2}{fx^2+3}}}-\frac{x\sqrt{dx^2+2}(\ldots}{3d\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*Sqrt[2 + d*x^2])/Sqrt[3 + f*x^2],x]
```


```
[Out] -((6*b*d - 2*b*f - 3*a*d*f)*x*Sqrt[2 + d*x^2])/(3*d*f*Sqrt[3 + f*x^2]) + (b
*x*Sqrt[2 + d*x^2]*Sqrt[3 + f*x^2])/(3*f) + (Sqrt[2]*(6*b*d - 2*b*f - 3*a*d
*f)*Sqrt[2 + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[3]], 1 - (3*d)/(2*f)]
)/(3*d*f^(3/2)*Sqrt[(2 + d*x^2)/(3 + f*x^2)]*Sqrt[3 + f*x^2]) - (Sqrt[2]*(b
 - a*f)*Sqrt[2 + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[3]], 1 - (3*d)/(2
*f)])/(f^(3/2)*Sqrt[(2 + d*x^2)/(3 + f*x^2)]*Sqrt[3 + f*x^2])
```


Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
```

```
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)\sqrt{2+dx^2}}{\sqrt{3+fx^2}}\,dx = \frac{bx\sqrt{2+dx^2}\sqrt{3+fx^2}}{3f} + \frac{\int \frac{-6(b-af)+(-6bd+2bf+3adf)x^2}{\sqrt{2+dx^2}\sqrt{3+fx^2}}\,dx}{3f}$$

$$= \frac{bx\sqrt{2+dx^2}\sqrt{3+fx^2}}{3f} - \frac{(2(b-af))\int \frac{1}{\sqrt{2+dx^2}\sqrt{3+fx^2}}\,dx}{f} - \frac{(6bd-2bf-3adf)\int \frac{x^2}{\sqrt{2+dx^2}\sqrt{3+}}$$

$$= -\frac{(6bd-2bf-3adf)x\sqrt{2+dx^2}}{3df\sqrt{3+fx^2}} + \frac{bx\sqrt{2+dx^2}\sqrt{3+fx^2}}{3f} - \frac{\sqrt{2}(b-af)\sqrt{2+dx^2}F\left(\tan^{-1}\right.}{f^{3/2}\sqrt{\frac{2+dx^2}{3+fx^2}}\sqrt{3+}}$$

$$= -\frac{(6bd-2bf-3adf)x\sqrt{2+dx^2}}{3df\sqrt{3+fx^2}} + \frac{bx\sqrt{2+dx^2}\sqrt{3+fx^2}}{3f} + \frac{\sqrt{2}(6bd-2bf-3adf)\sqrt{2+}}{3df^{3/2}\sqrt{\frac{2+dx}{3+fx}}}$$

Mathematica [C] time = 0.201503, size = 142, normalized size = 0.54

$$\frac{i\sqrt{3}(3d-2f)(af-2b)\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{2}}\right),\frac{2f}{3d}\right)+i\sqrt{3}(-3adf+6bd-2bf)E\left(i\sinh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{2}}\right)|\frac{2f}{3d}\right)+b\sqrt{d}fx\sqrt{dx^2}}{3\sqrt{d}f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*Sqrt[2 + d*x^2])/Sqrt[3 + f*x^2],x]
```

```
[Out] (b*Sqrt[d]*f*x*Sqrt[2 + d*x^2]*Sqrt[3 + f*x^2] + I*Sqrt[3]*(6*b*d - 2*b*f -
 3*a*d*f)*EllipticE[I*ArcSinh[(Sqrt[d]*x)/Sqrt[2]], (2*f)/(3*d)] + I*Sqrt[3
]*(3*d - 2*f)*(-2*b + a*f)*EllipticF[I*ArcSinh[(Sqrt[d]*x)/Sqrt[2]], (2*f)/
(3*d)])/(3*Sqrt[d]*f^2)
```

Maple [A] time = 0.02, size = 367, normalized size = 1.4

$$\frac{1}{\left(3\,df{x}^{4}+9\,d{x}^{2}+6\,f{x}^{2}+18\right)fd}\sqrt{d{x}^{2}+2}\sqrt{f{x}^{2}+3}\left({x}^{5}b{d}^{2}f\sqrt{-f}+3\,\sqrt{2}\text{EllipticE}\left(1/3\,x\sqrt{3}\sqrt{-f},1/2\,\sqrt{2}\sqrt{3}\sqrt{\frac{d}{f}}\right)ad\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x^2+2)^(1/2)/(f*x^2+3)^(1/2),x)
```

```
[Out] 1/3*(d*x^2+2)^(1/2)*(f*x^2+3)^(1/2)*(x^5*b*d^2*f*(-f)^(1/2)+3*2^(1/2)*Ellip
ticE(1/3*x*3^(1/2)*(-f)^(1/2),1/2*2^(1/2)*3^(1/2)*(1/f*d)^(1/2))*a*d*f*(d*x
^2+2)^(1/2)*(f*x^2+3)^(1/2)+3*x^3*b*d^2*(-f)^(1/2)+2*x^3*b*d*f*(-f)^(1/2)-6
*2^(1/2)*EllipticE(1/3*x*3^(1/2)*(-f)^(1/2),1/2*2^(1/2)*3^(1/2)*(1/f*d)^(1/
2))*b*d*(d*x^2+2)^(1/2)*(f*x^2+3)^(1/2)+2*2^(1/2)*EllipticE(1/3*x*3^(1/2)*(
-f)^(1/2),1/2*2^(1/2)*3^(1/2)*(1/f*d)^(1/2))*b*f*(d*x^2+2)^(1/2)*(f*x^2+3)^
(1/2)+3*2^(1/2)*EllipticF(1/3*x*3^(1/2)*(-f)^(1/2),1/2*2^(1/2)*3^(1/2)*(1/f
*d)^(1/2))*b*d*(d*x^2+2)^(1/2)*(f*x^2+3)^(1/2)-2*2^(1/2)*EllipticF(1/3*x*3^
(1/2)*(-f)^(1/2),1/2*2^(1/2)*3^(1/2)*(1/f*d)^(1/2))*b*f*(d*x^2+2)^(1/2)*(f*
x^2+3)^(1/2)+6*x*b*d*(-f)^(1/2))/(d*f*x^4+3*d*x^2+2*f*x^2+6)/f/(-f)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)\sqrt{dx^2 + 2}}{\sqrt{fx^2 + 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+2)^(1/2)/(f*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)*sqrt(d*x^2 + 2)/sqrt(f*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2 + a)\sqrt{dx^2 + 2}}{\sqrt{fx^2 + 3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+2)^(1/2)/(f*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)*sqrt(d*x^2 + 2)/sqrt(f*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + bx^2)\sqrt{dx^2 + 2}}{\sqrt{fx^2 + 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+2)**(1/2)/(f*x**2+3)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)*sqrt(d*x**2 + 2)/sqrt(f*x**2 + 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)\sqrt{dx^2 + 2}}{\sqrt{fx^2 + 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+2)^(1/2)/(f*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)*sqrt(d*x^2 + 2)/sqrt(f*x^2 + 3), x)
```

3.54 $\int \left(a+bx^2\right) \sqrt{2+dx^2}\sqrt{3+fx^2}\, dx$

Optimal. Leaf size=356

$$-\frac{\sqrt{2}\sqrt{dx^2+2}(-10adf+3bd+2bf)\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{3}}\right),1-\frac{3d}{2f}\right)}{5df^{3/2}\sqrt{fx^2+3}\sqrt{\frac{dx^2+2}{fx^2+3}}}+\frac{x\sqrt{dx^2+2}\left(5adf(3d+2f)-2b\left(9d^2-6df+4f\right.\right.}{15d^2f\sqrt{fx^2+3}}$$

```
[Out] ((5*a*d*f*(3*d + 2*f) - 2*b*(9*d^2 - 6*d*f + 4*f^2))*x*Sqrt[2 + d*x^2])/(15
*d^2*f*Sqrt[3 + f*x^2]) + ((3*b*d - 4*b*f + 5*a*d*f)*x*Sqrt[2 + d*x^2]*Sqrt
[3 + f*x^2])/(15*d*f) + (b*x*(2 + d*x^2)^(3/2)*Sqrt[3 + f*x^2])/(5*d) - (Sq
rt[2]*(5*a*d*f*(3*d + 2*f) - 2*b*(9*d^2 - 6*d*f + 4*f^2))*Sqrt[2 + d*x^2]*E
llipticE[ArcTan[(Sqrt[f]*x)/Sqrt[3]], 1 - (3*d)/(2*f)])/(15*d^2*f^(3/2)*Sqr
t[(2 + d*x^2)/(3 + f*x^2)]*Sqrt[3 + f*x^2]) - (Sqrt[2]*(3*b*d + 2*b*f - 10*
a*d*f)*Sqrt[2 + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[3]], 1 - (3*d)/(2*
f)])/(5*d*f^(3/2)*Sqrt[(2 + d*x^2)/(3 + f*x^2)]*Sqrt[3 + f*x^2])
```

Rubi [A] time = 0.322451, antiderivative size = 356, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {528, 531, 418, 492, 411}

$$\frac{x\sqrt{dx^2+2}\left(5adf(3d+2f)-2b\left(9d^2-6df+4f^2\right)\right)}{15d^2f\sqrt{fx^2+3}}-\frac{\sqrt{2}\sqrt{dx^2+2}\left(5adf(3d+2f)-2b\left(9d^2-6df+4f^2\right)\right)E\left(\tan^{-1}\left(\right.\right.}{15d^2f^{3/2}\sqrt{fx^2+3}\sqrt{\frac{dx^2+2}{fx^2+3}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)*Sqrt[2 + d*x^2]*Sqrt[3 + f*x^2],x]
```


```
[Out] ((5*a*d*f*(3*d + 2*f) - 2*b*(9*d^2 - 6*d*f + 4*f^2))*x*Sqrt[2 + d*x^2])/(15
*d^2*f*Sqrt[3 + f*x^2]) + ((3*b*d - 4*b*f + 5*a*d*f)*x*Sqrt[2 + d*x^2]*Sqrt
[3 + f*x^2])/(15*d*f) + (b*x*(2 + d*x^2)^(3/2)*Sqrt[3 + f*x^2])/(5*d) - (Sq
rt[2]*(5*a*d*f*(3*d + 2*f) - 2*b*(9*d^2 - 6*d*f + 4*f^2))*Sqrt[2 + d*x^2]*E
llipticE[ArcTan[(Sqrt[f]*x)/Sqrt[3]], 1 - (3*d)/(2*f)])/(15*d^2*f^(3/2)*Sqr
t[(2 + d*x^2)/(3 + f*x^2)]*Sqrt[3 + f*x^2]) - (Sqrt[2]*(3*b*d + 2*b*f - 10*
a*d*f)*Sqrt[2 + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[3]], 1 - (3*d)/(2*
f)])/(5*d*f^(3/2)*Sqrt[(2 + d*x^2)/(3 + f*x^2)]*Sqrt[3 + f*x^2])
```


Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \left(a+bx^2\right)\sqrt{2+dx^2}\sqrt{3+fx^2}\,dx = \frac{bx\left(2+dx^2\right)^{3/2}\sqrt{3+fx^2}}{5d} + \frac{\int \frac{\sqrt{2+dx^2}\left(-3(2b-5ad)+(3bd-4bf+5adf)x^2\right)}{\sqrt{3+fx^2}}\,dx}{5d}$$

$$= \frac{(3bd-4bf+5adf)x\sqrt{2+dx^2}\sqrt{3+fx^2}}{15df} + \frac{bx\left(2+dx^2\right)^{3/2}\sqrt{3+fx^2}}{5d} + \frac{\int \frac{-6(3bd\cdots}{\cdots}}{\cdots}$$

$$= \frac{(3bd-4bf+5adf)x\sqrt{2+dx^2}\sqrt{3+fx^2}}{15df} + \frac{bx\left(2+dx^2\right)^{3/2}\sqrt{3+fx^2}}{5d} - \frac{(2(3bd+\cdots}{\cdots}$$

$$= \frac{\left(5adf(3d+2f)-2b\left(9d^2-6df+4f^2\right)\right)x\sqrt{2+dx^2}}{15d^2f\sqrt{3+fx^2}} + \frac{(3bd-4bf+5adf)x\sqrt{2\cdots}}{15df}$$

$$= \frac{\left(5adf(3d+2f)-2b\left(9d^2-6df+4f^2\right)\right)x\sqrt{2+dx^2}}{15d^2f\sqrt{3+fx^2}} + \frac{(3bd-4bf+5adf)x\sqrt{2\cdots}}{15df}$$

Mathematica [C] time = 0.323886, size = 186, normalized size = 0.52

$$\frac{i\sqrt{3}(3d-2f)(5adf-6bd+2bf)\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{2}}\right),\frac{2f}{3d}\right)+i\sqrt{3}\left(2b\left(9d^2-6df+4f^2\right)-5adf(3d+2f)\right)E\left(i\,\text{si}\cdots\right.}{15d^{3/2}f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)*Sqrt[2 + d*x^2]*Sqrt[3 + f*x^2],x]
```

```
[Out] (Sqrt[d]*f*x*Sqrt[2 + d*x^2]*Sqrt[3 + f*x^2]*(2*b*f + 5*a*d*f + 3*b*d*(1 +
f*x^2)) + I*Sqrt[3]*(-5*a*d*f*(3*d + 2*f) + 2*b*(9*d^2 - 6*d*f + 4*f^2))*El
lipticE[I*ArcSinh[(Sqrt[d]*x)/Sqrt[2]], (2*f)/(3*d)] + I*Sqrt[3]*(3*d - 2*f
)*(-6*b*d + 2*b*f + 5*a*d*f)*EllipticF[I*ArcSinh[(Sqrt[d]*x)/Sqrt[2]], (2*f
)/(3*d)])/(15*d^(3/2)*f^2)
```

Maple [B] time = 0.023, size = 775, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x^2+2)^(1/2)*(f*x^2+3)^(1/2),x)
```

```
[Out] 1/15*(d*x^2+2)^(1/2)*(f*x^2+3)^(1/2)*(3*x^7*b*d^3*f^2*(-f)^(1/2)+5*x^5*a*d^
3*f^2*(-f)^(1/2)+12*x^5*b*d^3*f*(-f)^(1/2)+8*x^5*b*d^2*f^2*(-f)^(1/2)+15*x^
3*a*d^3*f*(-f)^(1/2)+10*x^3*a*d^2*f^2*(-f)^(1/2)+15*2^(1/2)*EllipticF(1/3*x
*3^(1/2)*(-f)^(1/2),1/2*2^(1/2)*3^(1/2)*(1/f*d)^(1/2))*a*d^2*f*(f*x^2+3)^(1
/2)*(d*x^2+2)^(1/2)-10*2^(1/2)*EllipticF(1/3*x*3^(1/2)*(-f)^(1/2),1/2*2^(1/
2)*3^(1/2)*(1/f*d)^(1/2))*a*d*f^2*(f*x^2+3)^(1/2)*(d*x^2+2)^(1/2)+15*2^(1/2
)*EllipticE(1/3*x*3^(1/2)*(-f)^(1/2),1/2*2^(1/2)*3^(1/2)*(1/f*d)^(1/2))*a*d
^2*f*(f*x^2+3)^(1/2)*(d*x^2+2)^(1/2)+10*2^(1/2)*EllipticE(1/3*x*3^(1/2)*(-f
)^(1/2),1/2*2^(1/2)*3^(1/2)*(1/f*d)^(1/2))*a*d*f^2*(f*x^2+3)^(1/2)*(d*x^2+2
)^(1/2)+9*x^3*b*d^3*(-f)^(1/2)+30*x^3*b*d^2*f*(-f)^(1/2)+4*x^3*b*d*f^2*(-f)
^(1/2)+9*2^(1/2)*EllipticF(1/3*x*3^(1/2)*(-f)^(1/2),1/2*2^(1/2)*3^(1/2)*(1/
f*d)^(1/2))*b*d^2*(f*x^2+3)^(1/2)*(d*x^2+2)^(1/2)-18*2^(1/2)*EllipticF(1/3*
x*3^(1/2)*(-f)^(1/2),1/2*2^(1/2)*3^(1/2)*(1/f*d)^(1/2))*b*d*f*(f*x^2+3)^(1/
2)*(d*x^2+2)^(1/2)+8*2^(1/2)*EllipticF(1/3*x*3^(1/2)*(-f)^(1/2),1/2*2^(1/2)
*3^(1/2)*(1/f*d)^(1/2))*b*f^2*(f*x^2+3)^(1/2)*(d*x^2+2)^(1/2)-18*2^(1/2)*El
lipticE(1/3*x*3^(1/2)*(-f)^(1/2),1/2*2^(1/2)*3^(1/2)*(1/f*d)^(1/2))*b*d^2*(
f*x^2+3)^(1/2)*(d*x^2+2)^(1/2)+12*2^(1/2)*EllipticE(1/3*x*3^(1/2)*(-f)^(1/2
),1/2*2^(1/2)*3^(1/2)*(1/f*d)^(1/2))*b*d*f*(f*x^2+3)^(1/2)*(d*x^2+2)^(1/2)-
8*2^(1/2)*EllipticE(1/3*x*3^(1/2)*(-f)^(1/2),1/2*2^(1/2)*3^(1/2)*(1/f*d)^(1
/2))*b*f^2*(f*x^2+3)^(1/2)*(d*x^2+2)^(1/2)+30*x*a*d^2*f*(-f)^(1/2)+18*x*b*d
^2*(-f)^(1/2)+12*x*b*d*f*(-f)^(1/2))/(d*f*x^4+3*d*x^2+2*f*x^2+6)/d^2/f/(-f)
^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2 + a\right)\sqrt{dx^2 + 2}\sqrt{fx^2 + 3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+2)^(1/2)*(f*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)*sqrt(d*x^2 + 2)*sqrt(f*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(bx^2 + a\right)\sqrt{dx^2 + 2}\sqrt{fx^2 + 3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+2)^(1/2)*(f*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)*sqrt(d*x^2 + 2)*sqrt(f*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + bx^2\right)\sqrt{dx^2 + 2}\sqrt{fx^2 + 3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+2)**(1/2)*(f*x**2+3)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)*sqrt(d*x**2 + 2)*sqrt(f*x**2 + 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2 + a\right)\sqrt{dx^2 + 2}\sqrt{fx^2 + 3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+2)^(1/2)*(f*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)*sqrt(d*x^2 + 2)*sqrt(f*x^2 + 3), x)
```

3.55 $\int \frac{-b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{1+\frac{2cx^2}{-b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}} \, dx$

Optimal. Leaf size=113

$$-\frac{\sqrt{b-\sqrt{b^2-4ac}}\left(\sqrt{b^2-4ac}+b\right)E\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)|\frac{b-\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}}$$

```
[Out] -((Sqrt[b - Sqrt[b^2 - 4*a*c]]*(b + Sqrt[b^2 - 4*a*c])*EllipticE[ArcSin[(Sq
rt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]], (b - Sqrt[b^2 - 4*a*c])/(b +
 Sqrt[b^2 - 4*a*c])])/(Sqrt[2]*Sqrt[c]))
```

Rubi [A] time = 0.203864, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 87, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.023, Rules used = {21, 424}

$$-\frac{\sqrt{b-\sqrt{b^2-4ac}}\left(\sqrt{b^2-4ac}+b\right)E\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)|\frac{b-\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(-b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(Sqrt[1 + (2*c*x^2)/(-b - Sqrt[b^2 -
 4*a*c])]*Sqrt[1 + (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]),x]
```

```
[Out] -((Sqrt[b - Sqrt[b^2 - 4*a*c]]*(b + Sqrt[b^2 - 4*a*c])*EllipticE[ArcSin[(Sq
rt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]], (b - Sqrt[b^2 - 4*a*c])/(b +
 Sqrt[b^2 - 4*a*c])])/(Sqrt[2]*Sqrt[c]))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{-b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{1+\frac{2cx^2}{-b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}} \, dx &= \left(-b-\sqrt{b^2-4ac}\right)\int \frac{\sqrt{1+\frac{2cx^2}{-b-\sqrt{b^2-4ac}}}}{\sqrt{1+\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}} \, dx\\ &= -\frac{\sqrt{b-\sqrt{b^2-4ac}}\left(b+\sqrt{b^2-4ac}\right)E\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)|\frac{b-\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}}\end{aligned}$$

Mathematica [C] time = 0.383008, size = 104, normalized size = 0.92

$$-2i\sqrt{2}a\sqrt{\frac{c}{\sqrt{b^2-4ac}-b}}E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{\sqrt{b^2-4ac}-b}}x\right)|\frac{b-\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(Sqrt[1 + (2*c*x^2)/(-b - Sqrt
[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]),x]
```

```
[Out] (-2*I)*Sqrt[2]*a*Sqrt[c/(-b + Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[
2]*Sqrt[c/(-b + Sqrt[b^2 - 4*a*c])]*x], (b - Sqrt[b^2 - 4*a*c])/(b + Sqrt[b
^2 - 4*a*c])]
```

Maple [F] time = 0.236, size = 0, normalized size = 0.

$$\int\left(2\,cx^2-\sqrt{-4\,ac+b^2}-b\right)\frac{1}{\sqrt{1+2\,\frac{cx^2}{-b-\sqrt{-4\,ac+b^2}}}}\frac{1}{\sqrt{1+2\,\frac{cx^2}{-b+\sqrt{-4\,ac+b^2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x^2-(-4*a*c+b^2)^(1/2)-b)/(1+2*c*x^2/(-b-(-4*a*c+b^2)^(1/2)))^(1/2
)/(1+2*c*x^2/(-b+(-4*a*c+b^2)^(1/2)))^(1/2),x)
```

```
[Out] int((2*c*x^2-(-4*a*c+b^2)^(1/2)-b)/(1+2*c*x^2/(-b-(-4*a*c+b^2)^(1/2)))^(1/2
)/(1+2*c*x^2/(-b+(-4*a*c+b^2)^(1/2)))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{2\,cx^2-b-\sqrt{b^2-4\,ac}}{\sqrt{-\frac{2\,cx^2}{b+\sqrt{b^2-4\,ac}}+1}\sqrt{-\frac{2\,cx^2}{b-\sqrt{b^2-4\,ac}}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b+2*c*x^2-(-4*a*c+b^2)^(1/2))/(1+2*c*x^2/(-b-(-4*a*c+b^2)^(1/2)
))^(1/2)/(1+2*c*x^2/(-b+(-4*a*c+b^2)^(1/2)))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((2*c*x^2 - b - sqrt(b^2 - 4*a*c))/(sqrt(-2*c*x^2/(b + sqrt(b^2 -
4*a*c)) + 1)*sqrt(-2*c*x^2/(b - sqrt(b^2 - 4*a*c)) + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(2\,acx^2-ab-\sqrt{b^2-4\,ac}a\right)\sqrt{-\frac{bx^2+\sqrt{b^2-4\,ac}x^2-2\,a}{a}}\sqrt{-\frac{bx^2-\sqrt{b^2-4\,ac}x^2-2\,a}{a}}}{2\left(cx^4-bx^2+a\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b+2*c*x^2-(-4*a*c+b^2)^(1/2))/(1+2*c*x^2/(-b-(-4*a*c+b^2)^(1/2)
))^(1/2)/(1+2*c*x^2/(-b+(-4*a*c+b^2)^(1/2)))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/2*(2*a*c*x^2 - a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(-(b*x^2 + sqrt(b^
2 - 4*a*c)*x^2 - 2*a)/a)*sqrt(-(b*x^2 - sqrt(b^2 - 4*a*c)*x^2 - 2*a)/a)/(c*
x^4 - b*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{-b+2cx^2-\sqrt{-4ac+b^2}}{\sqrt{\frac{-b+2cx^2-\sqrt{-4ac+b^2}}{-b-\sqrt{-4ac+b^2}}}\sqrt{\frac{-b+2cx^2+\sqrt{-4ac+b^2}}{-b+\sqrt{-4ac+b^2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b+2*c*x**2-(-4*a*c+b**2)**(1/2))/(1+2*c*x**2/(-b-(-4*a*c+b**2)*
*(1/2)))**(1/2)/(1+2*c*x**2/(-b+(-4*a*c+b**2)**(1/2)))**(1/2),x)
```

```
[Out] Integral((-b + 2*c*x**2 - sqrt(-4*a*c + b**2))/(sqrt((-b + 2*c*x**2 - sqrt(
-4*a*c + b**2))/(-b - sqrt(-4*a*c + b**2)))*sqrt((-b + 2*c*x**2 + sqrt(-4*a
*c + b**2))/(-b + sqrt(-4*a*c + b**2)))), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b+2*c*x^2-(-4*a*c+b^2)^(1/2))/(1+2*c*x^2/(-b-(-4*a*c+b^2)^(1/2)
))^(1/2)/(1+2*c*x^2/(-b+(-4*a*c+b^2)^(1/2)))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.56 $\int \frac{b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx$

Optimal. Leaf size=526

$$\frac{\left(b-\sqrt{b^2-4ac}\right)\sqrt{\sqrt{b^2-4ac}+b}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\,\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right),-\frac{2\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\frac{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}+\frac{x\left(b-\sqrt{b^2-4ac}\right)\sqrt{\cdots}}{\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+\cdots}}$$

```
[Out] ((b - Sqrt[b^2 - 4*a*c])*x*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])])/Sqr
t[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])] - ((b - Sqrt[b^2 - 4*a*c])*Sqrt[b
+ Sqrt[b^2 - 4*a*c]]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[
ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]], (-2*Sqrt[b^2 - 4*a
*c])/(b - Sqrt[b^2 - 4*a*c])])/(Sqrt[2]*Sqrt[c]*Sqrt[(1 + (2*c*x^2)/(b - Sq
rt[b^2 - 4*a*c]))/(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))]*Sqrt[1 + (2*c*x^
2)/(b + Sqrt[b^2 - 4*a*c])]) + ((b - Sqrt[b^2 - 4*a*c])*Sqrt[b + Sqrt[b^2 -
 4*a*c]]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[ArcTan[(Sqrt
[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]], (-2*Sqrt[b^2 - 4*a*c])/(b - Sq
rt[b^2 - 4*a*c])])/(Sqrt[2]*Sqrt[c]*Sqrt[(1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a
*c]))/(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))]*Sqrt[1 + (2*c*x^2)/(b + Sqrt
[b^2 - 4*a*c])])
```

Rubi [A] time = 0.630035, antiderivative size = 526, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 81, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {21, 422, 418, 492, 411}

$$\frac{x\left(b-\sqrt{b^2-4ac}\right)\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}}{\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}+\frac{\left(b-\sqrt{b^2-4ac}\right)\sqrt{\sqrt{b^2-4ac}+b}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\,F\left(\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)\middle|-\frac{2\sqrt{b^2-4\cdots}}{b-\sqrt{b^2-\cdots}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\frac{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4
*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]),x]
```



```
[Out] ((b - Sqrt[b^2 - 4*a*c])*x*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])])/Sqr
t[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])] - ((b - Sqrt[b^2 - 4*a*c])*Sqrt[b
+ Sqrt[b^2 - 4*a*c]]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[
ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]], (-2*Sqrt[b^2 - 4*a
*c])/(b - Sqrt[b^2 - 4*a*c])])/(Sqrt[2]*Sqrt[c]*Sqrt[(1 + (2*c*x^2)/(b - Sq
rt[b^2 - 4*a*c]))/(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))]*Sqrt[1 + (2*c*x^
2)/(b + Sqrt[b^2 - 4*a*c])]) + ((b - Sqrt[b^2 - 4*a*c])*Sqrt[b + Sqrt[b^2 -
 4*a*c]]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[ArcTan[(Sqrt
[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]], (-2*Sqrt[b^2 - 4*a*c])/(b - Sq
rt[b^2 - 4*a*c])])/(Sqrt[2]*Sqrt[c]*Sqrt[(1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a
*c]))/(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))]*Sqrt[1 + (2*c*x^2)/(b + Sqrt
[b^2 - 4*a*c])])
```


Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx = \left(b-\sqrt{b^2-4ac}\right)\int \frac{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx$$

$$= (2c)\int \frac{x^2}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx + \left(b-\sqrt{b^2-4ac}\right)\int \frac{1}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\cdots}$$

$$= \frac{\left(b-\sqrt{b^2-4ac}\right)x\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}} + \frac{\left(b-\sqrt{b^2-4ac}\right)\sqrt{b+\sqrt{b^2-4ac}}\sqrt{1+\frac{2c\cdots}{b-\sqrt{b\cdots}}}\cdots}{\sqrt{2}\sqrt{c}\sqrt{\frac{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\cdots}$$

$$= \frac{\left(b-\sqrt{b^2-4ac}\right)x\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}} - \frac{\left(b-\sqrt{b^2-4ac}\right)\sqrt{b+\sqrt{b^2-4ac}}\sqrt{1+\frac{2c\cdots}{b-\sqrt{b\cdots}}}\cdots}{\sqrt{2}\sqrt{c}\sqrt{\frac{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\cdots}$$

Mathematica [C] time = 0.4025, size = 203, normalized size = 0.39

$$-\frac{i\left(\left(\sqrt{b^2-4ac}+b\right)E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b-\sqrt{b^2-4ac}}}x\right)|\frac{b-\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}\right)-2\sqrt{b^2-4ac}\,\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{2}x\sqrt{\frac{c}{b-\sqrt{b^2-4ac}}}\right),\frac{b-\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}\right)\right)}{\sqrt{2}\sqrt{\frac{c}{b-\sqrt{b^2-4ac}}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(Sqrt[1 + (2*c*x^2)/(b - Sqrt[b
^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]),x]
```

```
[Out] ((-I)*((b + Sqrt[b^2 - 4*a*c])*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b - Sqrt
[b^2 - 4*a*c])]*x], (b - Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c])] - 2*Sq
rt[b^2 - 4*a*c]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b - Sqrt[b^2 - 4*a*c])]
*x], (b - Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c])]))/(Sqrt[2]*Sqrt[c/(b
- Sqrt[b^2 - 4*a*c])])
```

Maple [F] time = 0.154, size = 0, normalized size = 0.

$$\int \left(2\,cx^2-\sqrt{-4\,ac+b^2}+b\right)\frac{1}{\sqrt{1+2\,\frac{cx^2}{b-\sqrt{-4\,ac+b^2}}}}\frac{1}{\sqrt{1+2\,\frac{cx^2}{b+\sqrt{-4\,ac+b^2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x^2-(-4*a*c+b^2)^(1/2)+b)/(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)
/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2),x)
```

```
[Out] int((2*c*x^2-(-4*a*c+b^2)^(1/2)+b)/(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)
/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2\,cx^2+b-\sqrt{b^2-4\,ac}}{\sqrt{\frac{2\,cx^2}{b+\sqrt{b^2-4\,ac}}+1}\sqrt{\frac{2\,cx^2}{b-\sqrt{b^2-4\,ac}}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^2-(-4*a*c+b^2)^(1/2)+b)/(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))
^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((2*c*x^2 + b - sqrt(b^2 - 4*a*c))/(sqrt(2*c*x^2/(b + sqrt(b^2 - 4
*a*c)) + 1)*sqrt(2*c*x^2/(b - sqrt(b^2 - 4*a*c)) + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(2\,acx^2+ab-\sqrt{b^2-4\,ac}a\right)\sqrt{\frac{bx^2+\sqrt{b^2-4\,ac}x^2+2\,a}{a}}\sqrt{\frac{bx^2-\sqrt{b^2-4\,ac}x^2+2\,a}{a}}}{2\left(cx^4+bx^2+a\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^2-(-4*a*c+b^2)^(1/2)+b)/(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))
^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/2*(2*a*c*x^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt((b*x^2 + sqrt(b^2
 - 4*a*c)*x^2 + 2*a)/a)*sqrt((b*x^2 - sqrt(b^2 - 4*a*c)*x^2 + 2*a)/a)/(c*x^
4 + b*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b + 2cx^2 - \sqrt{-4ac + b^2}}{\sqrt{\frac{b+2cx^2-\sqrt{-4ac+b^2}}{b-\sqrt{-4ac+b^2}}}\sqrt{\frac{b+2cx^2+\sqrt{-4ac+b^2}}{b+\sqrt{-4ac+b^2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x**2-(-4*a*c+b**2)**(1/2)+b)/(1+2*c*x**2/(b-(-4*a*c+b**2)**(
1/2)))**(1/2)/(1+2*c*x**2/(b+(-4*a*c+b**2)**(1/2)))**(1/2),x)
```

```
[Out] Integral((b + 2*c*x**2 - sqrt(-4*a*c + b**2))/(sqrt((b + 2*c*x**2 - sqrt(-4
*a*c + b**2))/(b - sqrt(-4*a*c + b**2)))*sqrt((b + 2*c*x**2 + sqrt(-4*a*c +
 b**2))/(b + sqrt(-4*a*c + b**2)))), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^2-(-4*a*c+b^2)^(1/2)+b)/(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))
^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.57 $\int \frac{(a+bx^2)\sqrt{c+dx^2}}{e+fx^2}\,dx$

Optimal. Leaf size=128

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)(-2adf-bcf+2bde)}{2\sqrt{d}f^2}+\frac{(be-af)\sqrt{de-cf}\tanh^{-1}\left(\frac{x\sqrt{de-cf}}{\sqrt{e}\sqrt{c+dx^2}}\right)}{\sqrt{e}f^2}+\frac{bx\sqrt{c+dx^2}}{2f}$$

```
[Out] (b*x*Sqrt[c + d*x^2])/(2*f) - ((2*b*d*e - b*c*f - 2*a*d*f)*ArcTanh[(Sqrt[d]
*x)/Sqrt[c + d*x^2]])/(2*Sqrt[d]*f^2) + ((b*e - a*f)*Sqrt[d*e - c*f]*ArcTan
h[(Sqrt[d*e - c*f]*x)/(Sqrt[e]*Sqrt[c + d*x^2])])/(Sqrt[e]*f^2)
```

Rubi [A] time = 0.142179, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {528, 523, 217, 206, 377, 208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)(-2adf-bcf+2bde)}{2\sqrt{d}f^2}+\frac{(be-af)\sqrt{de-cf}\tanh^{-1}\left(\frac{x\sqrt{de-cf}}{\sqrt{e}\sqrt{c+dx^2}}\right)}{\sqrt{e}f^2}+\frac{bx\sqrt{c+dx^2}}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*Sqrt[c + d*x^2])/(e + f*x^2),x]
```

```
[Out] (b*x*Sqrt[c + d*x^2])/(2*f) - ((2*b*d*e - b*c*f - 2*a*d*f)*ArcTanh[(Sqrt[d]
*x)/Sqrt[c + d*x^2]])/(2*Sqrt[d]*f^2) + ((b*e - a*f)*Sqrt[d*e - c*f]*ArcTan
h[(Sqrt[d*e - c*f]*x)/(Sqrt[e]*Sqrt[c + d*x^2])])/(Sqrt[e]*f^2)
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)\sqrt{c+dx^2}}{e+fx^2}\,dx = \frac{bx\sqrt{c+dx^2}}{2f} + \frac{\int \frac{-c(be-2af)+(-2bde+bcf+2adf)x^2}{\sqrt{c+dx^2}\left(e+fx^2\right)}\,dx}{2f}$$

$$= \frac{bx\sqrt{c+dx^2}}{2f} + \frac{((be-af)(de-cf))\int \frac{1}{\sqrt{c+dx^2}\left(e+fx^2\right)}\,dx}{f^2} - \frac{(2bde-bcf-2adf)\int \frac{1}{\sqrt{c+dx^2}}\,dx}{2f^2}$$

$$= \frac{bx\sqrt{c+dx^2}}{2f} + \frac{((be-af)(de-cf))\,\text{Subst}\left(\int \frac{1}{e-(de-cf)x^2}\,dx, x, \frac{x}{\sqrt{c+dx^2}}\right)}{f^2} - \frac{(2bde-bcf-2adf)}{}$$

$$= \frac{bx\sqrt{c+dx^2}}{2f} - \frac{(2bde-bcf-2adf)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2\sqrt{d}f^2} + \frac{(be-af)\sqrt{de-cf}\tanh^{-1}\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{c+dx^2}}\right)}{\sqrt{e}f^2}$$

Mathematica [A] time = 0.245454, size = 124, normalized size = 0.97

$$\frac{\frac{\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)(2adf+bcf-2bde)}{\sqrt{d}} - \frac{2(be-af)\sqrt{cf-de}\tan^{-1}\left(\frac{x\sqrt{cf-de}}{\sqrt{e}\sqrt{c+dx^2}}\right)}{\sqrt{e}} + bfx\sqrt{c+dx^2}}{2f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*Sqrt[c + d*x^2])/(e + f*x^2),x]
```

```
[Out] (b*f*x*Sqrt[c + d*x^2] - (2*(b*e - a*f)*Sqrt[-(d*e) + c*f]*ArcTan[(Sqrt[-(d
*e) + c*f]*x)/(Sqrt[e]*Sqrt[c + d*x^2])])/Sqrt[e] + ((-2*b*d*e + b*c*f + 2*
a*d*f)*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/Sqrt[d])/(2*f^2)
```

Maple [B] time = 0.042, size = 1942, normalized size = 15.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x^2+c)^(1/2)/(f*x^2+e),x)
```

```
[Out] 1/2*b*x*(d*x^2+c)^(1/2)/f+1/2*b/f*c/d^(1/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))+1
/2/(-e*f)^(1/2)*((x-(-e*f)^(1/2)/f)^2*d+2*d*(-e*f)^(1/2)/f*(x-(-e*f)^(1/2)/
f)+(c*f-d*e)/f)^(1/2)*a-1/2/(-e*f)^(1/2)/f*((x-(-e*f)^(1/2)/f)^2*d+2*d*(-e*
f)^(1/2)/f*(x-(-e*f)^(1/2)/f)+(c*f-d*e)/f)^(1/2)*b*e+1/2/f*d^(1/2)*ln((d*(-
e*f)^(1/2)/f+(x-(-e*f)^(1/2)/f)*d)/d^(1/2)+((x-(-e*f)^(1/2)/f)^2*d+2*d*(-e*
```

```
f)^(1/2)/f*(x-(-e*f)^(1/2)/f)+(c*f-d*e)/f)^(1/2))*a-1/2/f^2*d^(1/2)*ln((d*(
-e*f)^(1/2)/f+(x-(-e*f)^(1/2)/f)*d)/d^(1/2)+((x-(-e*f)^(1/2)/f)^2*d+2*d*(-e
*f)^(1/2)/f*(x-(-e*f)^(1/2)/f)+(c*f-d*e)/f)^(1/2))*b*e-1/2/(-e*f)^(1/2)/((c
*f-d*e)/f)^(1/2)*ln((2*(c*f-d*e)/f+2*d*(-e*f)^(1/2)/f*(x-(-e*f)^(1/2)/f)+2*
((c*f-d*e)/f)^(1/2)*((x-(-e*f)^(1/2)/f)^2*d+2*d*(-e*f)^(1/2)/f*(x-(-e*f)^(1
/2)/f)+(c*f-d*e)/f)^(1/2))/(x-(-e*f)^(1/2)/f))*c*a+1/2/(-e*f)^(1/2)/f/((c*f
-d*e)/f)^(1/2)*ln((2*(c*f-d*e)/f+2*d*(-e*f)^(1/2)/f*(x-(-e*f)^(1/2)/f)+2*((
c*f-d*e)/f)^(1/2)*((x-(-e*f)^(1/2)/f)^2*d+2*d*(-e*f)^(1/2)/f*(x-(-e*f)^(1/2
)/f)+(c*f-d*e)/f)^(1/2))/(x-(-e*f)^(1/2)/f))*c*b*e+1/2/(-e*f)^(1/2)/f/((c*f
-d*e)/f)^(1/2)*ln((2*(c*f-d*e)/f+2*d*(-e*f)^(1/2)/f*(x-(-e*f)^(1/2)/f)+2*((
c*f-d*e)/f)^(1/2)*((x-(-e*f)^(1/2)/f)^2*d+2*d*(-e*f)^(1/2)/f*(x-(-e*f)^(1/2
)/f)+(c*f-d*e)/f)^(1/2))/(x-(-e*f)^(1/2)/f))*d*e*a-1/2/(-e*f)^(1/2)/f^2/((c
*f-d*e)/f)^(1/2)*ln((2*(c*f-d*e)/f+2*d*(-e*f)^(1/2)/f*(x-(-e*f)^(1/2)/f)+2*
((c*f-d*e)/f)^(1/2)*((x-(-e*f)^(1/2)/f)^2*d+2*d*(-e*f)^(1/2)/f*(x-(-e*f)^(1
/2)/f)+(c*f-d*e)/f)^(1/2))/(x-(-e*f)^(1/2)/f))*d*e^2*b-1/2/(-e*f)^(1/2)*((x
+(-e*f)^(1/2)/f)^2*d-2*d*(-e*f)^(1/2)/f*(x+(-e*f)^(1/2)/f)+(c*f-d*e)/f)^(1/
2)*a+1/2/(-e*f)^(1/2)/f*((x+(-e*f)^(1/2)/f)^2*d-2*d*(-e*f)^(1/2)/f*(x+(-e*f
)^(1/2)/f)+(c*f-d*e)/f)^(1/2)*b*e+1/2/f*d^(1/2)*ln((-d*(-e*f)^(1/2)/f+(x+(-
e*f)^(1/2)/f)*d)/d^(1/2)+((x+(-e*f)^(1/2)/f)^2*d-2*d*(-e*f)^(1/2)/f*(x+(-e*
f)^(1/2)/f)+(c*f-d*e)/f)^(1/2))*a-1/2/f^2*d^(1/2)*ln((-d*(-e*f)^(1/2)/f+(x+
(-e*f)^(1/2)/f)*d)/d^(1/2)+((x+(-e*f)^(1/2)/f)^2*d-2*d*(-e*f)^(1/2)/f*(x+(-
e*f)^(1/2)/f)+(c*f-d*e)/f)^(1/2))*b*e+1/2/(-e*f)^(1/2)/((c*f-d*e)/f)^(1/2)*
ln((2*(c*f-d*e)/f-2*d*(-e*f)^(1/2)/f*(x+(-e*f)^(1/2)/f)+2*((c*f-d*e)/f)^(1/
2)*((x+(-e*f)^(1/2)/f)^2*d-2*d*(-e*f)^(1/2)/f*(x+(-e*f)^(1/2)/f)+(c*f-d*e)/
f)^(1/2))/(x+(-e*f)^(1/2)/f))*c*a-1/2/(-e*f)^(1/2)/f/((c*f-d*e)/f)^(1/2)*ln
((2*(c*f-d*e)/f-2*d*(-e*f)^(1/2)/f*(x+(-e*f)^(1/2)/f)+2*((c*f-d*e)/f)^(1/2)
*((x+(-e*f)^(1/2)/f)^2*d-2*d*(-e*f)^(1/2)/f*(x+(-e*f)^(1/2)/f)+(c*f-d*e)/f)
^(1/2))/(x+(-e*f)^(1/2)/f))*c*b*e-1/2/(-e*f)^(1/2)/f/((c*f-d*e)/f)^(1/2)*ln
((2*(c*f-d*e)/f-2*d*(-e*f)^(1/2)/f*(x+(-e*f)^(1/2)/f)+2*((c*f-d*e)/f)^(1/2)
*((x+(-e*f)^(1/2)/f)^2*d-2*d*(-e*f)^(1/2)/f*(x+(-e*f)^(1/2)/f)+(c*f-d*e)/f)
^(1/2))/(x+(-e*f)^(1/2)/f))*d*e*a+1/2/(-e*f)^(1/2)/f^2/((c*f-d*e)/f)^(1/2)*
ln((2*(c*f-d*e)/f-2*d*(-e*f)^(1/2)/f*(x+(-e*f)^(1/2)/f)+2*((c*f-d*e)/f)^(1/
2)*((x+(-e*f)^(1/2)/f)^2*d-2*d*(-e*f)^(1/2)/f*(x+(-e*f)^(1/2)/f)+(c*f-d*e)/
f)^(1/2))/(x+(-e*f)^(1/2)/f))*d*e^2*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 7.51925, size = 1701, normalized size = 13.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(d*x^2 + c)*b*d*f*x - (2*b*d*e - (b*c + 2*a*d)*f)*sqrt(d)*log(-
2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) - (b*d*e - a*d*f)*sqrt((d*e - c*
```

```
f)/e)*log(((8*d^2*e^2 - 8*c*d*e*f + c^2*f^2)*x^4 + c^2*e^2 + 2*(4*c*d*e^2 -
 3*c^2*e*f)*x^2 - 4*(c*e^2*x + (2*d*e^2 - c*e*f)*x^3)*sqrt(d*x^2 + c)*sqrt(
(d*e - c*f)/e))/(f^2*x^4 + 2*e*f*x^2 + e^2)))/(d*f^2), 1/4*(2*sqrt(d*x^2 +
c)*b*d*f*x + 2*(2*b*d*e - (b*c + 2*a*d)*f)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(
d*x^2 + c)) - (b*d*e - a*d*f)*sqrt((d*e - c*f)/e)*log(((8*d^2*e^2 - 8*c*d*e
*f + c^2*f^2)*x^4 + c^2*e^2 + 2*(4*c*d*e^2 - 3*c^2*e*f)*x^2 - 4*(c*e^2*x +
(2*d*e^2 - c*e*f)*x^3)*sqrt(d*x^2 + c)*sqrt((d*e - c*f)/e))/(f^2*x^4 + 2*e*
f*x^2 + e^2)))/(d*f^2), 1/4*(2*sqrt(d*x^2 + c)*b*d*f*x - 2*(b*d*e - a*d*f)*
sqrt(-(d*e - c*f)/e)*arctan(1/2*((2*d*e - c*f)*x^2 + c*e)*sqrt(d*x^2 + c)*s
qrt(-(d*e - c*f)/e)/((d^2*e - c*d*f)*x^3 + (c*d*e - c^2*f)*x)) - (2*b*d*e -
 (b*c + 2*a*d)*f)*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c))/
(d*f^2), 1/2*(sqrt(d*x^2 + c)*b*d*f*x + (2*b*d*e - (b*c + 2*a*d)*f)*sqrt(-d
)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - (b*d*e - a*d*f)*sqrt(-(d*e - c*f)/e)
*arctan(1/2*((2*d*e - c*f)*x^2 + c*e)*sqrt(d*x^2 + c)*sqrt(-(d*e - c*f)/e)/
((d^2*e - c*d*f)*x^3 + (c*d*e - c^2*f)*x)))/(d*f^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^2\right)\sqrt{c+dx^2}}{e+fx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+c)**(1/2)/(f*x**2+e),x)
```

```
[Out] Integral((a + b*x**2)*sqrt(c + d*x**2)/(e + f*x**2), x)
```

Giac [A] time = 1.47179, size = 227, normalized size = 1.77

$$\frac{\sqrt{dx^2+c}\,bx}{2f} - \frac{\left(ac\sqrt{d}f^2 - bc\sqrt{d}fe - ad^{\frac{3}{2}}fe + bd^{\frac{3}{2}}e^2\right)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2 f - cf + 2de}{2\sqrt{cdfe-d^2e^2}}\right)}{\sqrt{cdfe-d^2e^2}\,f^2} - \frac{\left(bcf+2adf-2bde\right)\log\left(\left(\sqrt{d}x - \right.\right.}{4\sqrt{d}f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(d*x^2 + c)*b*x/f - (a*c*sqrt(d)*f^2 - b*c*sqrt(d)*f*e - a*d^(3/2)*
f*e + b*d^(3/2)*e^2)*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*f - c*f +
2*d*e)/sqrt(c*d*f*e - d^2*e^2))/(sqrt(c*d*f*e - d^2*e^2)*f^2) - 1/4*(b*c*f
+ 2*a*d*f - 2*b*d*e)*log((sqrt(d)*x - sqrt(d*x^2 + c))^2)/(sqrt(d)*f^2)
```

3.58 $\int \frac{\left(a+bx^2\right)^3}{\left(c+dx^2\right)\sqrt{e+fx^2}}\,dx$

Optimal. Leaf size=304

$$\frac{b\left(8a^2f^2-8abef+3b^2e^2\right)\tanh^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e+fx^2}}\right)}{8df^{5/2}}-\frac{b^2x\sqrt{e+fx^2}(bc-ad)}{2d^2f}-\frac{3b^2x\sqrt{e+fx^2}(be-2af)}{8df^2}+\frac{b^2x\left(a+bx^2\right)\sqrt{e+}}{4df}$$

```
[Out] -(b^2*(b*c - a*d)*x*Sqrt[e + f*x^2])/(2*d^2*f) - (3*b^2*(b*e - 2*a*f)*x*Sqr
t[e + f*x^2])/(8*d*f^2) + (b^2*x*(a + b*x^2)*Sqrt[e + f*x^2])/(4*d*f) - ((b
*c - a*d)^3*ArcTan[(Sqrt[d*e - c*f]*x)/(Sqrt[c]*Sqrt[e + f*x^2])])/(Sqrt[c]
*d^3*Sqrt[d*e - c*f]) + (b*(b*c - a*d)^2*ArcTanh[(Sqrt[f]*x)/Sqrt[e + f*x^2
]])/(d^3*Sqrt[f]) + (b*(b*c - a*d)*(b*e - 2*a*f)*ArcTanh[(Sqrt[f]*x)/Sqrt[e
 + f*x^2]])/(2*d^2*f^(3/2)) + (b*(3*b^2*e^2 - 8*a*b*e*f + 8*a^2*f^2)*ArcTan
h[(Sqrt[f]*x)/Sqrt[e + f*x^2]])/(8*d*f^(5/2))
```

Rubi [A] time = 0.299454, antiderivative size = 304, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {545, 416, 388, 217, 206, 523, 377, 205}

$$\frac{b\left(8a^2f^2-8abef+3b^2e^2\right)\tanh^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e+fx^2}}\right)}{8df^{5/2}}-\frac{b^2x\sqrt{e+fx^2}(bc-ad)}{2d^2f}-\frac{3b^2x\sqrt{e+fx^2}(be-2af)}{8df^2}+\frac{b^2x\left(a+bx^2\right)\sqrt{e+}}{4df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3/((c + d*x^2)*Sqrt[e + f*x^2]),x]
```

```
[Out] -(b^2*(b*c - a*d)*x*Sqrt[e + f*x^2])/(2*d^2*f) - (3*b^2*(b*e - 2*a*f)*x*Sqr
t[e + f*x^2])/(8*d*f^2) + (b^2*x*(a + b*x^2)*Sqrt[e + f*x^2])/(4*d*f) - ((b
*c - a*d)^3*ArcTan[(Sqrt[d*e - c*f]*x)/(Sqrt[c]*Sqrt[e + f*x^2])])/(Sqrt[c]
*d^3*Sqrt[d*e - c*f]) + (b*(b*c - a*d)^2*ArcTanh[(Sqrt[f]*x)/Sqrt[e + f*x^2
]])/(d^3*Sqrt[f]) + (b*(b*c - a*d)*(b*e - 2*a*f)*ArcTanh[(Sqrt[f]*x)/Sqrt[e
 + f*x^2]])/(2*d^2*f^(3/2)) + (b*(3*b^2*e^2 - 8*a*b*e*f + 8*a^2*f^2)*ArcTan
h[(Sqrt[f]*x)/Sqrt[e + f*x^2]])/(8*d*f^(5/2))
```

Rule 545

```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Dist[d/b, Int[(c + d*x^2)^(q - 1)*(e + f*x^2)^r, x], x
] + Dist[(b*c - a*d)/b, Int[((c + d*x^2)^(q - 1)*(e + f*x^2)^r)/(a + b*x^2)
, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && GtQ[q, 1]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^3}{\left(c+dx^2\right)\sqrt{e+fx^2}}\,dx = \frac{b\int \frac{\left(a+bx^2\right)^2}{\sqrt{e+fx^2}}\,dx}{d} + \frac{(-bc+ad)\int \frac{\left(a+bx^2\right)^2}{\left(c+dx^2\right)\sqrt{e+fx^2}}\,dx}{d}$$

$$= \frac{b^2x\left(a+bx^2\right)\sqrt{e+fx^2}}{4df} - \frac{(b(bc-ad))\int \frac{a+bx^2}{\sqrt{e+fx^2}}\,dx}{d^2} + \frac{(bc-ad)^2\int \frac{a+bx^2}{\left(c+dx^2\right)\sqrt{e+fx^2}}\,dx}{d^2} + \frac{b\int \frac{-a(b\cdots}{\cdots}}{\cdots}$$

$$= -\frac{b^2(bc-ad)x\sqrt{e+fx^2}}{2d^2f} - \frac{3b^2(be-2af)x\sqrt{e+fx^2}}{8df^2} + \frac{b^2x\left(a+bx^2\right)\sqrt{e+fx^2}}{4df} + \frac{\left(b(bc-ad)^2\cdots\right.}{\cdots}$$

$$= -\frac{b^2(bc-ad)x\sqrt{e+fx^2}}{2d^2f} - \frac{3b^2(be-2af)x\sqrt{e+fx^2}}{8df^2} + \frac{b^2x\left(a+bx^2\right)\sqrt{e+fx^2}}{4df} + \frac{\left(b(bc-ad)^2\cdots\right.}{\cdots}$$

$$= -\frac{b^2(bc-ad)x\sqrt{e+fx^2}}{2d^2f} - \frac{3b^2(be-2af)x\sqrt{e+fx^2}}{8df^2} + \frac{b^2x\left(a+bx^2\right)\sqrt{e+fx^2}}{4df} - \frac{(bc-ad)^3\,\mathrm{ta}\cdots}{\sqrt{c}d^3\cdots}$$

Mathematica [A] time = 0.342896, size = 265, normalized size = 0.87

$$\frac{\frac{8b\left(3a^2d^2-3abcd+b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e+fx^2}}\right)}{\sqrt{f}}+\frac{4b^2de(bc-3ad)\tanh^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e+fx^2}}\right)}{f^{3/2}}-\frac{4b^2dx\sqrt{e+fx^2}(bc-3ad)}{f}+\frac{8(ad-bc)^3\tan^{-1}\left(\frac{x\sqrt{de-cf}}{\sqrt{c}\sqrt{e+fx^2}}\right)}{\sqrt{c}\sqrt{de-cf}}+\frac{3b^3d^2e\left(e\tanh \cdots\right.}{\cdots}}{8d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3/((c + d*x^2)*Sqrt[e + f*x^2]),x]
```

```
[Out] ((-4*b^2*d*(b*c - 3*a*d)*x*Sqrt[e + f*x^2])/f + (2*b^3*d^2*x^3*Sqrt[e + f*x
^2])/f + (8*(-(b*c) + a*d)^3*ArcTan[(Sqrt[d*e - c*f]*x)/(Sqrt[c]*Sqrt[e + f
*x^2])])/(Sqrt[c]*Sqrt[d*e - c*f]) + (4*b^2*d*(b*c - 3*a*d)*e*ArcTanh[(Sqrt
[f]*x)/Sqrt[e + f*x^2]])/f^(3/2) + (8*b*(b^2*c^2 - 3*a*b*c*d + 3*a^2*d^2)*A
rcTanh[(Sqrt[f]*x)/Sqrt[e + f*x^2]])/Sqrt[f] + (3*b^3*d^2*e*(-(Sqrt[f]*x*Sq
rt[e + f*x^2]) + e*ArcTanh[(Sqrt[f]*x)/Sqrt[e + f*x^2]]))/f^(5/2))/(8*d^3)
```

Maple [B] time = 0.039, size = 1541, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3/(d*x^2+c)/(f*x^2+e)^(1/2),x)
```

```
[Out] 1/4*b^3/d*x^3/f*(f*x^2+e)^(1/2)-3/8*b^3/d*e/f^2*x*(f*x^2+e)^(1/2)+3/8*b^3/d
*e^2/f^(5/2)*ln(x*f^(1/2)+(f*x^2+e)^(1/2))+3/2*b^2/d*x/f*(f*x^2+e)^(1/2)*a-
1/2*b^3/d^2*x/f*(f*x^2+e)^(1/2)*c-3/2*b^2/d*e/f^(3/2)*ln(x*f^(1/2)+(f*x^2+e
)^(1/2))*a+1/2*b^3/d^2*e/f^(3/2)*ln(x*f^(1/2)+(f*x^2+e)^(1/2))*c+3*b/d*a^2*
ln(x*f^(1/2)+(f*x^2+e)^(1/2))/f^(1/2)-3*b^2/d^2*c*a*ln(x*f^(1/2)+(f*x^2+e)^
(1/2))/f^(1/2)+b^3/d^3*c^2*ln(x*f^(1/2)+(f*x^2+e)^(1/2))/f^(1/2)-1/2/(-c*d)
^(1/2)/(-(c*f-d*e)/d)^(1/2)*ln((-2*(c*f-d*e)/d+2*f*(-c*d)^(1/2)/d*(x-(-c*d)
^(1/2)/d)+2*(-(c*f-d*e)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*f+2*f*(-c*d)^(1/2)/d
*(x-(-c*d)^(1/2)/d)-(c*f-d*e)/d)^(1/2))/(x-(-c*d)^(1/2)/d))*a^3+3/2/d/(-c*d
)^(1/2)/(-(c*f-d*e)/d)^(1/2)*ln((-2*(c*f-d*e)/d+2*f*(-c*d)^(1/2)/d*(x-(-c*d
)^(1/2)/d)+2*(-(c*f-d*e)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*f+2*f*(-c*d)^(1/2)/
d*(x-(-c*d)^(1/2)/d)-(c*f-d*e)/d)^(1/2))/(x-(-c*d)^(1/2)/d))*a^2*c*b-3/2/d^
2/(-c*d)^(1/2)/(-(c*f-d*e)/d)^(1/2)*ln((-2*(c*f-d*e)/d+2*f*(-c*d)^(1/2)/d*(
x-(-c*d)^(1/2)/d)+2*(-(c*f-d*e)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*f+2*f*(-c*d)
^(1/2)/d*(x-(-c*d)^(1/2)/d)-(c*f-d*e)/d)^(1/2))/(x-(-c*d)^(1/2)/d))*a*c^2*b
^2+1/2/d^3/(-c*d)^(1/2)/(-(c*f-d*e)/d)^(1/2)*ln((-2*(c*f-d*e)/d+2*f*(-c*d)^
(1/2)/d*(x-(-c*d)^(1/2)/d)+2*(-(c*f-d*e)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*f+2
*f*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)-(c*f-d*e)/d)^(1/2))/(x-(-c*d)^(1/2)/d)
)*c^3*b^3+1/2/(-c*d)^(1/2)/(-(c*f-d*e)/d)^(1/2)*ln((-2*(c*f-d*e)/d-2*f*(-c*
d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+2*(-(c*f-d*e)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*
f-2*f*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)-(c*f-d*e)/d)^(1/2))/(x+(-c*d)^(1/2)
/d))*a^3-3/2/d/(-c*d)^(1/2)/(-(c*f-d*e)/d)^(1/2)*ln((-2*(c*f-d*e)/d-2*f*(-c
*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+2*(-(c*f-d*e)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2
*f-2*f*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)-(c*f-d*e)/d)^(1/2))/(x+(-c*d)^(1/2
)/d))*a^2*c*b+3/2/d^2/(-c*d)^(1/2)/(-(c*f-d*e)/d)^(1/2)*ln((-2*(c*f-d*e)/d-
2*f*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+2*(-(c*f-d*e)/d)^(1/2)*((x+(-c*d)^(1/
2)/d)^2*f-2*f*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)-(c*f-d*e)/d)^(1/2))/(x+(-c*
d)^(1/2)/d))*a*c^2*b^2-1/2/d^3/(-c*d)^(1/2)/(-(c*f-d*e)/d)^(1/2)*ln((-2*(c*
f-d*e)/d-2*f*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+2*(-(c*f-d*e)/d)^(1/2)*((x+(
-c*d)^(1/2)/d)^2*f-2*f*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)-(c*f-d*e)/d)^(1/2)
```

```
)/(x+(-c*d)^(1/2)/d))*c^3*b^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/(d*x^2+c)/(f*x^2+e)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 43.4153, size = 3518, normalized size = 11.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/(d*x^2+c)/(f*x^2+e)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(4*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(-c*d*e +
c^2*f)*f^3*log(((d^2*e^2 - 8*c*d*e*f + 8*c^2*f^2)*x^4 + c^2*e^2 - 2*(3*c*d*
e^2 - 4*c^2*e*f)*x^2 - 4*((d*e - 2*c*f)*x^3 - c*e*x)*sqrt(-c*d*e + c^2*f)*s
qrt(f*x^2 + e))/(d^2*x^4 + 2*c*d*x^2 + c^2)) + (3*b^3*c*d^3*e^3 + (b^3*c^2*
d^2 - 12*a*b^2*c*d^3)*e^2*f + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 6*a^2*b*c*d^
3)*e*f^2 - 8*(b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2)*f^3)*sqrt(f)*log(-
2*f*x^2 - 2*sqrt(f*x^2 + e)*sqrt(f)*x - e) + 2*(2*(b^3*c*d^3*e*f^2 - b^3*c^
2*d^2*f^3)*x^3 - (3*b^3*c*d^3*e^2*f + (b^3*c^2*d^2 - 12*a*b^2*c*d^3)*e*f^2
- 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2)*f^3)*x)*sqrt(f*x^2 + e))/(c*d^4*e*f^3 - c
^2*d^3*f^4), -1/16*(8*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*s
qrt(c*d*e - c^2*f)*f^3*arctan(1/2*sqrt(c*d*e - c^2*f)*((d*e - 2*c*f)*x^2 -
c*e)*sqrt(f*x^2 + e)/((c*d*e*f - c^2*f^2)*x^3 + (c*d*e^2 - c^2*e*f)*x)) - (
3*b^3*c*d^3*e^3 + (b^3*c^2*d^2 - 12*a*b^2*c*d^3)*e^2*f + 4*(b^3*c^3*d - 3*a
*b^2*c^2*d^2 + 6*a^2*b*c*d^3)*e*f^2 - 8*(b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*
c^2*d^2)*f^3)*sqrt(f)*log(-2*f*x^2 - 2*sqrt(f*x^2 + e)*sqrt(f)*x - e) - 2*(
2*(b^3*c*d^3*e*f^2 - b^3*c^2*d^2*f^3)*x^3 - (3*b^3*c*d^3*e^2*f + (b^3*c^2*d
^2 - 12*a*b^2*c*d^3)*e*f^2 - 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2)*f^3)*x)*sqrt(f
*x^2 + e))/(c*d^4*e*f^3 - c^2*d^3*f^4), 1/8*(2*(b^3*c^3 - 3*a*b^2*c^2*d + 3
*a^2*b*c*d^2 - a^3*d^3)*sqrt(-c*d*e + c^2*f)*f^3*log(((d^2*e^2 - 8*c*d*e*f
+ 8*c^2*f^2)*x^4 + c^2*e^2 - 2*(3*c*d*e^2 - 4*c^2*e*f)*x^2 - 4*((d*e - 2*c*
f)*x^3 - c*e*x)*sqrt(-c*d*e + c^2*f)*sqrt(f*x^2 + e))/(d^2*x^4 + 2*c*d*x^2
+ c^2)) - (3*b^3*c*d^3*e^3 + (b^3*c^2*d^2 - 12*a*b^2*c*d^3)*e^2*f + 4*(b^3*
c^3*d - 3*a*b^2*c^2*d^2 + 6*a^2*b*c*d^3)*e*f^2 - 8*(b^3*c^4 - 3*a*b^2*c^3*d
 + 3*a^2*b*c^2*d^2)*f^3)*sqrt(-f)*arctan(sqrt(-f)*x/sqrt(f*x^2 + e)) + (2*(
b^3*c*d^3*e*f^2 - b^3*c^2*d^2*f^3)*x^3 - (3*b^3*c*d^3*e^2*f + (b^3*c^2*d^2
- 12*a*b^2*c*d^3)*e*f^2 - 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2)*f^3)*x)*sqrt(f*x^
2 + e))/(c*d^4*e*f^3 - c^2*d^3*f^4), -1/8*(4*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a
^2*b*c*d^2 - a^3*d^3)*sqrt(c*d*e - c^2*f)*f^3*arctan(1/2*sqrt(c*d*e - c^2*f
)*((d*e - 2*c*f)*x^2 - c*e)*sqrt(f*x^2 + e)/((c*d*e*f - c^2*f^2)*x^3 + (c*d
*e^2 - c^2*e*f)*x)) + (3*b^3*c*d^3*e^3 + (b^3*c^2*d^2 - 12*a*b^2*c*d^3)*e^2
*f + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 6*a^2*b*c*d^3)*e*f^2 - 8*(b^3*c^4 - 3
*a*b^2*c^3*d + 3*a^2*b*c^2*d^2)*f^3)*sqrt(-f)*arctan(sqrt(-f)*x/sqrt(f*x^2
+ e)) - (2*(b^3*c*d^3*e*f^2 - b^3*c^2*d^2*f^3)*x^3 - (3*b^3*c*d^3*e^2*f + (
b^3*c^2*d^2 - 12*a*b^2*c*d^3)*e*f^2 - 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2)*f^3)*
```

```
x)*sqrt(f*x^2 + e))/(c*d^4*e*f^3 - c^2*d^3*f^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx^2)^3}{(c+dx^2)\sqrt{e+fx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3/(d*x**2+c)/(f*x**2+e)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)**3/((c + d*x**2)*sqrt(e + f*x**2)), x)
```

Giac [A] time = 1.7549, size = 392, normalized size = 1.29

$$\frac{1}{8}\left(\frac{2\,b^3x^2}{df}-\frac{4\,b^3cd^4f^2-12\,ab^2d^5f^2+3\,b^3d^5fe}{d^6f^3}\right)\sqrt{fx^2+ex}+\frac{\left(b^3c^3\sqrt{f}-3\,ab^2c^2d\sqrt{f}+3\,a^2bcd^2\sqrt{f}-a^3d^3\sqrt{f}\right)\operatorname{arctan}}{\sqrt{-c^2f^2+cdfe}d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/(d*x^2+c)/(f*x^2+e)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(2*b^3*x^2/(d*f) - (4*b^3*c*d^4*f^2 - 12*a*b^2*d^5*f^2 + 3*b^3*d^5*f*e)
/(d^6*f^3))*sqrt(f*x^2 + e)*x + (b^3*c^3*sqrt(f) - 3*a*b^2*c^2*d*sqrt(f) +
3*a^2*b*c*d^2*sqrt(f) - a^3*d^3*sqrt(f))*arctan(1/2*((sqrt(f)*x - sqrt(f*x^
2 + e))^2*d + 2*c*f - d*e)/sqrt(-c^2*f^2 + c*d*f*e))/(sqrt(-c^2*f^2 + c*d*f
*e)*d^3) - 1/16*(8*b^3*c^2*f^2 - 24*a*b^2*c*d*f^2 + 24*a^2*b*d^2*f^2 + 4*b^
3*c*d*f*e - 12*a*b^2*d^2*f*e + 3*b^3*d^2*e^2)*log((sqrt(f)*x - sqrt(f*x^2 +
 e))^2)/(d^3*f^(5/2))
```

3.59 $\int \frac{\left(a+bx^2\right)^2}{\left(c+dx^2\right)\sqrt{e+fx^2}}\,dx$

Optimal. Leaf size=166

$$\frac{(bc-ad)^2\tan^{-1}\left(\frac{x\sqrt{de-cf}}{\sqrt{c}\sqrt{e+fx^2}}\right)}{\sqrt{c}d^2\sqrt{de-cf}}-\frac{b(bc-ad)\tanh^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e+fx^2}}\right)}{d^2\sqrt{f}}-\frac{b(be-2af)\tanh^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e+fx^2}}\right)}{2df^{3/2}}+\frac{b^2x\sqrt{e+fx^2}}{2df}$$

```
[Out] (b^2*x*Sqrt[e + f*x^2])/(2*d*f) + ((b*c - a*d)^2*ArcTan[(Sqrt[d*e - c*f]*x)
/(Sqrt[c]*Sqrt[e + f*x^2])])/(Sqrt[c]*d^2*Sqrt[d*e - c*f]) - (b*(b*c - a*d)
*ArcTanh[(Sqrt[f]*x)/Sqrt[e + f*x^2]])/(d^2*Sqrt[f]) - (b*(b*e - 2*a*f)*Arc
Tanh[(Sqrt[f]*x)/Sqrt[e + f*x^2]])/(2*d*f^(3/2))
```

Rubi [A] time = 0.105244, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {545, 388, 217, 206, 523, 377, 205}

$$\frac{(bc-ad)^2\tan^{-1}\left(\frac{x\sqrt{de-cf}}{\sqrt{c}\sqrt{e+fx^2}}\right)}{\sqrt{c}d^2\sqrt{de-cf}}-\frac{b(bc-ad)\tanh^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e+fx^2}}\right)}{d^2\sqrt{f}}-\frac{b(be-2af)\tanh^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e+fx^2}}\right)}{2df^{3/2}}+\frac{b^2x\sqrt{e+fx^2}}{2df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/((c + d*x^2)*Sqrt[e + f*x^2]),x]
```

```
[Out] (b^2*x*Sqrt[e + f*x^2])/(2*d*f) + ((b*c - a*d)^2*ArcTan[(Sqrt[d*e - c*f]*x)
/(Sqrt[c]*Sqrt[e + f*x^2])])/(Sqrt[c]*d^2*Sqrt[d*e - c*f]) - (b*(b*c - a*d)
*ArcTanh[(Sqrt[f]*x)/Sqrt[e + f*x^2]])/(d^2*Sqrt[f]) - (b*(b*e - 2*a*f)*Arc
Tanh[(Sqrt[f]*x)/Sqrt[e + f*x^2]])/(2*d*f^(3/2))
```

Rule 545

```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Dist[d/b, Int[(c + d*x^2)^(q - 1)*(e + f*x^2)^r, x], x
] + Dist[(b*c - a*d)/b, Int[((c + d*x^2)^(q - 1)*(e + f*x^2)^r)/(a + b*x^2)
, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && GtQ[q, 1]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2}{\left(c+dx^2\right)\sqrt{e+fx^2}}\,dx = \frac{b\int \frac{a+bx^2}{\sqrt{e+fx^2}}\,dx}{d} + \frac{(-bc+ad)\int \frac{a+bx^2}{\left(c+dx^2\right)\sqrt{e+fx^2}}\,dx}{d}$$

$$= \frac{b^2x\sqrt{e+fx^2}}{2df} - \frac{(b(bc-ad))\int \frac{1}{\sqrt{e+fx^2}}\,dx}{d^2} + \frac{(bc-ad)^2\int \frac{1}{\left(c+dx^2\right)\sqrt{e+fx^2}}\,dx}{d^2} - \frac{(b(be-2af))\int \cdots}{2df\cdots}$$

$$= \frac{b^2x\sqrt{e+fx^2}}{2df} - \frac{(b(bc-ad))\,\text{Subst}\left(\int \frac{1}{1-fx^2}\,dx, x, \frac{x}{\sqrt{e+fx^2}}\right)}{d^2} + \frac{(bc-ad)^2\,\text{Subst}\left(\int \frac{1}{c-(-de+c\cdots}\cdots\right.}{d^2}$$

$$= \frac{b^2x\sqrt{e+fx^2}}{2df} + \frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{de-cf}\,x}{\sqrt{c}\sqrt{e+fx^2}}\right)}{\sqrt{c}d^2\sqrt{de-cf}} - \frac{b(bc-ad)\tanh^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e+fx^2}}\right)}{d^2\sqrt{f}} - \frac{b(be-2af)\cdots}{2\cdots}$$

Mathematica [A] time = 0.305973, size = 150, normalized size = 0.9

$$\frac{\frac{2f(bc-ad)^2\tan^{-1}\left(\frac{x\sqrt{de-cf}}{\sqrt{c}\sqrt{e+fx^2}}\right)+b^2\sqrt{c}dx\sqrt{e+fx^2}\sqrt{de-cf}}{\sqrt{c}f\sqrt{de-cf}} - \frac{b\tanh^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e+fx^2}}\right)(-4adf+2bcf+bde)}{f^{3/2}}}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/((c + d*x^2)*Sqrt[e + f*x^2]),x]
```

```
[Out] ((b^2*Sqrt[c]*d*Sqrt[d*e - c*f]*x*Sqrt[e + f*x^2] + 2*(b*c - a*d)^2*f*ArcTa
n[(Sqrt[d*e - c*f]*x)/(Sqrt[c]*Sqrt[e + f*x^2])])/(Sqrt[c]*f*Sqrt[d*e - c*f
]) - (b*(b*d*e + 2*b*c*f - 4*a*d*f)*ArcTanh[(Sqrt[f]*x)/Sqrt[e + f*x^2]])/f
^(3/2))/(2*d^2)
```

Maple [B] time = 0.015, size = 1052, normalized size = 6.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/(d*x^2+c)/(f*x^2+e)^(1/2),x)
```

```
[Out] 1/2*b^2*x*(f*x^2+e)^(1/2)/d/f-1/2*b^2/d*e/f^(3/2)*ln(x*f^(1/2)+(f*x^2+e)^(1
/2))+2*b/d*a*ln(x*f^(1/2)+(f*x^2+e)^(1/2))/f^(1/2)-b^2/d^2*c*ln(x*f^(1/2)+(
f*x^2+e)^(1/2))/f^(1/2)-1/2/(-c*d)^(1/2)/(-(c*f-d*e)/d)^(1/2)*ln((-2*(c*f-d
*e)/d+2*f*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+2*(-(c*f-d*e)/d)^(1/2)*((x-(-c*
d)^(1/2)/d)^2*f+2*f*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)-(c*f-d*e)/d)^(1/2))/(
x-(-c*d)^(1/2)/d))*a^2+1/d/(-c*d)^(1/2)/(-(c*f-d*e)/d)^(1/2)*ln((-2*(c*f-d*
e)/d+2*f*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+2*(-(c*f-d*e)/d)^(1/2)*((x-(-c*d
)^(1/2)/d)^2*f+2*f*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)-(c*f-d*e)/d)^(1/2))/(x
-(-c*d)^(1/2)/d))*c*a*b-1/2/d^2/(-c*d)^(1/2)/(-(c*f-d*e)/d)^(1/2)*ln((-2*(c
*f-d*e)/d+2*f*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+2*(-(c*f-d*e)/d)^(1/2)*((x-
(-c*d)^(1/2)/d)^2*f+2*f*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)-(c*f-d*e)/d)^(1/2
))/(x-(-c*d)^(1/2)/d))*b^2*c^2+1/2/(-c*d)^(1/2)/(-(c*f-d*e)/d)^(1/2)*ln((-2
*(c*f-d*e)/d-2*f*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+2*(-(c*f-d*e)/d)^(1/2)*(
(x+(-c*d)^(1/2)/d)^2*f-2*f*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)-(c*f-d*e)/d)^(
1/2))/(x+(-c*d)^(1/2)/d))*a^2-1/d/(-c*d)^(1/2)/(-(c*f-d*e)/d)^(1/2)*ln((-2*
(c*f-d*e)/d-2*f*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+2*(-(c*f-d*e)/d)^(1/2)*((
x+(-c*d)^(1/2)/d)^2*f-2*f*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)-(c*f-d*e)/d)^(1
/2))/(x+(-c*d)^(1/2)/d))*c*a*b+1/2/d^2/(-c*d)^(1/2)/(-(c*f-d*e)/d)^(1/2)*ln
((-2*(c*f-d*e)/d-2*f*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+2*(-(c*f-d*e)/d)^(1/
2)*((x+(-c*d)^(1/2)/d)^2*f-2*f*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)-(c*f-d*e)/
d)^(1/2))/(x+(-c*d)^(1/2)/d))*b^2*c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)/(f*x^2+e)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 11.0741, size = 2346, normalized size = 14.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)/(f*x^2+e)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-c*d*e + c^2*f)*f^2*log(((d^2*e
^2 - 8*c*d*e*f + 8*c^2*f^2)*x^4 + c^2*e^2 - 2*(3*c*d*e^2 - 4*c^2*e*f)*x^2 -
 4*((d*e - 2*c*f)*x^3 - c*e*x)*sqrt(-c*d*e + c^2*f)*sqrt(f*x^2 + e))/(d^2*x
^4 + 2*c*d*x^2 + c^2)) - 2*(b^2*c*d^2*e*f - b^2*c^2*d*f^2)*sqrt(f*x^2 + e)*
x + (b^2*c*d^2*e^2 + (b^2*c^2*d - 4*a*b*c*d^2)*e*f - 2*(b^2*c^3 - 2*a*b*c^2
*d)*f^2)*sqrt(f)*log(-2*f*x^2 - 2*sqrt(f*x^2 + e)*sqrt(f)*x - e))/(c*d^3*e*
f^2 - c^2*d^2*f^3), 1/4*(2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(c*d*e - c^2
*f)*f^2*arctan(1/2*sqrt(c*d*e - c^2*f)*((d*e - 2*c*f)*x^2 - c*e)*sqrt(f*x^2
 + e)/((c*d*e*f - c^2*f^2)*x^3 + (c*d*e^2 - c^2*e*f)*x)) + 2*(b^2*c*d^2*e*f
 - b^2*c^2*d*f^2)*sqrt(f*x^2 + e)*x - (b^2*c*d^2*e^2 + (b^2*c^2*d - 4*a*b*c
*d^2)*e*f - 2*(b^2*c^3 - 2*a*b*c^2*d)*f^2)*sqrt(f)*log(-2*f*x^2 - 2*sqrt(f*
x^2 + e)*sqrt(f)*x - e))/(c*d^3*e*f^2 - c^2*d^2*f^3), -1/4*((b^2*c^2 - 2*a*
```

```
b*c*d + a^2*d^2)*sqrt(-c*d*e + c^2*f)*f^2*log(((d^2*e^2 - 8*c*d*e*f + 8*c^2
*f^2)*x^4 + c^2*e^2 - 2*(3*c*d*e^2 - 4*c^2*e*f)*x^2 - 4*((d*e - 2*c*f)*x^3
- c*e*x)*sqrt(-c*d*e + c^2*f)*sqrt(f*x^2 + e))/(d^2*x^4 + 2*c*d*x^2 + c^2))
 - 2*(b^2*c*d^2*e*f - b^2*c^2*d*f^2)*sqrt(f*x^2 + e)*x - 2*(b^2*c*d^2*e^2 +
 (b^2*c^2*d - 4*a*b*c*d^2)*e*f - 2*(b^2*c^3 - 2*a*b*c^2*d)*f^2)*sqrt(-f)*ar
ctan(sqrt(-f)*x/sqrt(f*x^2 + e)))/(c*d^3*e*f^2 - c^2*d^2*f^3), 1/2*((b^2*c^
2 - 2*a*b*c*d + a^2*d^2)*sqrt(c*d*e - c^2*f)*f^2*arctan(1/2*sqrt(c*d*e - c^
2*f)*((d*e - 2*c*f)*x^2 - c*e)*sqrt(f*x^2 + e)/((c*d*e*f - c^2*f^2)*x^3 + (
c*d*e^2 - c^2*e*f)*x)) + (b^2*c*d^2*e*f - b^2*c^2*d*f^2)*sqrt(f*x^2 + e)*x
+ (b^2*c*d^2*e^2 + (b^2*c^2*d - 4*a*b*c*d^2)*e*f - 2*(b^2*c^3 - 2*a*b*c^2*d
)*f^2)*sqrt(-f)*arctan(sqrt(-f)*x/sqrt(f*x^2 + e)))/(c*d^3*e*f^2 - c^2*d^2*
f^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^2\right)^2}{\left(c+dx^2\right)\sqrt{e+fx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/(d*x**2+c)/(f*x**2+e)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)**2/((c + d*x**2)*sqrt(e + f*x**2)), x)
```

Giac [A] time = 1.65003, size = 248, normalized size = 1.49

$$\frac{\sqrt{fx^2+e}\,b^2x}{2\,df} - \frac{\left(b^2c^2\sqrt{f} - 2\,abcd\sqrt{f} + a^2d^2\sqrt{f}\right)\arctan\left(\frac{\left(\sqrt{f}x-\sqrt{fx^2+e}\right)^2 d+2\,cf-de}{2\,\sqrt{-c^2f^2+cdfe}}\right)}{\sqrt{-c^2f^2+cdfe}\,d^2} + \frac{\left(2\,b^2cf^{\frac{3}{2}} - 4\,abdf^{\frac{3}{2}} + b^2d\sqrt{fe}\right)\log}{4\,d^2f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)/(f*x^2+e)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(f*x^2 + e)*b^2*x/(d*f) - (b^2*c^2*sqrt(f) - 2*a*b*c*d*sqrt(f) + a^
2*d^2*sqrt(f))*arctan(1/2*((sqrt(f)*x - sqrt(f*x^2 + e))^2*d + 2*c*f - d*e)
/sqrt(-c^2*f^2 + c*d*f*e))/(sqrt(-c^2*f^2 + c*d*f*e)*d^2) + 1/4*(2*b^2*c*f^
(3/2) - 4*a*b*d*f^(3/2) + b^2*d*sqrt(f)*e)*log((sqrt(f)*x - sqrt(f*x^2 + e)
)^2)/(d^2*f^2)
```

3.60 $\int \frac{a+bx^2}{(c+dx^2)\sqrt{e+fx^2}}\,dx$

Optimal. Leaf size=91

$$\frac{b\tanh^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e+fx^2}}\right)}{d\sqrt{f}}-\frac{(bc-ad)\tan^{-1}\left(\frac{x\sqrt{de-cf}}{\sqrt{c}\sqrt{e+fx^2}}\right)}{\sqrt{c}d\sqrt{de-cf}}$$

```
[Out] -(((b*c - a*d)*ArcTan[(Sqrt[d*e - c*f]*x)/(Sqrt[c]*Sqrt[e + f*x^2])])/(Sqrt
[c]*d*Sqrt[d*e - c*f])) + (b*ArcTanh[(Sqrt[f]*x)/Sqrt[e + f*x^2]])/(d*Sqrt[
f])
```

Rubi [A] time = 0.0481703, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {523, 217, 206, 377, 205}

$$\frac{b\tanh^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e+fx^2}}\right)}{d\sqrt{f}}-\frac{(bc-ad)\tan^{-1}\left(\frac{x\sqrt{de-cf}}{\sqrt{c}\sqrt{e+fx^2}}\right)}{\sqrt{c}d\sqrt{de-cf}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/((c + d*x^2)*Sqrt[e + f*x^2]),x]
```

```
[Out] -(((b*c - a*d)*ArcTan[(Sqrt[d*e - c*f]*x)/(Sqrt[c]*Sqrt[e + f*x^2])])/(Sqrt
[c]*d*Sqrt[d*e - c*f])) + (b*ArcTanh[(Sqrt[f]*x)/Sqrt[e + f*x^2]])/(d*Sqrt[
f])
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2}{(c+dx^2)\sqrt{e+fx^2}}\,dx &= \frac{b\int \frac{1}{\sqrt{e+fx^2}}\,dx}{d}+\frac{(-bc+ad)\int \frac{1}{(c+dx^2)\sqrt{e+fx^2}}\,dx}{d}\\ &= \frac{b\,\mathrm{Subst}\left(\int \frac{1}{1-fx^2}\,dx, x, \frac{x}{\sqrt{e+fx^2}}\right)}{d}+\frac{(-bc+ad)\,\mathrm{Subst}\left(\int \frac{1}{c-(-de+cf)x^2}\,dx, x, \frac{x}{\sqrt{e+fx^2}}\right)}{d}\\ &= -\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{de-cf}\,x}{\sqrt{c}\sqrt{e+fx^2}}\right)}{\sqrt{c}d\sqrt{de-cf}}+\frac{b\tanh^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e+fx^2}}\right)}{d\sqrt{f}}\end{aligned}$$

Mathematica [A] time = 0.109592, size = 88, normalized size = 0.97

$$\frac{\frac{(ad-bc)\tan^{-1}\left(\frac{x\sqrt{de-cf}}{\sqrt{c}\sqrt{e+fx^2}}\right)}{\sqrt{c}\sqrt{de-cf}}+\frac{b\tanh^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e+fx^2}}\right)}{\sqrt{f}}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/((c + d*x^2)*Sqrt[e + f*x^2]),x]
```

```
[Out] (((-(b*c) + a*d)*ArcTan[(Sqrt[d*e - c*f]*x)/(Sqrt[c]*Sqrt[e + f*x^2])])/(Sq
rt[c]*Sqrt[d*e - c*f]) + (b*ArcTanh[(Sqrt[f]*x)/Sqrt[e + f*x^2]])/Sqrt[f])/
d
```

Maple [B] time = 0.009, size = 646, normalized size = 7.1

$$\frac{b}{d}\ln\left(x\sqrt{f}+\sqrt{fx^2+e}\right)\frac{1}{\sqrt{f}}-\frac{a}{2}\ln\left(\left(-2\frac{cf-de}{d}+2\frac{f\sqrt{-cd}}{d}\left(x-\frac{\sqrt{-cd}}{d}\right)+2\sqrt{-\frac{cf-de}{d}}\sqrt{\left(x-\frac{\sqrt{-cd}}{d}\right)^2f+2\frac{f\sqrt{-c}}{d}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/(d*x^2+c)/(f*x^2+e)^(1/2),x)
```

```
[Out] 1/d*b*ln(x*f^(1/2)+(f*x^2+e)^(1/2))/f^(1/2)-1/2/(-c*d)^(1/2)/(-(c*f-d*e)/d)
^(1/2)*ln((-2*(c*f-d*e)/d+2*f*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+2*(-(c*f-d*
e)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*f+2*f*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)-(
c*f-d*e)/d)^(1/2))/(x-(-c*d)^(1/2)/d))*a+1/2/(-c*d)^(1/2)/d/(-(c*f-d*e)/d)^
(1/2)*ln((-2*(c*f-d*e)/d+2*f*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+2*(-(c*f-d*e
)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*f+2*f*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)-(c
*f-d*e)/d)^(1/2))/(x-(-c*d)^(1/2)/d))*b*c+1/2/(-c*d)^(1/2)/(-(c*f-d*e)/d)^(
1/2)*ln((-2*(c*f-d*e)/d-2*f*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+2*(-(c*f-d*e)
/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*f-2*f*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)-(c*
f-d*e)/d)^(1/2))/(x+(-c*d)^(1/2)/d))*a-1/2/(-c*d)^(1/2)/d/(-(c*f-d*e)/d)^(1
/2)*ln((-2*(c*f-d*e)/d-2*f*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+2*(-(c*f-d*e)/
d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*f-2*f*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)-(c*f
-d*e)/d)^(1/2))/(x+(-c*d)^(1/2)/d))*b*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+c)/(f*x^2+e)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 7.24016, size = 1596, normalized size = 17.54

$$\left[\frac{\sqrt{-cde+c^2f}(bc-ad)f\log\left(\frac{(d^2e^2-8\,cdef+8\,c^2f^2)x^4+c^2e^2-2\,(3\,cde^2-4\,c^2ef)x^2-4\,((de-2\,cf)x^3-cex)\sqrt{-cde+c^2f}\sqrt{fx^2+e}}{d^2x^4+2\,cdx^2+c^2}\right)+2\left(bcde-bc^2f\right)\sqrt{f}}{4\left(cd^2ef-c^2df^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+c)/(f*x^2+e)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(-c*d*e + c^2*f)*(b*c - a*d)*f*log(((d^2*e^2 - 8*c*d*e*f + 8*c^2*
f^2)*x^4 + c^2*e^2 - 2*(3*c*d*e^2 - 4*c^2*e*f)*x^2 - 4*((d*e - 2*c*f)*x^3 -
 c*e*x)*sqrt(-c*d*e + c^2*f)*sqrt(f*x^2 + e))/(d^2*x^4 + 2*c*d*x^2 + c^2))
+ 2*(b*c*d*e - b*c^2*f)*sqrt(f)*log(-2*f*x^2 - 2*sqrt(f*x^2 + e)*sqrt(f)*x
- e))/(c*d^2*e*f - c^2*d*f^2), -1/2*(sqrt(c*d*e - c^2*f)*(b*c - a*d)*f*arct
an(1/2*sqrt(c*d*e - c^2*f)*((d*e - 2*c*f)*x^2 - c*e)*sqrt(f*x^2 + e)/((c*d*
e*f - c^2*f^2)*x^3 + (c*d*e^2 - c^2*e*f)*x)) - (b*c*d*e - b*c^2*f)*sqrt(f)*
log(-2*f*x^2 - 2*sqrt(f*x^2 + e)*sqrt(f)*x - e))/(c*d^2*e*f - c^2*d*f^2), 1
/4*(sqrt(-c*d*e + c^2*f)*(b*c - a*d)*f*log(((d^2*e^2 - 8*c*d*e*f + 8*c^2*f^
2)*x^4 + c^2*e^2 - 2*(3*c*d*e^2 - 4*c^2*e*f)*x^2 - 4*((d*e - 2*c*f)*x^3 - c
*e*x)*sqrt(-c*d*e + c^2*f)*sqrt(f*x^2 + e))/(d^2*x^4 + 2*c*d*x^2 + c^2)) -
4*(b*c*d*e - b*c^2*f)*sqrt(-f)*arctan(sqrt(-f)*x/sqrt(f*x^2 + e)))/(c*d^2*e
*f - c^2*d*f^2), -1/2*(sqrt(c*d*e - c^2*f)*(b*c - a*d)*f*arctan(1/2*sqrt(c*
d*e - c^2*f)*((d*e - 2*c*f)*x^2 - c*e)*sqrt(f*x^2 + e)/((c*d*e*f - c^2*f^2)
*x^3 + (c*d*e^2 - c^2*e*f)*x)) + 2*(b*c*d*e - b*c^2*f)*sqrt(-f)*arctan(sqrt
(-f)*x/sqrt(f*x^2 + e)))/(c*d^2*e*f - c^2*d*f^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a+bx^2}{\left(c+dx^2\right)\sqrt{e+fx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/(d*x**2+c)/(f*x**2+e)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)/((c + d*x**2)*sqrt(e + f*x**2)), x)
```

Giac [A] time = 1.66463, size = 159, normalized size = 1.75

$$\frac{\left(bc\sqrt{f} - ad\sqrt{f}\right)\arctan\left(\frac{\left(\sqrt{f}x - \sqrt{fx^2+e}\right)^2 d + 2\,cf - de}{2\,\sqrt{-c^2f^2+cdfe}}\right)}{\sqrt{-c^2f^2 + cdfe}d} - \frac{b\log\left(\left(\sqrt{f}x - \sqrt{fx^2 + e}\right)^2\right)}{2\,d\sqrt{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x^2+c)/(f*x^2+e)^(1/2),x, algorithm="giac")
```

```
[Out] (b*c*sqrt(f) - a*d*sqrt(f))*arctan(1/2*((sqrt(f)*x - sqrt(f*x^2 + e))^2*d +
 2*c*f - d*e)/sqrt(-c^2*f^2 + c*d*f*e))/(sqrt(-c^2*f^2 + c*d*f*e)*d) - 1/2*
b*log((sqrt(f)*x - sqrt(f*x^2 + e))^2)/(d*sqrt(f))
```

3.61 $\int \frac{1}{(c+dx^2)\sqrt{e+fx^2}}\,dx$

Optimal. Leaf size=49

$$\frac{\tan^{-1}\left(\frac{x\sqrt{de-cf}}{\sqrt{c}\sqrt{e+fx^2}}\right)}{\sqrt{c}\sqrt{de-cf}}$$

```
[Out] ArcTan[(Sqrt[d*e - c*f]*x)/(Sqrt[c]*Sqrt[e + f*x^2])]/(Sqrt[c]*Sqrt[d*e - c
*f])
```

Rubi [A] time = 0.0200121, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {377, 205}

$$\frac{\tan^{-1}\left(\frac{x\sqrt{de-cf}}{\sqrt{c}\sqrt{e+fx^2}}\right)}{\sqrt{c}\sqrt{de-cf}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + d*x^2)*Sqrt[e + f*x^2]),x]
```

```
[Out] ArcTan[(Sqrt[d*e - c*f]*x)/(Sqrt[c]*Sqrt[e + f*x^2])]/(Sqrt[c]*Sqrt[d*e - c
*f])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(c+dx^2\right)\sqrt{e+fx^2}}\,dx &= \text{Subst}\left(\int \frac{1}{c-(-de+cf)x^2}\,dx, x, \frac{x}{\sqrt{e+fx^2}}\right)\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{de-cf}x}{\sqrt{c}\sqrt{e+fx^2}}\right)}{\sqrt{c}\sqrt{de-cf}}\end{aligned}$$

Mathematica [A] time = 0.0107023, size = 49, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{x\sqrt{de-cf}}{\sqrt{c}\sqrt{e+fx^2}}\right)}{\sqrt{c}\sqrt{de-cf}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + d*x^2)*Sqrt[e + f*x^2]),x]
```

```
[Out] ArcTan[(Sqrt[d*e - c*f]*x)/(Sqrt[c]*Sqrt[e + f*x^2])]/(Sqrt[c]*Sqrt[d*e - c
*f])
```

Maple [B] time = 0.009, size = 306, normalized size = 6.2

$$-\frac{1}{2}\ln\left(\left(-2\frac{cf-de}{d}+2\frac{f\sqrt{-cd}}{d}\left(x-\frac{\sqrt{-cd}}{d}\right)+2\sqrt{-\frac{cf-de}{d}}\sqrt{\left(x-\frac{\sqrt{-cd}}{d}\right)^2 f+2\frac{f\sqrt{-cd}}{d}\left(x-\frac{\sqrt{-cd}}{d}\right)-\frac{cf-de}{d}}\right)\left(x-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x^2+c)/(f*x^2+e)^(1/2),x)
```

```
[Out] -1/2/(-c*d)^(1/2)/(-(c*f-d*e)/d)^(1/2)*ln((-2*(c*f-d*e)/d+2*f*(-c*d)^(1/2)/
d*(x-(-c*d)^(1/2)/d)+2*(-(c*f-d*e)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*f+2*f*(-c
*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)-(c*f-d*e)/d)^(1/2))/(x-(-c*d)^(1/2)/d))+1/2/
(-c*d)^(1/2)/(-(c*f-d*e)/d)^(1/2)*ln((-2*(c*f-d*e)/d-2*f*(-c*d)^(1/2)/d*(x+
(-c*d)^(1/2)/d)+2*(-(c*f-d*e)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*f-2*f*(-c*d)^(
1/2)/d*(x+(-c*d)^(1/2)/d)-(c*f-d*e)/d)^(1/2))/(x+(-c*d)^(1/2)/d))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x^2+c)/(f*x^2+e)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.70522, size = 513, normalized size = 10.47

$$\left[-\frac{\sqrt{-cde+c^2f}\log\left(\frac{\left(d^2e^2-8\,cdef+8\,c^2f^2\right)x^4+c^2e^2-2\left(3\,cde^2-4\,c^2ef\right)x^2-4\left(\left(de-2\,cf\right)x^3-cex\right)\sqrt{-cde+c^2f}\sqrt{fx^2+e}}{d^2x^4+2\,cdx^2+c^2}\right)}{4\left(cde-c^2f\right)},\ \frac{\arctan\left(\frac{\sqrt{cde-c^2f}\left(\left(de-2\,cf\right)x\right.}{2\left(\left(cdef-c^2f^2\right)x^3+\left(\right.\right.}\right.}{2\sqrt{cde-c^2f}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x^2+c)/(f*x^2+e)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*sqrt(-c*d*e + c^2*f)*log(((d^2*e^2 - 8*c*d*e*f + 8*c^2*f^2)*x^4 + c^2
*e^2 - 2*(3*c*d*e^2 - 4*c^2*e*f)*x^2 - 4*((d*e - 2*c*f)*x^3 - c*e*x)*sqrt(-
c*d*e + c^2*f)*sqrt(f*x^2 + e))/(d^2*x^4 + 2*c*d*x^2 + c^2))/(c*d*e - c^2*f
), 1/2*arctan(1/2*sqrt(c*d*e - c^2*f)*((d*e - 2*c*f)*x^2 - c*e)*sqrt(f*x^2
+ e)/((c*d*e*f - c^2*f^2)*x^3 + (c*d*e^2 - c^2*e*f)*x))/sqrt(c*d*e - c^2*f)
]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(c+dx^2\right)\sqrt{e+fx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x**2+c)/(f*x**2+e)**(1/2),x)
```

```
[Out] Integral(1/((c + d*x**2)*sqrt(e + f*x**2)), x)
```

Giac [A] time = 1.54156, size = 100, normalized size = 2.04

$$-\frac{\sqrt{f}\arctan\left(\frac{\left(\sqrt{f}x-\sqrt{fx^2+e}\right)^2 d+2\,cf-de}{2\,\sqrt{-c^2f^2+cdfe}}\right)}{\sqrt{-c^2f^2+cdfe}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x^2+c)/(f*x^2+e)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(f)*arctan(1/2*((sqrt(f)*x - sqrt(f*x^2 + e))^2*d + 2*c*f - d*e)/sqrt(
-c^2*f^2 + c*d*f*e))/sqrt(-c^2*f^2 + c*d*f*e)
```

3.62 $\int \frac{1}{(a+bx^2)(c+dx^2)\sqrt{e+fx^2}}\,dx$

Optimal. Leaf size=122

$$\frac{b\tan^{-1}\left(\frac{x\sqrt{be-af}}{\sqrt{a}\sqrt{e+fx^2}}\right)}{\sqrt{a}(bc-ad)\sqrt{be-af}}-\frac{d\tan^{-1}\left(\frac{x\sqrt{de-cf}}{\sqrt{c}\sqrt{e+fx^2}}\right)}{\sqrt{c}(bc-ad)\sqrt{de-cf}}$$

```
[Out] (b*ArcTan[(Sqrt[b*e - a*f]*x)/(Sqrt[a]*Sqrt[e + f*x^2])])/(Sqrt[a]*(b*c - a
*d)*Sqrt[b*e - a*f]) - (d*ArcTan[(Sqrt[d*e - c*f]*x)/(Sqrt[c]*Sqrt[e + f*x^
2])])/(Sqrt[c]*(b*c - a*d)*Sqrt[d*e - c*f])
```

Rubi [A] time = 0.110828, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {532, 377, 205}

$$\frac{b\tan^{-1}\left(\frac{x\sqrt{be-af}}{\sqrt{a}\sqrt{e+fx^2}}\right)}{\sqrt{a}(bc-ad)\sqrt{be-af}}-\frac{d\tan^{-1}\left(\frac{x\sqrt{de-cf}}{\sqrt{c}\sqrt{e+fx^2}}\right)}{\sqrt{c}(bc-ad)\sqrt{de-cf}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)*(c + d*x^2)*Sqrt[e + f*x^2]),x]
```

```
[Out] (b*ArcTan[(Sqrt[b*e - a*f]*x)/(Sqrt[a]*Sqrt[e + f*x^2])])/(Sqrt[a]*(b*c - a
*d)*Sqrt[b*e - a*f]) - (d*ArcTan[(Sqrt[d*e - c*f]*x)/(Sqrt[c]*Sqrt[e + f*x^
2])])/(Sqrt[c]*(b*c - a*d)*Sqrt[d*e - c*f])
```

Rule 532

```
Int[1/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)^2
]), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/((a + b*x^2)*Sqrt[e + f*x^2]), x
], x] - Dist[d/(b*c - a*d), Int[1/((c + d*x^2)*Sqrt[e + f*x^2]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+bx^2)(c+dx^2)\sqrt{e+fx^2}}\,dx = \frac{b\int \frac{1}{(a+bx^2)\sqrt{e+fx^2}}\,dx}{bc-ad} - \frac{d\int \frac{1}{(c+dx^2)\sqrt{e+fx^2}}\,dx}{bc-ad}$$

$$= \frac{b\,\text{Subst}\left(\int \frac{1}{a-(-be+af)x^2}\,dx, x, \frac{x}{\sqrt{e+fx^2}}\right)}{bc-ad} - \frac{d\,\text{Subst}\left(\int \frac{1}{c-(-de+cf)x^2}\,dx, x, \frac{x}{\sqrt{e+fx^2}}\right)}{bc-ad}$$

$$= \frac{b\tan^{-1}\left(\frac{\sqrt{be-af}\,x}{\sqrt{a}\sqrt{e+fx^2}}\right)}{\sqrt{a}(bc-ad)\sqrt{be-af}} - \frac{d\tan^{-1}\left(\frac{\sqrt{de-cf}\,x}{\sqrt{c}\sqrt{e+fx^2}}\right)}{\sqrt{c}(bc-ad)\sqrt{de-cf}}$$

Mathematica [A] time = 0.176689, size = 113, normalized size = 0.93

$$\frac{\frac{b\tan^{-1}\left(\frac{x\sqrt{be-af}}{\sqrt{a}\sqrt{e+fx^2}}\right)}{\sqrt{a}\sqrt{be-af}} - \frac{d\tan^{-1}\left(\frac{x\sqrt{de-cf}}{\sqrt{c}\sqrt{e+fx^2}}\right)}{\sqrt{c}\sqrt{de-cf}}}{bc-ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)*(c + d*x^2)*Sqrt[e + f*x^2]),x]
```

```
[Out] ((b*ArcTan[(Sqrt[b*e - a*f]*x)/(Sqrt[a]*Sqrt[e + f*x^2])])/(Sqrt[a]*Sqrt[b*
e - a*f]) - (d*ArcTan[(Sqrt[d*e - c*f]*x)/(Sqrt[c]*Sqrt[e + f*x^2])])/(Sqrt
[c]*Sqrt[d*e - c*f]))/(b*c - a*d)
```

Maple [B] time = 0.037, size = 782, normalized size = 6.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)/(d*x^2+c)/(f*x^2+e)^(1/2),x)
```

```
[Out] 1/2*b^2*d/(-a*b)^(1/2)/((-a*b)^(1/2)*d+b*(-c*d)^(1/2))/((-a*b)^(1/2)*d-b*(-
c*d)^(1/2))/(-(a*f-b*e)/b)^(1/2)*ln((-2*(a*f-b*e)/b-2*f*(-a*b)^(1/2)/b*(x+1
/b*(-a*b)^(1/2))+2*(-(a*f-b*e)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*f-2*f*(-a*b
)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*f-b*e)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))-1/
2*b^2*d/(-a*b)^(1/2)/((-a*b)^(1/2)*d+b*(-c*d)^(1/2))/((-a*b)^(1/2)*d-b*(-c*
d)^(1/2))/(-(a*f-b*e)/b)^(1/2)*ln((-2*(a*f-b*e)/b+2*f*(-a*b)^(1/2)/b*(x-1/b
*(-a*b)^(1/2))+2*(-(a*f-b*e)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*f+2*f*(-a*b)^
(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*f-b*e)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))+1/2*
b*d^2/((-a*b)^(1/2)*d+b*(-c*d)^(1/2))/((-a*b)^(1/2)*d-b*(-c*d)^(1/2))/(-c*d
)^(1/2)/(-(c*f-d*e)/d)^(1/2)*ln((-2*(c*f-d*e)/d+2*f*(-c*d)^(1/2)/d*(x-(-c*d
)^(1/2)/d)+2*(-(c*f-d*e)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*f+2*f*(-c*d)^(1/2)/
d*(x-(-c*d)^(1/2)/d)-(c*f-d*e)/d)^(1/2))/(x-(-c*d)^(1/2)/d))-1/2*b*d^2/((-a
*b)^(1/2)*d+b*(-c*d)^(1/2))/((-a*b)^(1/2)*d-b*(-c*d)^(1/2))/(-c*d)^(1/2)/(-
(c*f-d*e)/d)^(1/2)*ln((-2*(c*f-d*e)/d-2*f*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)
+2*(-(c*f-d*e)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*f-2*f*(-c*d)^(1/2)/d*(x+(-c*d
)^(1/2)/d)-(c*f-d*e)/d)^(1/2))/(x+(-c*d)^(1/2)/d))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+a)(dx^2+c)\sqrt{fx^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)/(f*x^2+e)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)*(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)/(f*x^2+e)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx^2)(c+dx^2)\sqrt{e+fx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)/(d*x**2+c)/(f*x**2+e)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x**2)*(c + d*x**2)*sqrt(e + f*x**2)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)/(f*x^2+e)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.63 $\int \frac{1}{\left(a+bx^2\right)^2\left(c+dx^2\right)\sqrt{e+fx^2}}\,dx$

Optimal. Leaf size=203

$$\frac{b\left(4a^2df-2abcf-3abde+b^2ce\right)\tan^{-1}\left(\frac{x\sqrt{be-af}}{\sqrt{a}\sqrt{e+fx^2}}\right)}{2a^{3/2}(bc-ad)^2(be-af)^{3/2}}+\frac{b^2x\sqrt{e+fx^2}}{2a\left(a+bx^2\right)(bc-ad)(be-af)}+\frac{d^2\tan^{-1}\left(\frac{x\sqrt{de-cf}}{\sqrt{c}\sqrt{e+fx^2}}\right)}{\sqrt{c}(bc-ad)^2\sqrt{de-cf}}$$

```
[Out] (b^2*x*Sqrt[e + f*x^2])/(2*a*(b*c - a*d)*(b*e - a*f)*(a + b*x^2)) + (b*(b^2
*c*e - 3*a*b*d*e - 2*a*b*c*f + 4*a^2*d*f)*ArcTan[(Sqrt[b*e - a*f]*x)/(Sqrt[
a]*Sqrt[e + f*x^2])])/(2*a^(3/2)*(b*c - a*d)^2*(b*e - a*f)^(3/2)) + (d^2*Ar
cTan[(Sqrt[d*e - c*f]*x)/(Sqrt[c]*Sqrt[e + f*x^2])])/(Sqrt[c]*(b*c - a*d)^2
*Sqrt[d*e - c*f])
```

Rubi [A] time = 0.265115, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {546, 377, 205, 527, 12}

$$\frac{b\left(4a^2df-2abcf-3abde+b^2ce\right)\tan^{-1}\left(\frac{x\sqrt{be-af}}{\sqrt{a}\sqrt{e+fx^2}}\right)}{2a^{3/2}(bc-ad)^2(be-af)^{3/2}}+\frac{b^2x\sqrt{e+fx^2}}{2a\left(a+bx^2\right)(bc-ad)(be-af)}+\frac{d^2\tan^{-1}\left(\frac{x\sqrt{de-cf}}{\sqrt{c}\sqrt{e+fx^2}}\right)}{\sqrt{c}(bc-ad)^2\sqrt{de-cf}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^2*(c + d*x^2)*Sqrt[e + f*x^2]),x]
```

```
[Out] (b^2*x*Sqrt[e + f*x^2])/(2*a*(b*c - a*d)*(b*e - a*f)*(a + b*x^2)) + (b*(b^2
*c*e - 3*a*b*d*e - 2*a*b*c*f + 4*a^2*d*f)*ArcTan[(Sqrt[b*e - a*f]*x)/(Sqrt[
a]*Sqrt[e + f*x^2])])/(2*a^(3/2)*(b*c - a*d)^2*(b*e - a*f)^(3/2)) + (d^2*Ar
cTan[(Sqrt[d*e - c*f]*x)/(Sqrt[c]*Sqrt[e + f*x^2])])/(Sqrt[c]*(b*c - a*d)^2
*Sqrt[d*e - c*f])
```

Rule 546

```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Dist[b^2/(b*c - a*d)^2, Int[((c + d*x^2)^(q + 2)*(e +
f*x^2)^r)/(a + b*x^2), x], x] - Dist[d/(b*c - a*d)^2, Int[(c + d*x^2)^q*(e
+ f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r},
x] && LtQ[q, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
```

```
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)^2\left(c+dx^2\right)\sqrt{e+fx^2}}\,dx = -\frac{b\int \frac{-bc+2ad+bdx^2}{\left(a+bx^2\right)^2\sqrt{e+fx^2}}\,dx}{(bc-ad)^2}+\frac{d^2\int \frac{1}{\left(c+dx^2\right)\sqrt{e+fx^2}}\,dx}{(bc-ad)^2}$$

$$= \frac{b^2x\sqrt{e+fx^2}}{2a(bc-ad)(be-af)\left(a+bx^2\right)}+\frac{d^2\,\mathrm{Subst}\left(\int \frac{1}{c-(-de+cf)x^2}\,dx,x,\frac{x}{\sqrt{e+fx^2}}\right)}{(bc-ad)^2}+\frac{b\int}{2}$$

$$= \frac{b^2x\sqrt{e+fx^2}}{2a(bc-ad)(be-af)\left(a+bx^2\right)}+\frac{d^2\tan^{-1}\left(\frac{\sqrt{de-cf}\,x}{\sqrt{c}\sqrt{e+fx^2}}\right)}{\sqrt{c}(bc-ad)^2\sqrt{de-cf}}+\frac{\left(b\left(b^2ce-3abde-2\right.\right.}{2a(}$$

$$= \frac{b^2x\sqrt{e+fx^2}}{2a(bc-ad)(be-af)\left(a+bx^2\right)}+\frac{d^2\tan^{-1}\left(\frac{\sqrt{de-cf}\,x}{\sqrt{c}\sqrt{e+fx^2}}\right)}{\sqrt{c}(bc-ad)^2\sqrt{de-cf}}+\frac{\left(b\left(b^2ce-3abde-2\right.\right.}{}$$

$$= \frac{b^2x\sqrt{e+fx^2}}{2a(bc-ad)(be-af)\left(a+bx^2\right)}+\frac{b\left(b^2ce-3abde-2abcf+4a^2df\right)\tan^{-1}\left(\frac{\sqrt{b}}{\sqrt{a}}\right.}{2a^{3/2}(bc-ad)^2(be-af)^{3/2}}$$

Mathematica [C] time = 2.5579, size = 531, normalized size = 2.62

$$\frac{\frac{bx\sqrt{e+fx^2}(bc-ad)\left(-30fx^2\sqrt{\frac{ax^2\left(e+fx^2\right)(be-af)}{e^2\left(a+bx^2\right)^2}}-45e\sqrt{\frac{ax^2\left(e+fx^2\right)(be-af)}{e^2\left(a+bx^2\right)^2}}+16fx^2\sqrt{\frac{a\left(e+fx^2\right)}{e\left(a+bx^2\right)}}\left(\frac{x^2(be-af)}{e\left(a+bx^2\right)}\right)^{5/2}{}_2F_1\left(2,3;\frac{7}{2};\frac{(be-af)x^2}{e\left(bx^2+a\right)}\right)+16e\sqrt{\frac{a\left(e+fx^2\right)}{e\left(a+bx^2\right)}}\left(\frac{x^2(be-af)}{e\left(a+bx^2\right)}\right)^{5/2}{}_2F}{e^2\left(a+bx^2\right)^2\left(\frac{x^2(be-af)}{e\left(a+bx^2\right)}\right)^{3/2}\sqrt{\frac{a\left(e+fx^2\right)}{e\left(a+bx^2\right)}}}}{30(bc-ad)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^2)^2*(c + d*x^2)*Sqrt[e + f*x^2]),x]
```

```
[Out] ((-30*b*d*ArcTan[(Sqrt[b*e - a*f]*x)/(Sqrt[a]*Sqrt[e + f*x^2])])/(Sqrt[a]*S
qrt[b*e - a*f]) + (30*d^2*ArcTan[(Sqrt[d*e - c*f]*x)/(Sqrt[c]*Sqrt[e + f*x^
2])])/(Sqrt[c]*Sqrt[d*e - c*f]) + (b*(b*c - a*d)*x*Sqrt[e + f*x^2]*(-45*e*S
qrt[(a*(b*e - a*f)*x^2*(e + f*x^2))/(e^2*(a + b*x^2)^2)] - 30*f*x^2*Sqrt[(a
*(b*e - a*f)*x^2*(e + f*x^2))/(e^2*(a + b*x^2)^2)] + 45*e*ArcSin[Sqrt[((b*e
 - a*f)*x^2)/(e*(a + b*x^2))]] + 30*f*x^2*ArcSin[Sqrt[((b*e - a*f)*x^2)/(e*
(a + b*x^2))]] + 16*e*(((b*e - a*f)*x^2)/(e*(a + b*x^2)))^(5/2)*Sqrt[(a*(e
+ f*x^2))/(e*(a + b*x^2))]*Hypergeometric2F1[2, 3, 7/2, ((b*e - a*f)*x^2)/(
e*(a + b*x^2))] + 16*f*x^2*(((b*e - a*f)*x^2)/(e*(a + b*x^2)))^(5/2)*Sqrt[(
a*(e + f*x^2))/(e*(a + b*x^2))]*Hypergeometric2F1[2, 3, 7/2, ((b*e - a*f)*x
^2)/(e*(a + b*x^2))]))/(e^2*(((b*e - a*f)*x^2)/(e*(a + b*x^2)))^(3/2)*(a +
b*x^2)^2*Sqrt[(a*(e + f*x^2))/(e*(a + b*x^2))]))/(30*(b*c - a*d)^2)
```

Maple [B] time = 0.056, size = 1865, normalized size = 9.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^2/(d*x^2+c)/(f*x^2+e)^(1/2),x)
```

```
[Out] -1/4*b^2*d/a/((-a*b)^(1/2)*d+b*(-c*d)^(1/2))/((-a*b)^(1/2)*d-b*(-c*d)^(1/2)
)/(a*f-b*e)/(x+1/b*(-a*b)^(1/2))*((x+1/b*(-a*b)^(1/2))^2*f-2*f*(-a*b)^(1/2)
/b*(x+1/b*(-a*b)^(1/2))-(a*f-b*e)/b)^(1/2)-1/4*b*d/a/((-a*b)^(1/2)*d+b*(-c*
d)^(1/2))/((-a*b)^(1/2)*d-b*(-c*d)^(1/2))*f*(-a*b)^(1/2)/(a*f-b*e)/(-(a*f-b
*e)/b)^(1/2)*ln((-2*(a*f-b*e)/b-2*f*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(
-(a*f-b*e)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*f-2*f*(-a*b)^(1/2)/b*(x+1/b*(-a
*b)^(1/2))-(a*f-b*e)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))-3/4*b^3*d^3/(-a*b)^(1/
2)/((-a*b)^(1/2)*d+b*(-c*d)^(1/2))^2/((-a*b)^(1/2)*d-b*(-c*d)^(1/2))^2/(-(a
*f-b*e)/b)^(1/2)*ln((-2*(a*f-b*e)/b-2*f*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))
+2*(-(a*f-b*e)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*f-2*f*(-a*b)^(1/2)/b*(x+1/b
*(-a*b)^(1/2))-(a*f-b*e)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))+1/4*b^4*d^2/a/(-a*
b)^(1/2)/((-a*b)^(1/2)*d+b*(-c*d)^(1/2))^2/((-a*b)^(1/2)*d-b*(-c*d)^(1/2))^
2/(-(a*f-b*e)/b)^(1/2)*ln((-2*(a*f-b*e)/b-2*f*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^
(1/2))+2*(-(a*f-b*e)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*f-2*f*(-a*b)^(1/2)/b*
(x+1/b*(-a*b)^(1/2))-(a*f-b*e)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))*c+3/4*b^3*d^
3/(-a*b)^(1/2)/((-a*b)^(1/2)*d+b*(-c*d)^(1/2))^2/((-a*b)^(1/2)*d-b*(-c*d)^(
1/2))^2/(-(a*f-b*e)/b)^(1/2)*ln((-2*(a*f-b*e)/b+2*f*(-a*b)^(1/2)/b*(x-1/b*(
-a*b)^(1/2))+2*(-(a*f-b*e)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*f+2*f*(-a*b)^(1
/2)/b*(x-1/b*(-a*b)^(1/2))-(a*f-b*e)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))-1/4*b^
4*d^2/a/(-a*b)^(1/2)/((-a*b)^(1/2)*d+b*(-c*d)^(1/2))^2/((-a*b)^(1/2)*d-b*(-
c*d)^(1/2))^2/(-(a*f-b*e)/b)^(1/2)*ln((-2*(a*f-b*e)/b+2*f*(-a*b)^(1/2)/b*(x
-1/b*(-a*b)^(1/2))+2*(-(a*f-b*e)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*f+2*f*(-a
*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*f-b*e)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*
c-1/4*b^2*d/a/((-a*b)^(1/2)*d+b*(-c*d)^(1/2))/((-a*b)^(1/2)*d-b*(-c*d)^(1/2
))/(a*f-b*e)/(x-1/b*(-a*b)^(1/2))*((x-1/b*(-a*b)^(1/2))^2*f+2*f*(-a*b)^(1/2
)/b*(x-1/b*(-a*b)^(1/2))-(a*f-b*e)/b)^(1/2)+1/4*b*d/a/((-a*b)^(1/2)*d+b*(-c
*d)^(1/2))/((-a*b)^(1/2)*d-b*(-c*d)^(1/2))*f*(-a*b)^(1/2)/(a*f-b*e)/(-(a*f-
b*e)/b)^(1/2)*ln((-2*(a*f-b*e)/b+2*f*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*
(-(a*f-b*e)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*f+2*f*(-a*b)^(1/2)/b*(x-1/b*(-
a*b)^(1/2))-(a*f-b*e)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))-1/2*b^2*d^4/((-a*b)^(
1/2)*d+b*(-c*d)^(1/2))^2/((-a*b)^(1/2)*d-b*(-c*d)^(1/2))^2/(-c*d)^(1/2)/(-(
c*f-d*e)/d)^(1/2)*ln((-2*(c*f-d*e)/d+2*f*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+
2*(-(c*f-d*e)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*f+2*f*(-c*d)^(1/2)/d*(x-(-c*d)
^(1/2)/d)-(c*f-d*e)/d)^(1/2))/(x-(-c*d)^(1/2)/d))+1/2*b^2*d^4/((-a*b)^(1/2)
*d+b*(-c*d)^(1/2))^2/((-a*b)^(1/2)*d-b*(-c*d)^(1/2))^2/(-c*d)^(1/2)/(-(c*f-
d*e)/d)^(1/2)*ln((-2*(c*f-d*e)/d-2*f*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+2*(-
(c*f-d*e)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*f-2*f*(-c*d)^(1/2)/d*(x+(-c*d)^(1/
2)/d)-(c*f-d*e)/d)^(1/2))/(x+(-c*d)^(1/2)/d))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^2\left(dx^2+c\right)\sqrt{fx^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)/(f*x^2+e)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)/(f*x^2+e)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**2/(d*x**2+c)/(f*x**2+e)**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 16.9323, size = 647, normalized size = 3.19

$$-\frac{1}{2}\left(\frac{2\,d^2\arctan\left(\frac{\left(\sqrt{f}x-\sqrt{fx^2+e}\right)^2 d+2\,cf-de}{2\sqrt{-c^2f^2+cdfe}}\right)}{\left(b^2c^2f^2-2\,abcdf^2+a^2d^2f^2\right)\sqrt{-c^2f^2+cdfe}}+\frac{\left(2\,ab^2cf-4\,a^2bdf-b^3ce+3\,ab^2de\right)\arctan\left(\frac{\left(\sqrt{f}x-\sqrt{fx^2\cdots}\right.}{2\sqrt{-a^2\cdots}}\right.}{\left(a^2b^2c^2f^3-2\,a^3bcdf^3+a^4d^2f^3-ab^3c^2f^2e+2\,a^2b^2cdf^2e-a^3bd\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)/(f*x^2+e)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(2*d^2*arctan(1/2*((sqrt(f)*x - sqrt(f*x^2 + e))^2*d + 2*c*f - d*e)/sq
rt(-c^2*f^2 + c*d*f*e))/((b^2*c^2*f^2 - 2*a*b*c*d*f^2 + a^2*d^2*f^2)*sqrt(-
c^2*f^2 + c*d*f*e)) + (2*a*b^2*c*f - 4*a^2*b*d*f - b^3*c*e + 3*a*b^2*d*e)*a
rctan(1/2*((sqrt(f)*x - sqrt(f*x^2 + e))^2*b + 2*a*f - b*e)/sqrt(-a^2*f^2 +
 a*b*f*e))/((a^2*b^2*c^2*f^3 - 2*a^3*b*c*d*f^3 + a^4*d^2*f^3 - a*b^3*c^2*f^
2*e + 2*a^2*b^2*c*d*f^2*e - a^3*b*d^2*f^2*e)*sqrt(-a^2*f^2 + a*b*f*e)) + 2*
(2*(sqrt(f)*x - sqrt(f*x^2 + e))^2*a*b*f - (sqrt(f)*x - sqrt(f*x^2 + e))^2*
b^2*e + b^2*e^2)/((a^2*b*c*f^3 - a^3*d*f^3 - a*b^2*c*f^2*e + a^2*b*d*f^2*e)
*((sqrt(f)*x - sqrt(f*x^2 + e))^4*b + 4*(sqrt(f)*x - sqrt(f*x^2 + e))^2*a*f
 - 2*(sqrt(f)*x - sqrt(f*x^2 + e))^2*b*e + b*e^2)))*f^(5/2)
```

3.64 $\int \frac{\left(c+dx^2\right)^{5/2}\sqrt{e+fx^2}}{a+bx^2}\,dx$

Optimal. Leaf size=608

$$\frac{de^{3/2}\sqrt{c+dx^2}\left(15a^2d^2f-40abcdf+b^2c(34cf-de)\right)\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{15b^3cf^{3/2}\sqrt{e+fx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}-\frac{\sqrt{e}\sqrt{c+dx^2}\left(15a^2d^2f^2-5abdf(\right.}{}$$

```
[Out] (d*(7*c*e - (2*d*e^2)/f + (3*c^2*f)/d)*x*Sqrt[c + d*x^2])/(15*b*Sqrt[e + f*
x^2]) + ((b*c - a*d)*(b*d*e + 4*b*c*f - 3*a*d*f)*x*Sqrt[c + d*x^2])/(3*b^3*
Sqrt[e + f*x^2]) + (d*(b*c - a*d)*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(3*b^2
) - (2*d*(d*e - 3*c*f)*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(15*b*f) + (d^2*x
*Sqrt[c + d*x^2]*(e + f*x^2)^(3/2))/(5*b*f) - (Sqrt[e]*(15*a^2*d^2*f^2 - 5*
a*b*d*f*(d*e + 7*c*f) + b^2*(-2*d^2*e^2 + 12*c*d*e*f + 23*c^2*f^2))*Sqrt[c
+ d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(15*b^3*f
^(3/2)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (d*e^(3/2)*
(-40*a*b*c*d*f + 15*a^2*d^2*f + b^2*c*(-(d*e) + 34*c*f))*Sqrt[c + d*x^2]*El
lipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(15*b^3*c*f^(3/2)*Sq
rt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + ((b*c - a*d)^3*e^(3/
2)*Sqrt[c + d*x^2]*EllipticPi[1 - (b*e)/(a*f), ArcTan[(Sqrt[f]*x)/Sqrt[e]],
 1 - (d*e)/(c*f)])/(a*b^3*c*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*S
qrt[e + f*x^2])
```

Rubi [A] time = 0.762876, antiderivative size = 776, normalized size of antiderivative = 1.28, number of steps used = 14, number of rules used = 9, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.281, Rules used = {545, 416, 528, 531, 418, 492, 411, 543, 539}

$$\frac{de^{3/2}\sqrt{c+dx^2}(5bc-3ad)(bc-ad)F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3b^3c\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}+\frac{e^{3/2}\sqrt{c+dx^2}(bc-ad)^3\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{ab^3c\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}+\frac{dx}{}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*x^2)^(5/2)*Sqrt[e + f*x^2])/(a + b*x^2),x]
```

```
[Out] (d*(7*c*e - (2*d*e^2)/f + (3*c^2*f)/d)*x*Sqrt[c + d*x^2])/(15*b*Sqrt[e + f*
x^2]) + ((b*c - a*d)*(b*d*e + 4*b*c*f - 3*a*d*f)*x*Sqrt[c + d*x^2])/(3*b^3*
Sqrt[e + f*x^2]) + (d*(b*c - a*d)*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(3*b^2
) - (2*d*(d*e - 3*c*f)*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(15*b*f) + (d^2*x
*Sqrt[c + d*x^2]*(e + f*x^2)^(3/2))/(5*b*f) - ((b*c - a*d)*Sqrt[e]*(b*d*e +
 4*b*c*f - 3*a*d*f)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]],
1 - (d*e)/(c*f)])/(3*b^3*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt
[e + f*x^2]) + (Sqrt[e]*(2*d^2*e^2 - 7*c*d*e*f - 3*c^2*f^2)*Sqrt[c + d*x^2]
*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(15*b*f^(3/2)*Sqr
t[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (d*(5*b*c - 3*a*d)*(b
*c - a*d)*e^(3/2)*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1
- (d*e)/(c*f)])/(3*b^3*c*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt
[e + f*x^2]) - (d*e^(3/2)*(d*e - 9*c*f)*Sqrt[c + d*x^2]*EllipticF[ArcTan[(S
qrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(15*b*f^(3/2)*Sqrt[(e*(c + d*x^2))/(c
*(e + f*x^2))]*Sqrt[e + f*x^2]) + ((b*c - a*d)^3*e^(3/2)*Sqrt[c + d*x^2]*El
lipticPi[1 - (b*e)/(a*f), ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(a
*b^3*c*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rule 545

```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Dist[d/b, Int[(c + d*x^2)^(q - 1)*(e + f*x^2)^r, x], x
] + Dist[(b*c - a*d)/b, Int[((c + d*x^2)^(q - 1)*(e + f*x^2)^r)/(a + b*x^2)
, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && GtQ[q, 1]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 543

```
Int[(((c_) + (d_.)*(x_)^2)^(3/2)*Sqrt[(e_) + (f_.)*(x_)^2])/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Dist[(b*c - a*d)^2/b^2, Int[Sqrt[e + f*x^2]/((a + b*x^
2)*Sqrt[c + d*x^2]), x], x] + Dist[d/b^2, Int[((2*b*c - a*d + b*d*x^2)*Sqrt
[e + f*x^2])/Sqrt[c + d*x^2], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && Pos
```

```
Q[d/c] && PosQ[f/e]
```

Rule 539

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[(c*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcT
an[Rt[d/c, 2]*x], 1 - (c*f)/(d*e)])/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c
*(e + f*x^2))/(e*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rubi steps

$$\int \frac{\left(c+dx^2\right)^{5/2}\sqrt{e+fx^2}}{a+bx^2}\,dx = \frac{d\int \left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}\,dx}{b} + \frac{(bc-ad)\int \frac{\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}}{a+bx^2}\,dx}{b}$$

$$= \frac{d^2x\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}}{5bf} + \frac{(d(bc-ad))\int \frac{\left(2bc-ad+bdx^2\right)\sqrt{e+fx^2}}{\sqrt{c+dx^2}}\,dx}{b^3} + \frac{(bc-ad)^3\int \frac{\sqrt{e+fx^2}}{\left(a+bx^2\right)\sqrt{c+d}}}{b^3}$$

$$= \frac{d(bc-ad)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{3b^2} - \frac{2d(de-3cf)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{15bf} + \frac{d^2x\sqrt{c+dx^2}\left(e+fx^2\right.}{5bf}$$

$$= \frac{d(bc-ad)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{3b^2} - \frac{2d(de-3cf)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{15bf} + \frac{d^2x\sqrt{c+dx^2}\left(e+fx^2\right.}{5bf}$$

$$= \frac{(bc-ad)(bde+4bcf-3adf)x\sqrt{c+dx^2}}{3b^3\sqrt{e+fx^2}} - \frac{\left(2d^2e^2-7cdef-3c^2f^2\right)x\sqrt{c+dx^2}}{15bf\sqrt{e+fx^2}} + \frac{d(bc-a}{}$$

$$= \frac{(bc-ad)(bde+4bcf-3adf)x\sqrt{c+dx^2}}{3b^3\sqrt{e+fx^2}} - \frac{\left(2d^2e^2-7cdef-3c^2f^2\right)x\sqrt{c+dx^2}}{15bf\sqrt{e+fx^2}} + \frac{d(bc-a}{}$$

Mathematica [C] time = 2.63806, size = 456, normalized size = 0.75

$$-ia\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\left(45a^2bcd^2f^3-15a^3d^3f^3+5ab^2df\left(-9c^2f^2-cdef+d^2e^2\right)+b^3\left(11c^2def^2+15c^3f^3-13cd^2e^2f+\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x^2)^(5/2)*Sqrt[e + f*x^2])/(a + b*x^2),x]
```

```
[Out] ((-I)*a*b*d*e*(15*a^2*d^2*f^2 - 5*a*b*d*f*(d*e + 7*c*f) + b^2*(-2*d^2*e^2 +
 12*c*d*e*f + 23*c^2*f^2))*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*Elliptic
E[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - I*a*(45*a^2*b*c*d^2*f^3 - 15*a^3*d
^3*f^3 + 5*a*b^2*d*f*(d^2*e^2 - c*d*e*f - 9*c^2*f^2) + b^3*(2*d^3*e^3 - 13*
c*d^2*e^2*f + 11*c^2*d*e*f^2 + 15*c^3*f^3))*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f
*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + f*(a*b^2*d*Sqrt[d
/c]*x*(c + d*x^2)*(e + f*x^2)*(11*b*c*f - 5*a*d*f + b*d*(e + 3*f*x^2)) - (1
```

```
5*I)*(b*c - a*d)^3*f*(b*e - a*f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*El
lipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]))/(15*a*b^4*Sqrt
[d/c]*f^2*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Maple [B] time = 0.036, size = 1891, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(5/2)*(f*x^2+e)^(1/2)/(b*x^2+a),x)
```

```
[Out] -1/15*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)*(5*(-d/c)^(1/2)*x^5*a^2*b^2*d^3*f^3-2
3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e
)^(1/2))*a*b^3*c^2*d*e*f^2+13*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Ellip
ticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b^3*c*d^2*e^2*f+35*((d*x^2+c)/c)^(1/
2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*b^2*c*
d^2*e*f^2-12*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/
2),(c*f/d/e)^(1/2))*a*b^3*c*d^2*e^2*f-45*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^
(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a^2*b^2*
c*d^2*e*f^2+45*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^
(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a*b^3*c^2*d*e*f^2+5*((d*x^2+c)/c)^
(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*b^2
*c*d^2*e*f^2-11*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^
(1/2),(c*f/d/e)^(1/2))*a*b^3*c^2*d*e*f^2-14*(-d/c)^(1/2)*x^5*a*b^3*c*d^2*f^
3-15*(-d/c)^(1/2)*x^3*a*b^3*c*d^2*e*f^2+2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)
^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b^3*d^3*e^3-(-d/c)^(1/2)
*x^3*a*b^3*d^3*e^2*f-15*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x
*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b^3*c^3*f^3-2*((d*x^2+c)/c)^(1/2)*((f*x^2+
e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b^3*d^3*e^3-4*(-d/c
)^(1/2)*x^5*a*b^3*d^3*e*f^2+5*(-d/c)^(1/2)*x^3*a^2*b^2*c*d^2*f^3+5*(-d/c)^(
1/2)*x^3*a^2*b^2*d^3*e*f^2-11*(-d/c)^(1/2)*x^3*a*b^3*c^2*d*f^3-15*((d*x^2+c
)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/
2)/(-d/c)^(1/2))*a^4*d^3*f^3-3*(-d/c)^(1/2)*x^7*a*b^3*d^3*f^3+15*((d*x^2+c)
/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^4
*d^3*f^3+5*(-d/c)^(1/2)*x*a^2*b^2*c*d^2*e*f^2-11*(-d/c)^(1/2)*x*a*b^3*c^2*d
*e*f^2-(-d/c)^(1/2)*x*a*b^3*c*d^2*e^2*f+15*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e
)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a^3*b*
d^3*e*f^2-45*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1
/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a^2*b^2*c^2*d*f^3+5*((d*x^2+c)/c)^(1
/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*b^2*d
^3*e^2*f+45*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/
2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a^3*b*c*d^2*f^3-15*((d*x^2+c)/c)^(1/2
)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^3*b*d^3*e
*f^2-5*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*
f/d/e)^(1/2))*a^2*b^2*d^3*e^2*f+45*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*
EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*b^2*c^2*d*f^3-45*((d*x^2+c)/c
)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^3*b
*c*d^2*f^3+15*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(
1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a*b^3*c^3*f^3-15*((d*x^2+c)/c)^(1/2
)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)
^(1/2))*b^4*c^3*e*f^2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)/b^4/f^2/(-d/c)^(1/2)/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)^{\frac{5}{2}}\sqrt{fx^2+e}}{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)*(f*x^2+e)^(1/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(5/2)*sqrt(f*x^2 + e)/(b*x^2 + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)*(f*x^2+e)^(1/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(5/2)*(f*x**2+e)**(1/2)/(b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)^{\frac{5}{2}}\sqrt{fx^2+e}}{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)*(f*x^2+e)^(1/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)^(5/2)*sqrt(f*x^2 + e)/(b*x^2 + a), x)
```

3.65 $\int \frac{(c+dx^2)^{3/2}\sqrt{e+fx^2}}{a+bx^2}\,dx$

Optimal. Leaf size=400

$$\frac{de^{3/2}\sqrt{c+dx^2}(5bc-3ad)\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{3b^2c\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{e^{3/2}\sqrt{c+dx^2}(bc-ad)^2\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{ab^2c\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+$$

```
[Out] ((b*d*e + 4*b*c*f - 3*a*d*f)*x*Sqrt[c + d*x^2])/(3*b^2*Sqrt[e + f*x^2]) + (
d*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(3*b) - (Sqrt[e]*(b*d*e + 4*b*c*f - 3*
a*d*f)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*
f)])/(3*b^2*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
+ (d*(5*b*c - 3*a*d)*e^(3/2)*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/S
qrt[e]], 1 - (d*e)/(c*f)])/(3*b^2*c*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*
x^2))]*Sqrt[e + f*x^2]) + ((b*c - a*d)^2*e^(3/2)*Sqrt[c + d*x^2]*EllipticPi
[1 - (b*e)/(a*f), ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(a*b^2*c*S
qrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rubi [A] time = 0.289189, antiderivative size = 400, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {543, 539, 528, 531, 418, 492, 411}

$$\frac{de^{3/2}\sqrt{c+dx^2}(5bc-3ad)F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3b^2c\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{e^{3/2}\sqrt{c+dx^2}(bc-ad)^2\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{ab^2c\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{x\sqrt{c+dx}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*x^2)^(3/2)*Sqrt[e + f*x^2])/(a + b*x^2),x]
```


```
[Out] ((b*d*e + 4*b*c*f - 3*a*d*f)*x*Sqrt[c + d*x^2])/(3*b^2*Sqrt[e + f*x^2]) + (
d*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(3*b) - (Sqrt[e]*(b*d*e + 4*b*c*f - 3*
a*d*f)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*
f)])/(3*b^2*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
+ (d*(5*b*c - 3*a*d)*e^(3/2)*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/S
qrt[e]], 1 - (d*e)/(c*f)])/(3*b^2*c*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*
x^2))]*Sqrt[e + f*x^2]) + ((b*c - a*d)^2*e^(3/2)*Sqrt[c + d*x^2]*EllipticPi
[1 - (b*e)/(a*f), ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(a*b^2*c*S
qrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```


Rule 543

```
Int[(((c_) + (d_.)*(x_)^2)^(3/2)*Sqrt[(e_) + (f_.)*(x_)^2])/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Dist[(b*c - a*d)^2/b^2, Int[Sqrt[e + f*x^2]/((a + b*x^
2)*Sqrt[c + d*x^2]), x], x] + Dist[d/b^2, Int[((2*b*c - a*d + b*d*x^2)*Sqrt
[e + f*x^2])/Sqrt[c + d*x^2], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && Pos
Q[d/c] && PosQ[f/e]
```

Rule 539

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[(c*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcT
an[Rt[d/c, 2]*x], 1 - (c*f)/(d*e)])/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c
```

```
*(e + f*x^2))/(e*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}}{a+bx^2}\,dx = \frac{d\int \frac{\left(2bc-ad+bdx^2\right)\sqrt{e+fx^2}}{\sqrt{c+dx^2}}\,dx}{b^2} + \frac{(bc-ad)^2\int \frac{\sqrt{e+fx^2}}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{b^2}$$

$$= \frac{dx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3b} + \frac{(bc-ad)^2e^{3/2}\sqrt{c+dx^2}\,\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{ab^2c\sqrt{f}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}\sqrt{e+fx^2}} + \int \frac{d(5b}{}$$

$$= \frac{dx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3b} + \frac{(bc-ad)^2e^{3/2}\sqrt{c+dx^2}\,\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{ab^2c\sqrt{f}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}\sqrt{e+fx^2}} + \frac{(d(5b}{}$$

$$= \frac{(bde+4bcf-3adf)x\sqrt{c+dx^2}}{3b^2\sqrt{e+fx^2}} + \frac{dx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3b} + \frac{d(5bc-3ad)e^{3/2}\sqrt{c+dx^2}F\left(\right.}{3b^2c\sqrt{f}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}$$

$$= \frac{(bde+4bcf-3adf)x\sqrt{c+dx^2}}{3b^2\sqrt{e+fx^2}} + \frac{dx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3b} - \frac{\sqrt{e}(bde+4bcf-3adf)\sqrt{c+}}{3b^2\sqrt{f}\sqrt{\frac{e(c+}{c(e+}}}$$

Mathematica [C] time = 1.49034, size = 346, normalized size = 0.86

$$-ia\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\left(3a^2d^2f^2-6abcdf^2+b^2\left(3c^2f^2+cdef-d^2e^2\right)\right)\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{\frac{d}{c}}\right),\frac{cf}{de}\right)+f\left(ab^2dx\sqrt{\frac{d}{c}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x^2)^(3/2)*Sqrt[e + f*x^2])/(a + b*x^2),x]
```

```
[Out] ((-I)*a*b*d*e*(b*d*e + 4*b*c*f - 3*a*d*f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x
^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - I*a*(-6*a*b*c*d*f^2
 + 3*a^2*d^2*f^2 + b^2*(-(d^2*e^2) + c*d*e*f + 3*c^2*f^2))*Sqrt[1 + (d*x^2)
/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + f*
(a*b^2*d*Sqrt[d/c]*x*(c + d*x^2)*(e + f*x^2) - (3*I)*(b*c - a*d)^2*(b*e - a
*f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSi
nh[Sqrt[d/c]*x], (c*f)/(d*e)]))/(3*a*b^3*Sqrt[d/c]*f*Sqrt[c + d*x^2]*Sqrt[e
 + f*x^2])
```

Maple [B] time = 0.015, size = 1059, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(3/2)*(f*x^2+e)^(1/2)/(b*x^2+a),x)
```

```
[Out] 1/3*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)*((-d/c)^(1/2)*x^5*a*b^2*d^2*f^2+(-d/c)^
(1/2)*x^3*a*b^2*c*d*f^2+(-d/c)^(1/2)*x^3*a*b^2*d^2*e*f+3*EllipticF(x*(-d/c)
^(1/2),(c*f/d/e)^(1/2))*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*a^3*d^2*f^2
```

```
-6*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*((d*x^2+c)/c)^(1/2)*((f*x^2+e)
/e)^(1/2)*a^2*b*c*d*f^2+3*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*((d*x^2
+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*a*b^2*c^2*f^2+EllipticF(x*(-d/c)^(1/2),(c*
f/d/e)^(1/2))*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*a*b^2*c*d*e*f-Ellipti
cF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*
a*b^2*d^2*e^2-3*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*((d*x^2+c)/c)^(1/
2)*((f*x^2+e)/e)^(1/2)*a^2*b*d^2*e*f+4*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(
1/2))*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*a*b^2*c*d*e*f+EllipticE(x*(-d
/c)^(1/2),(c*f/d/e)^(1/2))*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*a*b^2*d^
2*e^2-3*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*((d*x^
2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*a^3*d^2*f^2+6*EllipticPi(x*(-d/c)^(1/2),b
*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*a
^2*b*c*d*f^2+3*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))
*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*a^2*b*d^2*e*f-3*EllipticPi(x*(-d/c
)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e
)^(1/2)*a*b^2*c^2*f^2-6*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/
c)^(1/2))*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*a*b^2*c*d*e*f+3*EllipticP
i(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*((d*x^2+c)/c)^(1/2)*((f
*x^2+e)/e)^(1/2)*b^3*c^2*e*f+(-d/c)^(1/2)*x*a*b^2*c*d*e*f)/(d*f*x^4+c*f*x^2
+d*e*x^2+c*e)/b^3/(-d/c)^(1/2)/f/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)^{\frac{3}{2}}\sqrt{fx^2+e}}{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)*(f*x^2+e)^(1/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(3/2)*sqrt(f*x^2 + e)/(b*x^2 + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)*(f*x^2+e)^(1/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c+dx^2\right)^{\frac{3}{2}}\sqrt{e+fx^2}}{a+bx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(3/2)*(f*x**2+e)**(1/2)/(b*x**2+a),x)
```

```
[Out] Integral((c + d*x**2)**(3/2)*sqrt(e + f*x**2)/(a + b*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)^{\frac{3}{2}}\sqrt{fx^2+e}}{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)*(f*x^2+e)^(1/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)^(3/2)*sqrt(f*x^2 + e)/(b*x^2 + a), x)
```

3.66 $\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{a+bx^2}\,dx$

Optimal. Leaf size=321

$$\frac{de^{3/2}\sqrt{c+dx^2}\,\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{bc\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{e^{3/2}\sqrt{c+dx^2}(bc-ad)\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{abc\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{fx\sqrt{c+dx^2}}{b\sqrt{e+fx^2}}-\cdots$$

```
[Out] (f*x*Sqrt[c + d*x^2])/(b*Sqrt[e + f*x^2]) - (Sqrt[e]*Sqrt[f]*Sqrt[c + d*x^2
]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(b*Sqrt[(e*(c +
d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (d*e^(3/2)*Sqrt[c + d*x^2]*Elli
pticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(b*c*Sqrt[f]*Sqrt[(e*(
c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + ((b*c - a*d)*e^(3/2)*Sqrt[c
 + d*x^2]*EllipticPi[1 - (b*e)/(a*f), ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e
)/(c*f)])/(a*b*c*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x
^2])
```

Rubi [A] time = 0.19095, antiderivative size = 321, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {535, 422, 418, 492, 411, 539}

$$\frac{e^{3/2}\sqrt{c+dx^2}(bc-ad)\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{abc\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{de^{3/2}\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{bc\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{fx\sqrt{c+dx^2}}{b\sqrt{e+fx^2}}-\frac{\sqrt{e}\sqrt{f}\sqrt{c}\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(a + b*x^2),x]
```


```
[Out] (f*x*Sqrt[c + d*x^2])/(b*Sqrt[e + f*x^2]) - (Sqrt[e]*Sqrt[f]*Sqrt[c + d*x^2
]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(b*Sqrt[(e*(c +
d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (d*e^(3/2)*Sqrt[c + d*x^2]*Elli
pticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(b*c*Sqrt[f]*Sqrt[(e*(
c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + ((b*c - a*d)*e^(3/2)*Sqrt[c
 + d*x^2]*EllipticPi[1 - (b*e)/(a*f), ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e
)/(c*f)])/(a*b*c*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x
^2])
```


Rule 535

```
Int[(Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)^2])/((a_) + (b_.)*(x_
)^2), x_Symbol] :> Dist[d/b, Int[Sqrt[e + f*x^2]/Sqrt[c + d*x^2], x], x] +
Dist[(b*c - a*d)/b, Int[Sqrt[e + f*x^2]/((a + b*x^2)*Sqrt[c + d*x^2]), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !SimplerSqrtQ[-(f/e), -(d/c)]
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 539

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[(c*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcT
an[Rt[d/c, 2]*x], 1 - (c*f)/(d*e)])/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c
*(e + f*x^2))/(e*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{a+bx^2}\,dx = \frac{d\int \frac{\sqrt{e+fx^2}}{\sqrt{c+dx^2}}\,dx}{b} + \frac{(bc-ad)\int \frac{\sqrt{e+fx^2}}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{b}$$

$$= \frac{(bc-ad)e^{3/2}\sqrt{c+dx^2}\,\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{abc\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} + \frac{(de)\int \frac{1}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{b} + \frac{(df)\int \frac{}{\sqrt{}}$$

$$= \frac{fx\sqrt{c+dx^2}}{b\sqrt{e+fx^2}} + \frac{de^{3/2}\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{bc\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} + \frac{(bc-ad)e^{3/2}\sqrt{c+dx^2}\,\Pi\left(1-\frac{be}{af};\tan\right.}{abc\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e-}}$$

$$= \frac{fx\sqrt{c+dx^2}}{b\sqrt{e+fx^2}} - \frac{\sqrt{e}\sqrt{f}\sqrt{c+dx^2}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{b\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} + \frac{de^{3/2}\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\right.}{bc\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

Mathematica [C] time = 0.366024, size = 184, normalized size = 0.57

$$-\frac{i\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\left((bc-ad)\left(af\,\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{\frac{d}{c}}\right),\frac{cf}{de}\right)+(be-af)\Pi\left(\frac{bc}{ad};i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)\right)+abdeE\left(i\right.\right.}{ab^2\sqrt{\frac{d}{c}}\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(a + b*x^2),x]
```

```
[Out] ((-I)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*(a*b*d*e*EllipticE[I*ArcSinh[
Sqrt[d/c]*x], (c*f)/(d*e)] + (b*c - a*d)*(a*f*EllipticF[I*ArcSinh[Sqrt[d/c]
*x], (c*f)/(d*e)] + (b*e - a*f)*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]
*x], (c*f)/(d*e)])))/(a*b^2*Sqrt[d/c]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Maple [A] time = 0.013, size = 340, normalized size = 1.1

$$\frac{1}{\left(dfx^4+cfx^2+dex^2+ce\right)b^2a}\left(-\mathrm{EllipticF}\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)a^2df+\mathrm{EllipticF}\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)abcf+\mathrm{EllipticE}\left(x\sqrt{-\frac{d}{c}},\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a),x)
```

```
[Out] (-EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*d*f+EllipticF(x*(-d/c)^(1/2
),(c*f/d/e)^(1/2))*a*b*c*f+EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b*d*
e+EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a^2*d*f-Elli
pticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a*b*c*f-EllipticPi
(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a*b*d*e+EllipticPi(x*(-d
/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*b^2*c*e)*(d*x^2+c)^(1/2)*(f*x^
2+e)^(1/2)*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2
+c*e)/b^2/(-d/c)^(1/2)/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2+c}\sqrt{fx^2+e}}{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b*x^2 + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{a+bx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)*(f*x**2+e)**(1/2)/(b*x**2+a),x)
```

```
[Out] Integral(sqrt(c + d*x**2)*sqrt(e + f*x**2)/(a + b*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2+c}\sqrt{fx^2+e}}{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b*x^2 + a), x)
```

3.67 $\int \frac{\sqrt{e+fx^2}}{(a+bx^2)\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=102

$$\frac{e^{3/2}\sqrt{c+dx^2}\,\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{ac\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

```
[Out] (e^(3/2)*Sqrt[c + d*x^2]*EllipticPi[1 - (b*e)/(a*f), ArcTan[(Sqrt[f]*x)/Sqr
t[e]], 1 - (d*e)/(c*f)])/(a*c*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]
*Sqrt[e + f*x^2])
```

Rubi [A] time = 0.033963, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.031, Rules used = {539}

$$\frac{e^{3/2}\sqrt{c+dx^2}\,\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{ac\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e + f*x^2]/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```


```
[Out] (e^(3/2)*Sqrt[c + d*x^2]*EllipticPi[1 - (b*e)/(a*f), ArcTan[(Sqrt[f]*x)/Sqr
t[e]], 1 - (d*e)/(c*f)])/(a*c*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]
*Sqrt[e + f*x^2])
```


Rule 539

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[(c*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcT
an[Rt[d/c, 2]*x], 1 - (c*f)/(d*e)])/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c
*(e + f*x^2))/(e*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rubi steps

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)\sqrt{c+dx^2}}\,dx = \frac{e^{3/2}\sqrt{c+dx^2}\,\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{ac\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

Mathematica [C] time = 0.230891, size = 143, normalized size = 1.4

$$-\frac{i\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\left(af\,\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{\frac{d}{c}}\right),\frac{cf}{de}\right)+(be-af)\Pi\left(\frac{bc}{ad};i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)\right)}{ab\sqrt{\frac{d}{c}}\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e + f*x^2]/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] ((-I)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*(a*f*EllipticF[I*ArcSinh[Sqrt
[d/c]*x], (c*f)/(d*e)] + (b*e - a*f)*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt
[d/c]*x], (c*f)/(d*e)]))/(a*b*Sqrt[d/c]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Maple [A] time = 0.017, size = 191, normalized size = 1.9

$$\frac{1}{ab\left(dfx^4+cfx^2+dex^2+ce\right)}\left(\mathrm{EllipticF}\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)af-\mathrm{EllipticPi}\left(x\sqrt{-\frac{d}{c}},\frac{bc}{ad},\sqrt{-\frac{f}{e}}\frac{1}{\sqrt{-\frac{d}{c}}}\right)af+\mathrm{EllipticPi}\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e)^(1/2)/(b*x^2+a)/(d*x^2+c)^(1/2),x)
```

```
[Out] (EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*f-EllipticPi(x*(-d/c)^(1/2),b*
c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a*f+EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/
e)^(1/2)/(-d/c)^(1/2))*b*e)/b*((f*x^2+e)/e)^(1/2)*((d*x^2+c)/c)^(1/2)*(d*x^
2+c)^(1/2)*(f*x^2+e)^(1/2)/a/(-d/c)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{fx^2+e}}{\left(bx^2+a\right)\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e)^(1/2)/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(f*x^2 + e)/((b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e)^(1/2)/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{e+fx^2}}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e)**(1/2)/(b*x**2+a)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(e + f*x**2)/((a + b*x**2)*sqrt(c + d*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{fx^2+e}}{(bx^2+a)\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e)^(1/2)/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(f*x^2 + e)/((b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

3.68 $\int \frac{\sqrt{e+fx^2}}{\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=209

$$\frac{be^{3/2}\sqrt{c+dx^2}\,\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{ac\sqrt{f}\sqrt{e+fx^2}(bc-ad)\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}-\frac{\sqrt{d}\sqrt{e+fx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(e+fx^2\right)}{e\left(c+dx^2\right)}}}$$

```
[Out] -((Sqrt[d]*Sqrt[e + f*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)
/(d*e)])/(Sqrt[c]*(b*c - a*d)*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c +
d*x^2))])) + (b*e^(3/2)*Sqrt[c + d*x^2]*EllipticPi[1 - (b*e)/(a*f), ArcTan[
(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(a*c*(b*c - a*d)*Sqrt[f]*Sqrt[(e*(c
 + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rubi [A] time = 0.105186, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {541, 539, 411}

$$\frac{be^{3/2}\sqrt{c+dx^2}\,\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{ac\sqrt{f}\sqrt{e+fx^2}(bc-ad)\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}-\frac{\sqrt{d}\sqrt{e+fx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(e+fx^2\right)}{e\left(c+dx^2\right)}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e + f*x^2]/((a + b*x^2)*(c + d*x^2)^(3/2)),x]
```


```
[Out] -((Sqrt[d]*Sqrt[e + f*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)
/(d*e)])/(Sqrt[c]*(b*c - a*d)*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c +
d*x^2))])) + (b*e^(3/2)*Sqrt[c + d*x^2]*EllipticPi[1 - (b*e)/(a*f), ArcTan[
(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(a*c*(b*c - a*d)*Sqrt[f]*Sqrt[(e*(c
 + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```


Rule 541

```
Int[Sqrt[(e_) + (f_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[b/(b*c - a*d), Int[Sqrt[e + f*x^2]/((a + b*x^2)*S
qrt[c + d*x^2]), x], x] - Dist[d/(b*c - a*d), Int[Sqrt[e + f*x^2]/(c + d*x^
2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[d/c] && PosQ[f/e]
```

Rule 539

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[(c*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcT
an[Rt[d/c, 2]*x], 1 - (c*f)/(d*e)])/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c
*(e + f*x^2))/(e*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
```

```
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\sqrt{e+fx^2}}{\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}\,dx = \frac{b\int \frac{\sqrt{e+fx^2}}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{bc-ad} - \frac{d\int \frac{\sqrt{e+fx^2}}{\left(c+dx^2\right)^{3/2}}\,dx}{bc-ad}$$

$$= -\frac{\sqrt{d}\sqrt{e+fx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}(bc-ad)\sqrt{c+dx^2}\sqrt{\frac{c\left(e+fx^2\right)}{e\left(c+dx^2\right)}}} + \frac{be^{3/2}\sqrt{c+dx^2}\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{ac(bc-ad)\sqrt{f}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}\sqrt{e+fx^2}}$$

Mathematica [C] time = 0.772547, size = 347, normalized size = 1.66

$$\frac{\sqrt{\frac{d}{c}}\left(ia\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}(cf-de)\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{\frac{d}{c}}\right),\frac{cf}{de}\right)+ibce\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\Pi\left(\frac{bc}{ad};i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)-i}{ad\sqrt{c+dx^2}\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e + f*x^2]/((a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[d/c]*(a*d*Sqrt[d/c]*e*x + a*d*Sqrt[d/c]*f*x^3 + I*a*d*e*Sqrt[1 + (d*x
^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] +
 I*a*(-(d*e) + c*f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*Arc
Sinh[Sqrt[d/c]*x], (c*f)/(d*e)] + I*b*c*e*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x
^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - I*a*c
*f*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSin
h[Sqrt[d/c]*x], (c*f)/(d*e)]))/(a*d*(-(b*c) + a*d)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2])
```

Maple [A] time = 0.024, size = 390, normalized size = 1.9

$$\frac{1}{ac\left(ad-bc\right)\left(dfx^4+cfx^2+dex^2+ce\right)}\left(x^3adf\sqrt{-\frac{d}{c}}-\text{EllipticF}\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)acf\sqrt{\frac{fx^2+e}{e}}\sqrt{\frac{dx^2+c}{c}}+\text{EllipticF}\left(x\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e)^(1/2)/(b*x^2+a)/(d*x^2+c)^(3/2),x)
```

```
[Out] (x^3*a*d*f*(-d/c)^(1/2)-EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c*f*((f
*x^2+e)/e)^(1/2)*((d*x^2+c)/c)^(1/2)+EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/
2))*a*d*e*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-EllipticE(x*(-d/c)^(1/2),
(c*f/d/e)^(1/2))*a*d*e*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+EllipticPi(x
*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a*c*f*((d*x^2+c)/c)^(1/2)*
((f*x^2+e)/e)^(1/2)-EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(
1/2))*b*c*e*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+x*a*d*e*(-d/c)^(1/2))*(
d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/a/c/(-d/c)^(1/2)/(a*d-b*c)/(d*f*x^4+c*f*x^2+
d*e*x^2+c*e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{fx^2+e}}{(bx^2+a)(dx^2+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e)^(1/2)/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(f*x^2 + e)/((b*x^2 + a)*(d*x^2 + c)^(3/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e)^(1/2)/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)\,(c+dx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e)**(1/2)/(b*x**2+a)/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral(sqrt(e + f*x**2)/((a + b*x**2)*(c + d*x**2)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{fx^2+e}}{(bx^2+a)(dx^2+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e)^(1/2)/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(f*x^2 + e)/((b*x^2 + a)*(d*x^2 + c)^(3/2)), x)
```

3.69 $\int \frac{\sqrt{e+fx^2}}{(a+bx^2)(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=401

$$\frac{de^{3/2}\sqrt{f}\sqrt{c+dx^2}\,\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{3c^2\sqrt{e+fx^2}(bc-ad)(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{b^2e^{3/2}\sqrt{c+dx^2}\,\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{ac\sqrt{f}\sqrt{e+fx^2}(bc-ad)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{\sqrt{d}\sqrt{e+fx^2}(bc(5de}{3c^{3/2}\sqrt{c}}$$

```
[Out] -(d*x*Sqrt[e + f*x^2])/(3*c*(b*c - a*d)*(c + d*x^2)^(3/2)) - (Sqrt[d]*(b*c*
(5*d*e - 4*c*f) - a*d*(2*d*e - c*f))*Sqrt[e + f*x^2]*EllipticE[ArcTan[(Sqrt
[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(3*c^(3/2)*(b*c - a*d)^2*(d*e - c*f)*Sqr
t[c + d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))]) + (d*e^(3/2)*Sqrt[f]*Sq
rt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(3*c
^2*(b*c - a*d)*(d*e - c*f)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f
*x^2]) + (b^2*e^(3/2)*Sqrt[c + d*x^2]*EllipticPi[1 - (b*e)/(a*f), ArcTan[(S
qrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(a*c*(b*c - a*d)^2*Sqrt[f]*Sqrt[(e*(c
 + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rubi [A] time = 0.366965, antiderivative size = 401, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {546, 539, 526, 525, 418, 411}

$$\frac{b^2e^{3/2}\sqrt{c+dx^2}\,\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{ac\sqrt{f}\sqrt{e+fx^2}(bc-ad)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{de^{3/2}\sqrt{f}\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3c^2\sqrt{e+fx^2}(bc-ad)(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{\sqrt{d}\sqrt{e+fx^2}(bc(5de-4cf)}{3c^{3/2}\sqrt{c+dx^2}(b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e + f*x^2]/((a + b*x^2)*(c + d*x^2)^(5/2)),x]
```



```
[Out] -(d*x*Sqrt[e + f*x^2])/(3*c*(b*c - a*d)*(c + d*x^2)^(3/2)) - (Sqrt[d]*(b*c*
(5*d*e - 4*c*f) - a*d*(2*d*e - c*f))*Sqrt[e + f*x^2]*EllipticE[ArcTan[(Sqrt
[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(3*c^(3/2)*(b*c - a*d)^2*(d*e - c*f)*Sqr
t[c + d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))]) + (d*e^(3/2)*Sqrt[f]*Sq
rt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(3*c
^2*(b*c - a*d)*(d*e - c*f)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f
*x^2]) + (b^2*e^(3/2)*Sqrt[c + d*x^2]*EllipticPi[1 - (b*e)/(a*f), ArcTan[(S
qrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(a*c*(b*c - a*d)^2*Sqrt[f]*Sqrt[(e*(c
 + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```


Rule 546

```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Dist[b^2/(b*c - a*d)^2, Int[((c + d*x^2)^(q + 2)*(e +
f*x^2)^r)/(a + b*x^2), x], x] - Dist[d/(b*c - a*d)^2, Int[(c + d*x^2)^q*(e
+ f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r},
x] && LtQ[q, -1]
```

Rule 539

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[(c*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcT
```

```
an[Rt[d/c, 2]*x], 1 - (c*f)/(d*e)])/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c
*(e + f*x^2))/(e*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 526

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^q)/(a*b*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p +
 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 525

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\sqrt{e+fx^2}}{\left(a+bx^2\right)\left(c+dx^2\right)^{5/2}}\,dx = \frac{b^2\int \frac{\sqrt{e+fx^2}}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{(bc-ad)^2} - \frac{d\int \frac{\left(2bc-ad+bdx^2\right)\sqrt{e+fx^2}}{\left(c+dx^2\right)^{5/2}}\,dx}{(bc-ad)^2}$$

$$= -\frac{dx\sqrt{e+fx^2}}{3c(bc-ad)\left(c+dx^2\right)^{3/2}} + \frac{b^2e^{3/2}\sqrt{c+dx^2}\,\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{ac(bc-ad)^2\sqrt{f}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}\sqrt{e+fx^2}} + \frac{\int \frac{-d(5bc-2\ldots}{\left(c+\ldots\right.}}{3c\ldots}$$

$$= -\frac{dx\sqrt{e+fx^2}}{3c(bc-ad)\left(c+dx^2\right)^{3/2}} + \frac{b^2e^{3/2}\sqrt{c+dx^2}\,\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{ac(bc-ad)^2\sqrt{f}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}\sqrt{e+fx^2}} + \frac{(def)\int \ldots}{3c(bc-\ldots}$$

$$= -\frac{dx\sqrt{e+fx^2}}{3c(bc-ad)\left(c+dx^2\right)^{3/2}} - \frac{\sqrt{d}(bc(5de-4cf)-ad(2de-cf))\sqrt{e+fx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\ldots\right.}{3c^{3/2}(bc-ad)^2(de-cf)\sqrt{c+dx^2}\sqrt{\frac{c\left(e+fx^2\right)}{e\left(c+dx^2\right)}}}$$

Mathematica [C] time = 3.43655, size = 427, normalized size = 1.06

$$-ia\left(c+dx^2\right)\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}(cf-de)\left(2ad^2e+bc(3cf-5de)\right)\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{\frac{d}{c}}\right),\frac{cf}{de}\right)+acx\left(\frac{d}{c}\right)^{3/2}\left(e+fx^2\right)($$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e + f*x^2]/((a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] (a*c*(d/c)^(3/2)*x*(e + f*x^2)*(b*c*(6*c*d*e - 5*c^2*f + 5*d^2*e*x^2 - 4*c*
d*f*x^2) + a*d*(-3*c*d*e + 2*c^2*f - 2*d^2*e*x^2 + c*d*f*x^2)) - I*a*d*e*(a
*d*(2*d*e - c*f) + b*c*(-5*d*e + 4*c*f))*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]*Sq
rt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - I*a*(-(d
*e) + c*f)*(2*a*d^2*e + b*c*(-5*d*e + 3*c*f))*(c + d*x^2)*Sqrt[1 + (d*x^2)/
c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - (3*
I)*b*c^2*(b*e - a*f)*(-(d*e) + c*f)*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[1
+ (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/
(3*a*c^2*Sqrt[d/c]*(b*c - a*d)^2*(-(d*e) + c*f)*(c + d*x^2)^(3/2)*Sqrt[e +
f*x^2])
```

Maple [B] time = 0.036, size = 2068, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e)^(1/2)/(b*x^2+a)/(d*x^2+c)^(5/2),x)
```

```
[Out] 1/3*(x^5*a^2*c*d^3*f^2*(-d/c)^(1/2)+2*x^3*a*b*c^2*d^2*e*f*(-d/c)^(1/2)-5*x*
a*b*c^3*d*e*f*(-d/c)^(1/2)-3*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)
/(-d/c)^(1/2))*x^2*b^2*c^2*d^2*e^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+
2*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*c^2*d^2*e*f*((d*x^2+c)/c)^(
1/2)*((f*x^2+e)/e)^(1/2)+5*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b*c^
2*d^2*e^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-EllipticE(x*(-d/c)^(1/2),
(c*f/d/e)^(1/2))*a^2*c^2*d^2*e*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-5*
EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b*c^2*d^2*e^2*((d*x^2+c)/c)^(1/
2)*((f*x^2+e)/e)^(1/2)-2*x^3*a^2*d^4*e^2*(-d/c)^(1/2)-5*EllipticE(x*(-d/c)^
(1/2),(c*f/d/e)^(1/2))*x^2*a*b*c*d^3*e^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^
(1/2)+3*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a*b*c^3*d*f^2*((d*x^2
+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+5*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2)
)*x^2*a*b*c*d^3*e^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-3*EllipticPi(x*
(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*x^2*a*b*c^3*d*f^2*((d*x^2+c
)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-2*x^5*a^2*d^4*e*f*(-d/c)^(1/2)+2*x^3*a^2*c^2
*d^2*f^2*(-d/c)^(1/2)-3*x*a^2*c*d^3*e^2*(-d/c)^(1/2)-4*x^5*a*b*c^2*d^2*f^2*
(-d/c)^(1/2)-5*x^3*a*b*c^3*d*f^2*(-d/c)^(1/2)+5*x^3*a*b*c*d^3*e^2*(-d/c)^(1
/2)+2*x*a^2*c^2*d^2*e*f*(-d/c)^(1/2)+6*x*a*b*c^2*d^2*e^2*(-d/c)^(1/2)-2*Ell
ipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a^2*d^4*e^2*((d*x^2+c)/c)^(1/2)*
((f*x^2+e)/e)^(1/2)+2*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a^2*d^4
*e^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-2*EllipticF(x*(-d/c)^(1/2),(c*
f/d/e)^(1/2))*a^2*c*d^3*e^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+3*Ellip
ticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b*c^4*f^2*((d*x^2+c)/c)^(1/2)*((f*x^
2+e)/e)^(1/2)+2*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*c*d^3*e^2*((d
*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-3*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-
f/e)^(1/2)/(-d/c)^(1/2))*a*b*c^4*f^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2
)+3*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*b^2*c^4*e*
```

```
f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-3*EllipticPi(x*(-d/c)^(1/2),b*c/a
/d,(-f/e)^(1/2)/(-d/c)^(1/2))*b^2*c^3*d*e^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/
e)^(1/2)-2*x^3*a^2*c*d^3*e*f*(-d/c)^(1/2)+5*x^5*a*b*c*d^3*e*f*(-d/c)^(1/2)+
3*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*x^2*b^2*c^3*
d*e*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-8*EllipticF(x*(-d/c)^(1/2),(c
*f/d/e)^(1/2))*a*b*c^3*d*e*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+4*Elli
pticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b*c^3*d*e*f*((d*x^2+c)/c)^(1/2)*((f
*x^2+e)/e)^(1/2)+3*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1
/2))*a*b*c^3*d*e*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+2*EllipticF(x*(-
d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a^2*c*d^3*e*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e
)/e)^(1/2)-EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a^2*c*d^3*e*f*((d*
x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+4*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1
/2))*x^2*a*b*c^2*d^2*e*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+3*Elliptic
Pi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*x^2*a*b*c^2*d^2*e*f*((
d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-8*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^
(1/2))*x^2*a*b*c^2*d^2*e*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2))/(f*x^2+
e)^(1/2)/(a*d-b*c)^2/(-d/c)^(1/2)/c^2/(c*f-d*e)/a/(d*x^2+c)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{fx^2+e}}{(bx^2+a)(dx^2+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e)^(1/2)/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(f*x^2 + e)/((b*x^2 + a)*(d*x^2 + c)^(5/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e)^(1/2)/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)(c+dx^2)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e)**(1/2)/(b*x**2+a)/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral(sqrt(e + f*x**2)/((a + b*x**2)*(c + d*x**2)**(5/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{fx^2 + e}}{(bx^2 + a)(dx^2 + c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e)^(1/2)/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(f*x^2 + e)/((b*x^2 + a)*(d*x^2 + c)^(5/2)), x)
```

3.70 $\int \frac{\sqrt{e+fx^2}}{(a+bx^2)(c+dx^2)^{7/2}}\,dx$

Optimal. Leaf size=630

$$\frac{de^{3/2}\sqrt{f}\sqrt{c+dx^2}(bc(9de-11cf)-2ad(2de-3cf))\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{15c^3\sqrt{e+fx^2}(bc-ad)^2(de-cf)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{b^3e^{3/2}\sqrt{c+dx^2}\,\Pi\left(1-\frac{be}{af};\tan^{-1}\right.}{ac\sqrt{f}\sqrt{e+fx^2}(bc-ad)^3}$$

```
[Out] -(d*x*Sqrt[e + f*x^2])/(5*c*(b*c - a*d)*(c + d*x^2)^(5/2)) - (d*(b*c*(9*d*e
 - 8*c*f) - a*d*(4*d*e - 3*c*f))*x*Sqrt[e + f*x^2])/(15*c^2*(b*c - a*d)^2*(
d*e - c*f)*(c + d*x^2)^(3/2)) - (b^2*Sqrt[d]*Sqrt[e + f*x^2]*EllipticE[ArcT
an[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(Sqrt[c]*(b*c - a*d)^3*Sqrt[c +
d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))]) + (Sqrt[d]*(a*d*(8*d^2*e^2 -
13*c*d*e*f + 3*c^2*f^2) - 2*b*c*(9*d^2*e^2 - 14*c*d*e*f + 4*c^2*f^2))*Sqrt[
e + f*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(15*c^(
5/2)*(b*c - a*d)^2*(d*e - c*f)^2*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c
 + d*x^2))]) + (d*e^(3/2)*Sqrt[f]*(b*c*(9*d*e - 11*c*f) - 2*a*d*(2*d*e - 3*
c*f))*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f
)])/(15*c^3*(b*c - a*d)^2*(d*e - c*f)^2*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2)
)]*Sqrt[e + f*x^2]) + (b^3*e^(3/2)*Sqrt[c + d*x^2]*EllipticPi[1 - (b*e)/(a*
f), ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(a*c*(b*c - a*d)^3*Sqrt[
f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rubi [A] time = 0.719061, antiderivative size = 630, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {546, 541, 539, 411, 526, 527, 525, 418}

$$\frac{b^3e^{3/2}\sqrt{c+dx^2}\,\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{ac\sqrt{f}\sqrt{e+fx^2}(bc-ad)^3\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{b^2\sqrt{d}\sqrt{e+fx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\sqrt{c+dx^2}(bc-ad)^3\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}+\frac{\sqrt{d}\sqrt{e+fx^2}\left(ad\left(3c^2f^2-1\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e + f*x^2]/((a + b*x^2)*(c + d*x^2)^(7/2)),x]
```


```
[Out] -(d*x*Sqrt[e + f*x^2])/(5*c*(b*c - a*d)*(c + d*x^2)^(5/2)) - (d*(b*c*(9*d*e
 - 8*c*f) - a*d*(4*d*e - 3*c*f))*x*Sqrt[e + f*x^2])/(15*c^2*(b*c - a*d)^2*(
d*e - c*f)*(c + d*x^2)^(3/2)) - (b^2*Sqrt[d]*Sqrt[e + f*x^2]*EllipticE[ArcT
an[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(Sqrt[c]*(b*c - a*d)^3*Sqrt[c +
d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))]) + (Sqrt[d]*(a*d*(8*d^2*e^2 -
13*c*d*e*f + 3*c^2*f^2) - 2*b*c*(9*d^2*e^2 - 14*c*d*e*f + 4*c^2*f^2))*Sqrt[
e + f*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(15*c^(
5/2)*(b*c - a*d)^2*(d*e - c*f)^2*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c
 + d*x^2))]) + (d*e^(3/2)*Sqrt[f]*(b*c*(9*d*e - 11*c*f) - 2*a*d*(2*d*e - 3*
c*f))*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f
)])/(15*c^3*(b*c - a*d)^2*(d*e - c*f)^2*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2)
)]*Sqrt[e + f*x^2]) + (b^3*e^(3/2)*Sqrt[c + d*x^2]*EllipticPi[1 - (b*e)/(a*
f), ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(a*c*(b*c - a*d)^3*Sqrt[
f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```


Rule 546

```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Dist[b^2/(b*c - a*d)^2, Int[((c + d*x^2)^(q + 2)*(e +
f*x^2)^r)/(a + b*x^2), x], x] - Dist[d/(b*c - a*d)^2, Int[(c + d*x^2)^q*(e
+ f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r},
x] && LtQ[q, -1]
```

Rule 541

```
Int[Sqrt[(e_) + (f_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[b/(b*c - a*d), Int[Sqrt[e + f*x^2]/((a + b*x^2)*S
qrt[c + d*x^2]), x], x] - Dist[d/(b*c - a*d), Int[Sqrt[e + f*x^2]/(c + d*x^
2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[d/c] && PosQ[f/e]
```

Rule 539

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[(c*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcT
an[Rt[d/c, 2]*x], 1 - (c*f)/(d*e)])/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c
*(e + f*x^2))/(e*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 526

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^q)/(a*b*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p +
 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 525

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
```

```
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\int \frac{\sqrt{e+fx^2}}{\left(a+bx^2\right)\left(c+dx^2\right)^{7/2}}\,dx = \frac{b^2\int \frac{\sqrt{e+fx^2}}{\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}\,dx}{(bc-ad)^2} - \frac{d\int \frac{\left(2bc-ad+bdx^2\right)\sqrt{e+fx^2}}{\left(c+dx^2\right)^{7/2}}\,dx}{(bc-ad)^2}$$

$$= -\frac{dx\sqrt{e+fx^2}}{5c(bc-ad)\left(c+dx^2\right)^{5/2}} + \frac{b^3\int \frac{\sqrt{e+fx^2}}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{(bc-ad)^3} - \frac{\left(b^2d\right)\int \frac{\sqrt{e+fx^2}}{\left(c+dx^2\right)^{3/2}}\,dx}{(bc-ad)^3} + \frac{\int \frac{-d(9bc-4ad)\cdots}{\left(c+dx^2\right)\cdots}}{5c(b\cdots}$$

$$= -\frac{dx\sqrt{e+fx^2}}{5c(bc-ad)\left(c+dx^2\right)^{5/2}} - \frac{d(bc(9de-8cf)-ad(4de-3cf))x\sqrt{e+fx^2}}{15c^2(bc-ad)^2(de-cf)\left(c+dx^2\right)^{3/2}} - \frac{b^2\sqrt{d}\sqrt{e+f\cdots}}{\sqrt{c}(bc-\cdots}$$

$$= -\frac{dx\sqrt{e+fx^2}}{5c(bc-ad)\left(c+dx^2\right)^{5/2}} - \frac{d(bc(9de-8cf)-ad(4de-3cf))x\sqrt{e+fx^2}}{15c^2(bc-ad)^2(de-cf)\left(c+dx^2\right)^{3/2}} - \frac{b^2\sqrt{d}\sqrt{e+f\cdots}}{\sqrt{c}(bc-\cdots}$$

$$= -\frac{dx\sqrt{e+fx^2}}{5c(bc-ad)\left(c+dx^2\right)^{5/2}} - \frac{d(bc(9de-8cf)-ad(4de-3cf))x\sqrt{e+fx^2}}{15c^2(bc-ad)^2(de-cf)\left(c+dx^2\right)^{3/2}} - \frac{b^2\sqrt{d}\sqrt{e+f\cdots}}{\sqrt{c}(bc-\cdots}$$

Mathematica [C] time = 3.07883, size = 584, normalized size = 0.93

$$\frac{-adx\sqrt{\frac{d}{c}}\left(e+fx^2\right)\left(\left(c+dx^2\right)^2\left(a^2d^2\left(3c^2f^2-13cdef+8d^2e^2\right)+abcd\left(-11c^2f^2+41cdef-26d^2e^2\right)+b^2c^2\left(23c^2f^2-\cdots\right.\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e + f*x^2]/((a + b*x^2)*(c + d*x^2)^(7/2)),x]
```

```
[Out] (-(a*d*Sqrt[d/c]*x*(e + f*x^2)*(3*c^2*(b*c - a*d)^2*(d*e - c*f)^2 + c*(b*c
- a*d)*(-(d*e) + c*f)*(a*d*(4*d*e - 3*c*f) + b*c*(-9*d*e + 8*c*f))*(c + d*x
^2) + (a*b*c*d*(-26*d^2*e^2 + 41*c*d*e*f - 11*c^2*f^2) + a^2*d^2*(8*d^2*e^2
 - 13*c*d*e*f + 3*c^2*f^2) + b^2*c^2*(33*d^2*e^2 - 58*c*d*e*f + 23*c^2*f^2)
)*(c + d*x^2)^2)) - I*(c + d*x^2)^2*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]
*(a*d*e*(a*b*c*d*(-26*d^2*e^2 + 41*c*d*e*f - 11*c^2*f^2) + a^2*d^2*(8*d^2*e
^2 - 13*c*d*e*f + 3*c^2*f^2) + b^2*c^2*(33*d^2*e^2 - 58*c*d*e*f + 23*c^2*f^
2))*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - (d*e - c*f)*(-(a*(2*a*
b*c*d^2*e*(13*d*e - 14*c*f) + a^2*d^3*e*(-8*d*e + 9*c*f) + b^2*c^2*(-33*d^2
*e^2 + 49*c*d*e*f - 15*c^2*f^2))*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d
*e)]) + 15*b^2*c^3*(b*e - a*f)*(-(d*e) + c*f)*EllipticPi[(b*c)/(a*d), I*Arc
Sinh[Sqrt[d/c]*x], (c*f)/(d*e)])))/(15*a*c^3*Sqrt[d/c]*(b*c - a*d)^3*(d*e -
 c*f)^2*(c + d*x^2)^(5/2)*Sqrt[e + f*x^2])
```

Maple [B] time = 0.063, size = 6245, normalized size = 9.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e)^(1/2)/(b*x^2+a)/(d*x^2+c)^(7/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{fx^2+e}}{(bx^2+a)(dx^2+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e)^(1/2)/(b*x^2+a)/(d*x^2+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(f*x^2 + e)/((b*x^2 + a)*(d*x^2 + c)^(7/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e)^(1/2)/(b*x^2+a)/(d*x^2+c)^(7/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e)**(1/2)/(b*x**2+a)/(d*x**2+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{fx^2+e}}{(bx^2+a)(dx^2+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e)^(1/2)/(b*x^2+a)/(d*x^2+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(f*x^2 + e)/((b*x^2 + a)*(d*x^2 + c)^(7/2)), x)
```

3.71 $\int \frac{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}{a+bx^2}\,dx$

Optimal. Leaf size=659

$$\frac{e^{3/2}\sqrt{c+dx^2}\left(15a^2d^2f-5abd(5cf+3de)+3b^2c(3cf+8de)\right)\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{15b^3c\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{\sqrt{e}\sqrt{c+dx^2}\left(15a^2d^2\right.}{}$$

```
[Out] ((b*c - a*d)^2*f^2*x*Sqrt[c + d*x^2])/(b^3*d*Sqrt[e + f*x^2]) + (2*(b*c - a
*d)*f*(2*d*e - c*f)*x*Sqrt[c + d*x^2])/(3*b^2*d*Sqrt[e + f*x^2]) + ((3*d^2*
e^2 + 7*c*d*e*f - 2*c^2*f^2)*x*Sqrt[c + d*x^2])/(15*b*d*Sqrt[e + f*x^2]) +
((b*c - a*d)*f*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(3*b^2) + (2*(3*d*e - c*f
)*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(15*b) + (f*x*(c + d*x^2)^(3/2)*Sqrt[e
 + f*x^2])/(5*b) - (Sqrt[e]*(15*a^2*d^2*f^2 - 20*a*b*d*f*(d*e + c*f) + 3*b^
2*(d^2*e^2 + 9*c*d*e*f + c^2*f^2))*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f
]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(15*b^3*d*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*
(e + f*x^2))]*Sqrt[e + f*x^2]) + (e^(3/2)*(15*a^2*d^2*f + 3*b^2*c*(8*d*e +
3*c*f) - 5*a*b*d*(3*d*e + 5*c*f))*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]
*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(15*b^3*c*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(
e + f*x^2))]*Sqrt[e + f*x^2]) + ((b*c - a*d)^2*e^(3/2)*(b*e - a*f)*Sqrt[c +
 d*x^2]*EllipticPi[1 - (b*e)/(a*f), ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/
(c*f)])/(a*b^3*c*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x
^2])
```

Rubi [A] time = 0.752705, antiderivative size = 784, normalized size of antiderivative = 1.19, number of steps used = 14, number of rules used = 9, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.281, Rules used = {545, 416, 528, 531, 418, 492, 411, 543, 539}

$$\frac{c^{3/2}\sqrt{e+fx^2}(bc-ad)(be-af)^2\Pi\left(1-\frac{bc}{ad};\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{ab^3\sqrt{d}e\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}+\frac{fx\sqrt{c+dx^2}\sqrt{e+fx^2}(bc-ad)}{3b^2}+\frac{fx\sqrt{c+dx^2}(bc-}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*x^2)^(3/2)*(e + f*x^2)^(3/2))/(a + b*x^2),x]
```

```
[Out] ((b*c - a*d)*f*(4*b*d*e + b*c*f - 3*a*d*f)*x*Sqrt[c + d*x^2])/(3*b^3*d*Sqrt
[e + f*x^2]) + ((3*d^2*e^2 + 7*c*d*e*f - 2*c^2*f^2)*x*Sqrt[c + d*x^2])/(15*
b*d*Sqrt[e + f*x^2]) + ((b*c - a*d)*f*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(3
*b^2) + (2*(3*d*e - c*f)*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(15*b) + (f*x*(
c + d*x^2)^(3/2)*Sqrt[e + f*x^2])/(5*b) - ((b*c - a*d)*Sqrt[e]*Sqrt[f]*(4*b
*d*e + b*c*f - 3*a*d*f)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e
]], 1 - (d*e)/(c*f)])/(3*b^3*d*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e
 + f*x^2]) - (Sqrt[e]*(3*d^2*e^2 + 7*c*d*e*f - 2*c^2*f^2)*Sqrt[c + d*x^2]*E
llipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(15*b*d*Sqrt[f]*Sqr
t[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + ((b*c - a*d)*Sqrt[e]*
Sqrt[f]*(5*b*e - 3*a*f)*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e
]], 1 - (d*e)/(c*f)])/(3*b^3*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e +
 f*x^2]) + (e^(3/2)*(9*d*e - c*f)*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]
*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(15*b*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e +
f*x^2))]*Sqrt[e + f*x^2]) + (c^(3/2)*(b*c - a*d)*(b*e - a*f)^2*Sqrt[e + f*x
^2]*EllipticPi[1 - (b*c)/(a*d), ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e
```

```
)])/(a*b^3*Sqrt[d]*e*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))])
```

Rule 545

```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Dist[d/b, Int[(c + d*x^2)^(q - 1)*(e + f*x^2)^r, x], x
] + Dist[(b*c - a*d)/b, Int[((c + d*x^2)^(q - 1)*(e + f*x^2)^r)/(a + b*x^2)
, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && GtQ[q, 1]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 543

```
Int[(((c_) + (d_.)*(x_)^2)^(3/2)*Sqrt[(e_) + (f_.)*(x_)^2])/((a_) + (b_.)*(
```

```
x_)^2), x_Symbol] :> Dist[(b*c - a*d)^2/b^2, Int[Sqrt[e + f*x^2]/((a + b*x^
2)*Sqrt[c + d*x^2]), x], x] + Dist[d/b^2, Int[((2*b*c - a*d + b*d*x^2)*Sqrt
[e + f*x^2])/Sqrt[c + d*x^2], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && Pos
Q[d/c] && PosQ[f/e]
```

Rule 539

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[(c*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcT
an[Rt[d/c, 2]*x], 1 - (c*f)/(d*e)])/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c
*(e + f*x^2))/(e*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rubi steps

$$\int \frac{\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^{3/2}}{a+bx^2}\,dx = \frac{d\int \sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}\,dx}{b} + \frac{(bc-ad)\int \frac{\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}}{a+bx^2}\,dx}{b}$$

$$= \frac{fx\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}}{5b} + \frac{\int \frac{\sqrt{c+dx^2}\left(e(5de-cf)+2f(3de-cf)x^2\right)}{\sqrt{e+fx^2}}\,dx}{5b} + \frac{((bc-ad)f)\int \frac{\sqrt{c+dx^2}\cdots}{\cdots}}{b^3}$$

$$= \frac{(bc-ad)fx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3b^2} + \frac{2(3de-cf)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{15b} + \frac{fx\left(c+dx^2\right)^{3/2}\cdots}{5b}$$

$$= \frac{(bc-ad)fx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3b^2} + \frac{2(3de-cf)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{15b} + \frac{fx\left(c+dx^2\right)^{3/2}\cdots}{5b}$$

$$= \frac{(bc-ad)f(4bde+bcf-3adf)x\sqrt{c+dx^2}}{3b^3d\sqrt{e+fx^2}} + \frac{\left(3d^2e^2+7cdef-2c^2f^2\right)x\sqrt{c+dx^2}}{15bd\sqrt{e+fx^2}} + \cdots$$

$$= \frac{(bc-ad)f(4bde+bcf-3adf)x\sqrt{c+dx^2}}{3b^3d\sqrt{e+fx^2}} + \frac{\left(3d^2e^2+7cdef-2c^2f^2\right)x\sqrt{c+dx^2}}{15bd\sqrt{e+fx^2}} + \cdots$$

Mathematica [C] time = 2.49514, size = 445, normalized size = 0.68

$$-ia\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\left(15a^2bdf^2(2cf+de)-15a^3d^2f^3+5ab^2f\left(-3c^2f^2-7cdef+d^2e^2\right)-3b^3e\left(-7c^2f^2+cdef+d^2\cdots\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x^2)^(3/2)*(e + f*x^2)^(3/2))/(a + b*x^2),x]
```

```
[Out] ((-I)*a*b*e*(15*a^2*d^2*f^2 - 20*a*b*d*f*(d*e + c*f) + 3*b^2*(d^2*e^2 + 9*c
*d*e*f + c^2*f^2))*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcS
inh[Sqrt[d/c]*x], (c*f)/(d*e)] - I*a*(-15*a^3*d^2*f^3 + 15*a^2*b*d*f^2*(d*e
 + 2*c*f) - 3*b^3*e*(d^2*e^2 + c*d*e*f - 7*c^2*f^2) + 5*a*b^2*f*(d^2*e^2 -
7*c*d*e*f - 3*c^2*f^2))*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I
```

```
*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + f*(a*b^2*Sqrt[d/c]*x*(c + d*x^2)*(e +
 f*x^2)*(-5*a*d*f + 3*b*(2*d*e + 2*c*f + d*f*x^2)) - (15*I)*(b*c - a*d)^2*(
b*e - a*f)^2*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d)
, I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]))/(15*a*b^4*Sqrt[d/c]*f*Sqrt[c + d*x
^2]*Sqrt[e + f*x^2])
```

Maple [B] time = 0.023, size = 1939, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(3/2)*(f*x^2+e)^(3/2)/(b*x^2+a),x)
```

```
[Out] -1/15*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)*(-15*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e
)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a^4*d^
2*f^3+35*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(
c*f/d/e)^(1/2))*a^2*b^2*c*d*e*f^2+30*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2
)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a*b^3*c*d*e^
2*f-27*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*
f/d/e)^(1/2))*a*b^3*c*d*e^2*f-60*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*El
lipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a^2*b^2*c*d*e*f^
2+3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d
/e)^(1/2))*a*b^3*d^2*e^3-3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Elliptic
E(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b^3*d^2*e^3-15*((d*x^2+c)/c)^(1/2)*((f*
x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2)
)*a^2*b^2*c^2*f^3-15*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(
-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*b^4*c^2*e^2*f+15*((d*x^2+c)/
c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*
b^2*c^2*f^3-9*(-d/c)^(1/2)*x^5*a*b^3*c*d*f^3-9*(-d/c)^(1/2)*x^5*a*b^3*d^2*e
*f^2+5*(-d/c)^(1/2)*x^3*a^2*b^2*c*d*f^3+5*(-d/c)^(1/2)*x^3*a^2*b^2*d^2*e*f^
2-6*(-d/c)^(1/2)*x^3*a*b^3*d^2*e^2*f-6*(-d/c)^(1/2)*x*a*b^3*c^2*e*f^2+15*((
d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1
/2))*a^4*d^2*f^3-3*(-d/c)^(1/2)*x^7*a*b^3*d^2*f^3+5*(-d/c)^(1/2)*x^5*a^2*b^
2*d^2*f^3-6*(-d/c)^(1/2)*x^3*a*b^3*c^2*f^3+3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)
/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b^3*c*d*e^2*f+20*((d*
x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2
))*a^2*b^2*c*d*e*f^2-15*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(
x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a^2*b^2*d^2*e^2*f+30*((d*
x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e
)^(1/2)/(-d/c)^(1/2))*a*b^3*c^2*e*f^2-15*(-d/c)^(1/2)*x^3*a*b^3*c*d*e*f^2+5
*(-d/c)^(1/2)*x*a^2*b^2*c*d*e*f^2-6*(-d/c)^(1/2)*x*a*b^3*c*d*e^2*f-30*((d*x
^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2)
)*a^3*b*c*d*f^3-15*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/
c)^(1/2),(c*f/d/e)^(1/2))*a^3*b*d^2*e*f^2-5*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/
e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*b^2*d^2*e^2*f-21*((d
*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/
2))*a*b^3*c^2*e*f^2-15*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*
(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^3*b*d^2*e*f^2+20*((d*x^2+c)/c)^(1/2)*((f*x^
2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*b^2*d^2*e^2*f-3
*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)
^(1/2))*a*b^3*c^2*e*f^2+30*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Elliptic
Pi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a^3*b*c*d*f^3+30*((d*x
^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)
^(1/2)/(-d/c)^(1/2))*a^3*b*d^2*e*f^2)/f/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)/b^4/(
-d/c)^(1/2)/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)^{\frac{3}{2}}\left(fx^2+e\right)^{\frac{3}{2}}}{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)*(f*x^2+e)^(3/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(3/2)*(f*x^2 + e)^(3/2)/(b*x^2 + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)*(f*x^2+e)^(3/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c+dx^2\right)^{\frac{3}{2}}\left(e+fx^2\right)^{\frac{3}{2}}}{a+bx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(3/2)*(f*x**2+e)**(3/2)/(b*x**2+a),x)
```

```
[Out] Integral((c + d*x**2)**(3/2)*(e + f*x**2)**(3/2)/(a + b*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)^{\frac{3}{2}}\left(fx^2+e\right)^{\frac{3}{2}}}{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)*(f*x^2+e)^(3/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)^(3/2)*(f*x^2 + e)^(3/2)/(b*x^2 + a), x)
```

3.72 $\int \frac{\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}}{a+bx^2}\,dx$

Optimal. Leaf size=403

$$\frac{\sqrt{e}\sqrt{f}\sqrt{c+dx^2}(5be-3af)\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{3b^2\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{c^{3/2}\sqrt{e+fx^2}(be-af)^2\Pi\left(1-\frac{bc}{ad};\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{ab^2\sqrt{d}e\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}+\frac{f}{}$$

```
[Out] (f*(4*b*d*e + b*c*f - 3*a*d*f)*x*Sqrt[c + d*x^2])/(3*b^2*d*Sqrt[e + f*x^2])
 + (f*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(3*b) - (Sqrt[e]*Sqrt[f]*(4*b*d*e
+ b*c*f - 3*a*d*f)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1
 - (d*e)/(c*f)])/(3*b^2*d*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*
x^2]) + (Sqrt[e]*Sqrt[f]*(5*b*e - 3*a*f)*Sqrt[c + d*x^2]*EllipticF[ArcTan[(
Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(3*b^2*Sqrt[(e*(c + d*x^2))/(c*(e +
f*x^2))]*Sqrt[e + f*x^2]) + (c^(3/2)*(b*e - a*f)^2*Sqrt[e + f*x^2]*Elliptic
Pi[1 - (b*c)/(a*d), ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(a*b^2*S
qrt[d]*e*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))])
```

Rubi [A] time = 0.304038, antiderivative size = 403, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {543, 539, 528, 531, 418, 492, 411}

$$\frac{c^{3/2}\sqrt{e+fx^2}(be-af)^2\Pi\left(1-\frac{bc}{ad};\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{ab^2\sqrt{d}e\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}+\frac{fx\sqrt{c+dx^2}(-3adf+bcf+4bde)}{3b^2d\sqrt{e+fx^2}}+\frac{\sqrt{e}\sqrt{f}\sqrt{c+dx^2}(5be-3af}{3b^2\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*x^2]*(e + f*x^2)^(3/2))/(a + b*x^2),x]
```


```
[Out] (f*(4*b*d*e + b*c*f - 3*a*d*f)*x*Sqrt[c + d*x^2])/(3*b^2*d*Sqrt[e + f*x^2])
 + (f*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(3*b) - (Sqrt[e]*Sqrt[f]*(4*b*d*e
+ b*c*f - 3*a*d*f)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1
 - (d*e)/(c*f)])/(3*b^2*d*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*
x^2]) + (Sqrt[e]*Sqrt[f]*(5*b*e - 3*a*f)*Sqrt[c + d*x^2]*EllipticF[ArcTan[(
Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(3*b^2*Sqrt[(e*(c + d*x^2))/(c*(e +
f*x^2))]*Sqrt[e + f*x^2]) + (c^(3/2)*(b*e - a*f)^2*Sqrt[e + f*x^2]*Elliptic
Pi[1 - (b*c)/(a*d), ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(a*b^2*S
qrt[d]*e*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))])
```


Rule 543

```
Int[(((c_) + (d_.)*(x_)^2)^(3/2)*Sqrt[(e_) + (f_.)*(x_)^2])/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Dist[(b*c - a*d)^2/b^2, Int[Sqrt[e + f*x^2]/((a + b*x^
2)*Sqrt[c + d*x^2]), x], x] + Dist[d/b^2, Int[((2*b*c - a*d + b*d*x^2)*Sqrt
[e + f*x^2])/Sqrt[c + d*x^2], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && Pos
Q[d/c] && PosQ[f/e]
```

Rule 539

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[(c*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcT
an[Rt[d/c, 2]*x], 1 - (c*f)/(d*e)])/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c
```

```
*(e + f*x^2))/(e*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}}{a+bx^2}\,dx = \frac{f\int \frac{\sqrt{c+dx^2}\left(2be-af+bfx^2\right)}{\sqrt{e+fx^2}}\,dx}{b^2} + \frac{(be-af)^2\int \frac{\sqrt{c+dx^2}}{\left(a+bx^2\right)\sqrt{e+fx^2}}\,dx}{b^2}$$

$$= \frac{fx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3b} + \frac{c^{3/2}(be-af)^2\sqrt{e+fx^2}\,\Pi\left(1-\frac{bc}{ad};\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{ab^2\sqrt{d}e\sqrt{c+dx^2}\sqrt{\frac{c\left(e+fx^2\right)}{e\left(c+dx^2\right)}}} + \int \frac{cf(5be-$$

$$= \frac{fx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3b} + \frac{c^{3/2}(be-af)^2\sqrt{e+fx^2}\,\Pi\left(1-\frac{bc}{ad};\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{ab^2\sqrt{d}e\sqrt{c+dx^2}\sqrt{\frac{c\left(e+fx^2\right)}{e\left(c+dx^2\right)}}} + \frac{(cf(5be-$$

$$= \frac{f(4bde+bcf-3adf)x\sqrt{c+dx^2}}{3b^2d\sqrt{e+fx^2}} + \frac{fx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3b} + \frac{\sqrt{e}\sqrt{f}(5be-3af)\sqrt{c+dx^2}F\big(}{3b^2\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}\sqrt{e}$$

$$= \frac{f(4bde+bcf-3adf)x\sqrt{c+dx^2}}{3b^2d\sqrt{e+fx^2}} + \frac{fx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3b} - \frac{\sqrt{e}\sqrt{f}(4bde+bcf-3adf)\sqrt{c}}{3b^2d\sqrt{\frac{e\left(c+dx\right.}{c\left(e+f\right.}}}$$

Mathematica [C] time = 1.10426, size = 739, normalized size = 1.83

$$\frac{-ia\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\left(3a^2df^2-3abf(cf+de)+b^2e(4cf-de)\right)\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{\frac{d}{c}}\right),\frac{cf}{de}\right)-3ia^2bcf^2\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx}{e}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + d*x^2]*(e + f*x^2)^(3/2))/(a + b*x^2),x]
```

```
[Out] (a*b^2*c*Sqrt[d/c]*e*f*x + a*b^2*d*Sqrt[d/c]*e*f*x^3 + a*b^2*c*Sqrt[d/c]*f^
2*x^3 + a*b^2*d*Sqrt[d/c]*f^2*x^5 - I*a*b*e*(4*b*d*e + b*c*f - 3*a*d*f)*Sqr
t[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f
)/(d*e)] - I*a*(3*a^2*d*f^2 - 3*a*b*f*(d*e + c*f) + b^2*e*(-(d*e) + 4*c*f))
*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x],
(c*f)/(d*e)] - (3*I)*b^3*c*e^2*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*Elli
pticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + (3*I)*a*b^2*d*e^
2*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh
[Sqrt[d/c]*x], (c*f)/(d*e)] + (6*I)*a*b^2*c*e*f*Sqrt[1 + (d*x^2)/c]*Sqrt[1
+ (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] -
 (6*I)*a^2*b*d*e*f*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)
/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - (3*I)*a^2*b*c*f^2*Sqrt[1 + (
d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x
], (c*f)/(d*e)] + (3*I)*a^3*d*f^2*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*E
llipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/(3*a*b^3*Sqrt[
d/c]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Maple [B] time = 0.017, size = 1028, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)*(f*x^2+e)^(3/2)/(b*x^2+a),x)
```

```
[Out] 1/3*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)*((-d/c)^(1/2)*x^5*a*b^2*d*f^2+(-d/c)^(1
/2)*x^3*a*b^2*c*f^2+(-d/c)^(1/2)*x^3*a*b^2*d*e*f+3*((d*x^2+c)/c)^(1/2)*((f*
x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^3*d*f^2-3*((d*x
^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2)
)*a^2*b*c*f^2-3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^
(1/2),(c*f/d/e)^(1/2))*a^2*b*d*e*f+4*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2
)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b^2*c*e*f-((d*x^2+c)/c)^(1/2)
*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b^2*d*e^2-
3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e
)^(1/2))*a^2*b*d*e*f+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-
d/c)^(1/2),(c*f/d/e)^(1/2))*a*b^2*c*e*f+4*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)
^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b^2*d*e^2-3*((d*x^2+c)/c
)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/
(-d/c)^(1/2))*a^3*d*f^2+3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticP
i(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a^2*b*c*f^2+6*((d*x^2+c
)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/
2)/(-d/c)^(1/2))*a^2*b*d*e*f-6*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Elli
pticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a*b^2*c*e*f-3*((d*
x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e
)^(1/2)/(-d/c)^(1/2))*a*b^2*d*e^2+3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)
*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*b^3*c*e^2+(-d
/c)^(1/2)*x*a*b^2*c*e*f)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)/b^3/(-d/c)^(1/2)/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2 + c}\left(fx^2 + e\right)^{\frac{3}{2}}}{bx^2 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*(f*x^2+e)^(3/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)/(b*x^2 + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*(f*x^2+e)^(3/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c + dx^2}\left(e + fx^2\right)^{\frac{3}{2}}}{a + bx^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)*(f*x**2+e)**(3/2)/(b*x**2+a),x)
```

```
[Out] Integral(sqrt(c + d*x**2)*(e + f*x**2)**(3/2)/(a + b*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2+c}\left(fx^2+e\right)^{\frac{3}{2}}}{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*(f*x^2+e)^(3/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)/(b*x^2 + a), x)
```

3.73 $\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)\sqrt{c+dx^2}} dx$

Optimal. Leaf size=328

$$\frac{e^{3/2}\sqrt{f}\sqrt{c+dx^2}\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{bc\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{e^{3/2}\sqrt{c+dx^2}(be-af)\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{abc\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{f^2x\sqrt{c+d}}{bd\sqrt{e+f}}$$

```
[Out] (f^2*x*Sqrt[c + d*x^2])/(b*d*Sqrt[e + f*x^2]) - (Sqrt[e]*f^(3/2)*Sqrt[c + d
*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(b*d*Sqrt[(e
*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (e^(3/2)*Sqrt[f]*Sqrt[c +
 d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(b*c*Sqrt[
(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (e^(3/2)*(b*e - a*f)*Sq
rt[c + d*x^2]*EllipticPi[1 - (b*e)/(a*f), ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 -
(d*e)/(c*f)])/(a*b*c*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e +
 f*x^2])
```

Rubi [A] time = 0.191238, antiderivative size = 328, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {545, 422, 418, 492, 411, 539}

$$\frac{e^{3/2}\sqrt{c+dx^2}(be-af)\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{abc\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{e^{3/2}\sqrt{f}\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{bc\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{f^2x\sqrt{c+dx^2}}{bd\sqrt{e+fx^2}}-\frac{\sqrt{e}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x^2)^(3/2)/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```


```
[Out] (f^2*x*Sqrt[c + d*x^2])/(b*d*Sqrt[e + f*x^2]) - (Sqrt[e]*f^(3/2)*Sqrt[c + d
*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(b*d*Sqrt[(e
*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (e^(3/2)*Sqrt[f]*Sqrt[c +
 d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(b*c*Sqrt[
(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (e^(3/2)*(b*e - a*f)*Sq
rt[c + d*x^2]*EllipticPi[1 - (b*e)/(a*f), ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 -
(d*e)/(c*f)])/(a*b*c*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e +
 f*x^2])
```


Rule 545

```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Dist[d/b, Int[(c + d*x^2)^(q - 1)*(e + f*x^2)^r, x], x
] + Dist[(b*c - a*d)/b, Int[((c + d*x^2)^(q - 1)*(e + f*x^2)^r)/(a + b*x^2)
, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && GtQ[q, 1]
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 539

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[(c*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcT
an[Rt[d/c, 2]*x], 1 - (c*f)/(d*e)])/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c
*(e + f*x^2))/(e*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rubi steps

$$\int \frac{\left(e+fx^2\right)^{3/2}}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx = \frac{f\int \frac{\sqrt{e+fx^2}}{\sqrt{c+dx^2}}\,dx}{b} + \frac{(be-af)\int \frac{\sqrt{e+fx^2}}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{b}$$

$$= \frac{e^{3/2}(be-af)\sqrt{c+dx^2}\,\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{abc\sqrt{f}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}\sqrt{e+fx^2}} + \frac{(ef)\int \frac{1}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{b} + \frac{f^2\int \frac{}{\sqrt{c+dx^2}}}{b}$$

$$= \frac{f^2x\sqrt{c+dx^2}}{bd\sqrt{e+fx^2}} + \frac{e^{3/2}\sqrt{f}\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{bc\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}\sqrt{e+fx^2}} + \frac{e^{3/2}(be-af)\sqrt{c+dx^2}\,\Pi\left(1-\frac{be}{af};\right.}{abc\sqrt{f}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}\sqrt{e}}$$

$$= \frac{f^2x\sqrt{c+dx^2}}{bd\sqrt{e+fx^2}} - \frac{\sqrt{e}f^{3/2}\sqrt{c+dx^2}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{bd\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}\sqrt{e+fx^2}} + \frac{e^{3/2}\sqrt{f}\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1\right.}{bc\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}\sqrt{e+fx^2}}$$

Mathematica [C] time = 0.355811, size = 184, normalized size = 0.56

$$-\frac{i\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\left((be-af)\left(af\,\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{\frac{d}{c}}\right),\frac{cf}{de}\right)+(be-af)\Pi\left(\frac{bc}{ad};i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)\right)+abefE\left(i\sin\right.\right.}{ab^2\sqrt{\frac{d}{c}}\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x^2)^(3/2)/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] ((-I)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*(a*b*e*f*EllipticE[I*ArcSinh[
Sqrt[d/c]*x], (c*f)/(d*e)] + (b*e - a*f)*(a*f*EllipticF[I*ArcSinh[Sqrt[d/c]
*x], (c*f)/(d*e)] + (b*e - a*f)*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]
*x], (c*f)/(d*e)])))/(a*b^2*Sqrt[d/c]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Maple [A] time = 0.018, size = 300, normalized size = 0.9

$$\frac{1}{ab^2\left(dfx^4+cfx^2+dex^2+ce\right)}\left(-\text{EllipticF}\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)a^2f^2+\text{EllipticF}\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)abef+\text{EllipticE}\left(x\sqrt{-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e)^(3/2)/(b*x^2+a)/(d*x^2+c)^(1/2),x)
```

```
[Out] (-EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*f^2+EllipticF(x*(-d/c)^(1/2
),(c*f/d/e)^(1/2))*a*b*e*f+EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b*e*
f+EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a^2*f^2-2*El
lipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a*b*e*f+Elliptic
Pi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*b^2*e^2)*((f*x^2+e)/e)
^(1/2)*((d*x^2+c)/c)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/a/(-d/c)^(1/2)/b
^2/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx^2+e\right)^{\frac{3}{2}}}{\left(bx^2+a\right)\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e)^(3/2)/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x^2 + e)^(3/2)/((b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e)^(3/2)/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(e+fx^2\right)^{\frac{3}{2}}}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e)**(3/2)/(b*x**2+a)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral((e + f*x**2)**(3/2)/((a + b*x**2)*sqrt(c + d*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx^2+e\right)^{\frac{3}{2}}}{\left(bx^2+a\right)\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e)^(3/2)/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((f*x^2 + e)^(3/2)/((b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

3.74 $\int \frac{\left(e+fx^2\right)^{3/2}}{\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=224

$$\frac{e^{3/2}\sqrt{c+dx^2}(be-af)\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{ac\sqrt{f}\sqrt{e+fx^2}(bc-ad)\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}-\frac{\sqrt{e+fx^2}(de-cf)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(e+fx^2\right)}{e\left(c+dx^2\right)}}}$$

```
[Out] -(((d*e - c*f)*Sqrt[e + f*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (
c*f)/(d*e)])/(Sqrt[c]*Sqrt[d]*(b*c - a*d)*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*x^
2))/(e*(c + d*x^2))])) + (e^(3/2)*(b*e - a*f)*Sqrt[c + d*x^2]*EllipticPi[1
- (b*e)/(a*f), ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(a*c*(b*c - a
*d)*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rubi [A] time = 0.111799, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {542, 539, 411}

$$\frac{e^{3/2}\sqrt{c+dx^2}(be-af)\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{ac\sqrt{f}\sqrt{e+fx^2}(bc-ad)\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}-\frac{\sqrt{e+fx^2}(de-cf)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(e+fx^2\right)}{e\left(c+dx^2\right)}}}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x^2)^(3/2)/((a + b*x^2)*(c + d*x^2)^(3/2)),x]
```


```
[Out] -(((d*e - c*f)*Sqrt[e + f*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (
c*f)/(d*e)])/(Sqrt[c]*Sqrt[d]*(b*c - a*d)*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*x^
2))/(e*(c + d*x^2))])) + (e^(3/2)*(b*e - a*f)*Sqrt[c + d*x^2]*EllipticPi[1
- (b*e)/(a*f), ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(a*c*(b*c - a
*d)*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```


Rule 542

```
Int[((e_) + (f_.)*(x_)^2)^(3/2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2
)^(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[Sqrt[e + f*x^2]/((
a + b*x^2)*Sqrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqr
t[e + f*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && P
osQ[d/c] && PosQ[f/e]
```

Rule 539

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[(c*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcT
an[Rt[d/c, 2]*x], 1 - (c*f)/(d*e)])/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c
*(e + f*x^2))/(e*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
```

```
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\left(e+fx^2\right)^{3/2}}{\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}\,dx = \frac{(be-af)\int \frac{\sqrt{e+fx^2}}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{bc-ad} - \frac{(de-cf)\int \frac{\sqrt{e+fx^2}}{\left(c+dx^2\right)^{3/2}}\,dx}{bc-ad}$$

$$= -\frac{(de-cf)\sqrt{e+fx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\sqrt{d}(bc-ad)\sqrt{c+dx^2}\sqrt{\frac{c\left(e+fx^2\right)}{e\left(c+dx^2\right)}}} + \frac{e^{3/2}(be-af)\sqrt{c+dx^2}\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right.\right.}{ac(bc-ad)\sqrt{f}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}\sqrt{e+fx^2}}$$

Mathematica [C] time = 1.27748, size = 492, normalized size = 2.2

$$\frac{\sqrt{\frac{d}{c}}\left(ia\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\left(be(2cf-de)-acf^2\right)\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{\frac{d}{c}}\right),\frac{cf}{de}\right)+ia^2cf^2\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\Pi\left(\frac{bc}{ad};i\sinh^{-}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x^2)^(3/2)/((a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[d/c]*(a*b*d*Sqrt[d/c]*e^2*x - a*b*c*Sqrt[d/c]*e*f*x + a*b*d*Sqrt[d/c]
*e*f*x^3 - a*b*c*Sqrt[d/c]*f^2*x^3 - I*a*b*e*(-(d*e) + c*f)*Sqrt[1 + (d*x^2
)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + I
*a*(-(a*c*f^2) + b*e*(-(d*e) + 2*c*f))*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)
/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + I*b^2*c*e^2*Sqrt[1 + (
d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x
], (c*f)/(d*e)] - (2*I)*a*b*c*e*f*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*E
llipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + I*a^2*c*f^2*S
qrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sq
rt[d/c]*x], (c*f)/(d*e)]))/(a*b*d*(-(b*c) + a*d)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2])
```

Maple [B] time = 0.03, size = 630, normalized size = 2.8

$$\frac{1}{abc\,(ad-bc)\left(dfx^4+cfx^2+dex^2+ce\right)}\left(-x^3abcf^2\sqrt{-\frac{d}{c}}+x^3abdef\sqrt{-\frac{d}{c}}+\text{EllipticF}\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)a^2cf^2\sqrt{\frac{dx^2+c}{c}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e)^(3/2)/(b*x^2+a)/(d*x^2+c)^(3/2),x)
```

```
[Out] (-x^3*a*b*c*f^2*(-d/c)^(1/2)+x^3*a*b*d*e*f*(-d/c)^(1/2)+EllipticF(x*(-d/c)^
(1/2),(c*f/d/e)^(1/2))*a^2*c*f^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-2*
EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b*c*e*f*((d*x^2+c)/c)^(1/2)*((f
*x^2+e)/e)^(1/2)+EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b*d*e^2*((d*x^
2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))
*a*b*c*e*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-EllipticE(x*(-d/c)^(1/2)
,(c*f/d/e)^(1/2))*a*b*d*e^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-Ellipti
```

```
cPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a^2*c*f^2*((d*x^2+c)/
c)^(1/2)*((f*x^2+e)/e)^(1/2)+2*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/
2)/(-d/c)^(1/2))*a*b*c*e*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-Elliptic
Pi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*b^2*c*e^2*((d*x^2+c)/c
)^(1/2)*((f*x^2+e)/e)^(1/2)-x*a*b*c*e*f*(-d/c)^(1/2)+x*a*b*d*e^2*(-d/c)^(1/
2))*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/b/a/c/(-d/c)^(1/2)/(a*d-b*c)/(d*f*x^4+c
*f*x^2+d*e*x^2+c*e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx^2+e\right)^{\frac{3}{2}}}{\left(bx^2+a\right)\left(dx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e)^(3/2)/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x^2 + e)^(3/2)/((b*x^2 + a)*(d*x^2 + c)^(3/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e)^(3/2)/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e)**(3/2)/(b*x**2+a)/(d*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx^2+e\right)^{\frac{3}{2}}}{\left(bx^2+a\right)\left(dx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e)^(3/2)/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((f*x^2 + e)^(3/2)/((b*x^2 + a)*(d*x^2 + c)^(3/2)), x)
```

3.75 $\int \frac{\left(e+fx^2\right)^{3/2}}{\left(a+bx^2\right)\left(c+dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=391

$$\frac{e^{3/2}\sqrt{f}\sqrt{c+dx^2}\,\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{3c^2\sqrt{e+fx^2}(bc-ad)\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}-\frac{\sqrt{e+fx^2}(bc(5de-cf)-2ad(cf+de))E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{3c^{3/2}\sqrt{d}\sqrt{c+dx^2}(bc-ad)^2\sqrt{\frac{c\left(e+fx^2\right)}{e\left(c+dx^2\right)}}}+\frac{be^{3/2}\cdots}{\cdots}$$

```
[Out] -((d*e - c*f)*x*Sqrt[e + f*x^2])/(3*c*(b*c - a*d)*(c + d*x^2)^(3/2)) - ((b*
c*(5*d*e - c*f) - 2*a*d*(d*e + c*f))*Sqrt[e + f*x^2]*EllipticE[ArcTan[(Sqrt
[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(3*c^(3/2)*Sqrt[d]*(b*c - a*d)^2*Sqrt[c
+ d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))]) + (e^(3/2)*Sqrt[f]*Sqrt[c +
 d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(3*c^2*(b*
c - a*d)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (b*e^(3/2
)*(b*e - a*f)*Sqrt[c + d*x^2]*EllipticPi[1 - (b*e)/(a*f), ArcTan[(Sqrt[f]*x
)/Sqrt[e]], 1 - (d*e)/(c*f)])/(a*c*(b*c - a*d)^2*Sqrt[f]*Sqrt[(e*(c + d*x^2
))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rubi [A] time = 0.404117, antiderivative size = 391, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {544, 539, 526, 525, 418, 411}

$$\frac{e^{3/2}\sqrt{f}\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3c^2\sqrt{e+fx^2}(bc-ad)\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}-\frac{\sqrt{e+fx^2}(bc(5de-cf)-2ad(cf+de))E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{3c^{3/2}\sqrt{d}\sqrt{c+dx^2}(bc-ad)^2\sqrt{\frac{c\left(e+fx^2\right)}{e\left(c+dx^2\right)}}}+\frac{be^{3/2}\sqrt{c+dx^2}\cdots}{ac\sqrt{\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x^2)^(3/2)/((a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] -((d*e - c*f)*x*Sqrt[e + f*x^2])/(3*c*(b*c - a*d)*(c + d*x^2)^(3/2)) - ((b*
c*(5*d*e - c*f) - 2*a*d*(d*e + c*f))*Sqrt[e + f*x^2]*EllipticE[ArcTan[(Sqrt
[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(3*c^(3/2)*Sqrt[d]*(b*c - a*d)^2*Sqrt[c
+ d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))]) + (e^(3/2)*Sqrt[f]*Sqrt[c +
 d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(3*c^2*(b*
c - a*d)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (b*e^(3/2
)*(b*e - a*f)*Sqrt[c + d*x^2]*EllipticPi[1 - (b*e)/(a*f), ArcTan[(Sqrt[f]*x
)/Sqrt[e]], 1 - (d*e)/(c*f)])/(a*c*(b*c - a*d)^2*Sqrt[f]*Sqrt[(e*(c + d*x^2
))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rule 544

```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Dist[(b*(b*e - a*f))/(b*c - a*d)^2, Int[((c + d*x^2)^(
q + 2)*(e + f*x^2)^(r - 1))/(a + b*x^2), x], x] - Dist[1/(b*c - a*d)^2, Int
[(c + d*x^2)^q*(e + f*x^2)^(r - 1)*(2*b*c*d*e - a*d^2*e - b*c^2*f + d^2*(b*
e - a*f)*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[q, -1] && GtQ[
r, 1]
```

Rule 539

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[(c*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcT
```

```
an[Rt[d/c, 2]*x], 1 - (c*f)/(d*e)])/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c
*(e + f*x^2))/(e*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 526

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^q)/(a*b*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p +
 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 525

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\left(e+fx^2\right)^{3/2}}{\left(a+bx^2\right)\left(c+dx^2\right)^{5/2}}\,dx = -\frac{\int \frac{\sqrt{e+fx^2}\left(2bcde-ad^2e-bc^2f+d^2(be-af)x^2\right)}{\left(c+dx^2\right)^{5/2}}\,dx}{(bc-ad)^2} + \frac{(b(be-af))\int \frac{\sqrt{e+fx^2}}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{(bc-ad)^2}$$

$$= -\frac{(de-cf)x\sqrt{e+fx^2}}{3c(bc-ad)\left(c+dx^2\right)^{3/2}} + \frac{be^{3/2}(be-af)\sqrt{c+dx^2}\,\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{ac(bc-ad)^2\sqrt{f}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}\sqrt{e+fx^2}} + \ldots$$

$$= -\frac{(de-cf)x\sqrt{e+fx^2}}{3c(bc-ad)\left(c+dx^2\right)^{3/2}} + \frac{be^{3/2}(be-af)\sqrt{c+dx^2}\,\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{ac(bc-ad)^2\sqrt{f}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}\sqrt{e+fx^2}} + \ldots$$

$$= -\frac{(de-cf)x\sqrt{e+fx^2}}{3c(bc-ad)\left(c+dx^2\right)^{3/2}} - \frac{(bc(5de-cf)-2ad(de+cf))\sqrt{e+fx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\ldots\right.}{3c^{3/2}\sqrt{d}(bc-ad)^2\sqrt{c+dx^2}\sqrt{\frac{c\left(e+fx^2\right)}{e\left(c+dx^2\right)}}}$$

Mathematica [C] time = 1.81255, size = 999, normalized size = 2.55

$$\frac{abc^3\left(\frac{d}{c}\right)^{3/2}f^2x^5+2a^2cd^2\sqrt{\frac{d}{c}}f^2x^5+2a^2d^3\sqrt{\frac{d}{c}}efx^5-5abcd^2\sqrt{\frac{d}{c}}efx^5+2a^2d^3\sqrt{\frac{d}{c}}e^2x^3-5abcd^2\sqrt{\frac{d}{c}}e^2x^3+a^2c^3\left(\frac{d}{c}\right)^{3/2}f^2x^3}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x^2)^(3/2)/((a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] (3*a^2*c*d^2*Sqrt[d/c]*e^2*x - 6*a*b*c^3*(d/c)^(3/2)*e^2*x + 2*a*b*c^3*Sqrt
[d/c]*e*f*x + a^2*c^3*(d/c)^(3/2)*e*f*x - 5*a*b*c*d^2*Sqrt[d/c]*e^2*x^3 + 2
*a^2*d^3*Sqrt[d/c]*e^2*x^3 + 5*a^2*c*d^2*Sqrt[d/c]*e*f*x^3 - 5*a*b*c^3*(d/c
)^(3/2)*e*f*x^3 + 2*a*b*c^3*Sqrt[d/c]*f^2*x^3 + a^2*c^3*(d/c)^(3/2)*f^2*x^3
 - 5*a*b*c*d^2*Sqrt[d/c]*e*f*x^5 + 2*a^2*d^3*Sqrt[d/c]*e*f*x^5 + 2*a^2*c*d^
2*Sqrt[d/c]*f^2*x^5 + a*b*c^3*(d/c)^(3/2)*f^2*x^5 + I*a*e*(b*c*(-5*d*e + c*
f) + 2*a*d*(d*e + c*f))*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]
*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - I*a*(-(d*e) + c*f)*(5*b*c
*e - 2*a*d*e - 3*a*c*f)*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]
*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - (3*I)*b^2*c^3*e^2*Sqrt[1
+ (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c
]*x], (c*f)/(d*e)] + (6*I)*a*b*c^3*e*f*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)
/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - (3*I)*a^
2*c^3*f^2*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I
*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - (3*I)*b^2*c^2*d*e^2*x^2*Sqrt[1 + (d*x
^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x],
(c*f)/(d*e)] + (6*I)*a*b*c^2*d*e*f*x^2*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)
/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - (3*I)*a^
2*c^2*d*f^2*x^2*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a
*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/(3*a*c^2*Sqrt[d/c]*(b*c - a*d)^2
*(c + d*x^2)^(3/2)*Sqrt[e + f*x^2])
```

Maple [B] time = 0.036, size = 1876, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e)^(3/2)/(b*x^2+a)/(d*x^2+c)^(5/2),x)
```

```
[Out] 1/3*(2*x^5*a^2*c*d^2*f^2*(-d/c)^(1/2)+3*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(
-f/e)^(1/2)/(-d/c)^(1/2))*b^2*c^3*e^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/
2)-3*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*c^3*f^2*((d*x^2+c)/c)^(1
/2)*((f*x^2+e)/e)^(1/2)+2*x^3*a^2*d^3*e^2*(-d/c)^(1/2)-5*x^3*a*b*c^2*d*e*f*
(-d/c)^(1/2)+3*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))
*x^2*b^2*c^2*d*e^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-3*EllipticF(x*(-
d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a^2*c^2*d*f^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e
)/e)^(1/2)-6*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a
*b*c^3*e*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+EllipticF(x*(-d/c)^(1/2)
,(c*f/d/e)^(1/2))*a^2*c^2*d*e*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+5*E
llipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b*c^3*e*f*((d*x^2+c)/c)^(1/2)*((
f*x^2+e)/e)^(1/2)-5*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b*c^2*d*e^2
*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-2*EllipticE(x*(-d/c)^(1/2),(c*f/d/
e)^(1/2))*a^2*c^2*d*e*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-EllipticE(x
*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b*c^3*e*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e
)^(1/2)+5*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b*c^2*d*e^2*((d*x^2+c
```

```
)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-5*x^5*a*b*c*d^2*e*f*(-d/c)^(1/2)+3*EllipticP
i(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*x^2*a^2*c^2*d*f^2*((d*x
^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+2*x^5*a^2*d^3*e*f*(-d/c)^(1/2)+x^3*a^2*c
^2*d*f^2*(-d/c)^(1/2)+2*x^3*a*b*c^3*f^2*(-d/c)^(1/2)+3*x*a^2*c*d^2*e^2*(-d/
c)^(1/2)+3*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a^2
*c^3*f^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+5*x^3*a^2*c*d^2*e*f*(-d/c)
^(1/2)+2*x*a*b*c^3*e*f*(-d/c)^(1/2)-6*x*a*b*c^2*d*e^2*(-d/c)^(1/2)+2*Ellipt
icF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a^2*d^3*e^2*((d*x^2+c)/c)^(1/2)*((f
*x^2+e)/e)^(1/2)-2*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a^2*d^3*e^
2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+2*EllipticF(x*(-d/c)^(1/2),(c*f/d
/e)^(1/2))*a^2*c*d^2*e^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-2*Elliptic
E(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*c*d^2*e^2*((d*x^2+c)/c)^(1/2)*((f*x^2
+e)/e)^(1/2)-5*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a*b*c*d^2*e^2*
((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-2*EllipticE(x*(-d/c)^(1/2),(c*f/d/e
)^(1/2))*x^2*a^2*c*d^2*e*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+5*Ellipt
icE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a*b*c*d^2*e^2*((d*x^2+c)/c)^(1/2)*(
(f*x^2+e)/e)^(1/2)+EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a^2*c*d^2*
e*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-5*x^3*a*b*c*d^2*e^2*(-d/c)^(1/2
)+x*a^2*c^2*d*e*f*(-d/c)^(1/2)+x^5*a*b*c^2*d*f^2*(-d/c)^(1/2)+5*EllipticF(x
*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a*b*c^2*d*e*f*((d*x^2+c)/c)^(1/2)*((f*x^
2+e)/e)^(1/2)-EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a*b*c^2*d*e*f*(
(d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-6*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,
(-f/e)^(1/2)/(-d/c)^(1/2))*x^2*a*b*c^2*d*e*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)
/e)^(1/2))/(f*x^2+e)^(1/2)/(a*d-b*c)^2/(-d/c)^(1/2)/c^2/a/(d*x^2+c)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx^2+e\right)^{\frac{3}{2}}}{\left(bx^2+a\right)\left(dx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e)^(3/2)/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x^2 + e)^(3/2)/((b*x^2 + a)*(d*x^2 + c)^(5/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e)^(3/2)/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e)**(3/2)/(b*x**2+a)/(d*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx^2+e\right)^{\frac{3}{2}}}{\left(bx^2+a\right)\left(dx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e)^(3/2)/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((f*x^2 + e)^(3/2)/((b*x^2 + a)*(d*x^2 + c)^(5/2)), x)
```

3.76 $\int \frac{\left(e+fx^2\right)^{3/2}}{\left(a+bx^2\right)\left(c+dx^2\right)^{7/2}}\,dx$

Optimal. Leaf size=639

$$\frac{e^{3/2}\sqrt{f}\sqrt{c+dx^2}(3bc(3de-2cf)-ad(4de-cf))\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{15c^3\sqrt{e+fx^2}(bc-ad)^2(de-cf)\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}+\frac{b^2e^{3/2}\sqrt{c+dx^2}(be-af)\Pi\left(1-\frac{be}{af};\right.}{ac\sqrt{f}\sqrt{e+fx^2}(bc-ad)^3}$$

```
[Out] -((d*e - c*f)*x*Sqrt[e + f*x^2])/(5*c*(b*c - a*d)*(c + d*x^2)^(5/2)) - ((3*
b*c*(3*d*e - c*f) - 2*a*d*(2*d*e + c*f))*x*Sqrt[e + f*x^2])/(15*c^2*(b*c -
a*d)^2*(c + d*x^2)^(3/2)) - (b*Sqrt[d]*(b*e - a*f)*Sqrt[e + f*x^2]*Elliptic
E[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(Sqrt[c]*(b*c - a*d)^3*Sqr
t[c + d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))]) + ((a*d*(8*d^2*e^2 - 3*
c*d*e*f - 2*c^2*f^2) - 3*b*c*(6*d^2*e^2 - 6*c*d*e*f + c^2*f^2))*Sqrt[e + f*
x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(15*c^(5/2)*S
qrt[d]*(b*c - a*d)^2*(d*e - c*f)*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c
 + d*x^2))]) + (e^(3/2)*Sqrt[f]*(3*b*c*(3*d*e - 2*c*f) - a*d*(4*d*e - c*f))
*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(
15*c^3*(b*c - a*d)^2*(d*e - c*f)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt
[e + f*x^2]) + (b^2*e^(3/2)*(b*e - a*f)*Sqrt[c + d*x^2]*EllipticPi[1 - (b*e
)/(a*f), ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(a*c*(b*c - a*d)^3*
Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rubi [A] time = 0.779407, antiderivative size = 639, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {544, 541, 539, 411, 526, 527, 525, 418}

$$\frac{b^2e^{3/2}\sqrt{c+dx^2}(be-af)\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{ac\sqrt{f}\sqrt{e+fx^2}(bc-ad)^3\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}+\frac{\sqrt{e+fx^2}\left(ad\left(-2c^2f^2-3cdef+8d^2e^2\right)-3bc\left(c^2f^2-6cde\right.\right.}{15c^{5/2}\sqrt{d}\sqrt{c+dx^2}(bc-ad)^2(de-c}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x^2)^(3/2)/((a + b*x^2)*(c + d*x^2)^(7/2)),x]
```

```
[Out] -((d*e - c*f)*x*Sqrt[e + f*x^2])/(5*c*(b*c - a*d)*(c + d*x^2)^(5/2)) - ((3*
b*c*(3*d*e - c*f) - 2*a*d*(2*d*e + c*f))*x*Sqrt[e + f*x^2])/(15*c^2*(b*c -
a*d)^2*(c + d*x^2)^(3/2)) - (b*Sqrt[d]*(b*e - a*f)*Sqrt[e + f*x^2]*Elliptic
E[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(Sqrt[c]*(b*c - a*d)^3*Sqr
t[c + d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))]) + ((a*d*(8*d^2*e^2 - 3*
c*d*e*f - 2*c^2*f^2) - 3*b*c*(6*d^2*e^2 - 6*c*d*e*f + c^2*f^2))*Sqrt[e + f*
x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(15*c^(5/2)*S
qrt[d]*(b*c - a*d)^2*(d*e - c*f)*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c
 + d*x^2))]) + (e^(3/2)*Sqrt[f]*(3*b*c*(3*d*e - 2*c*f) - a*d*(4*d*e - c*f))
*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(
15*c^3*(b*c - a*d)^2*(d*e - c*f)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt
[e + f*x^2]) + (b^2*e^(3/2)*(b*e - a*f)*Sqrt[c + d*x^2]*EllipticPi[1 - (b*e
)/(a*f), ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(a*c*(b*c - a*d)^3*
Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rule 544

```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Dist[(b*(b*e - a*f))/(b*c - a*d)^2, Int[((c + d*x^2)^(
q + 2)*(e + f*x^2)^(r - 1))/(a + b*x^2), x], x] - Dist[1/(b*c - a*d)^2, Int
[(c + d*x^2)^q*(e + f*x^2)^(r - 1)*(2*b*c*d*e - a*d^2*e - b*c^2*f + d^2*(b*
e - a*f)*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[q, -1] && GtQ[
r, 1]
```

Rule 541

```
Int[Sqrt[(e_) + (f_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[b/(b*c - a*d), Int[Sqrt[e + f*x^2]/((a + b*x^2)*S
qrt[c + d*x^2]), x], x] - Dist[d/(b*c - a*d), Int[Sqrt[e + f*x^2]/(c + d*x^
2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[d/c] && PosQ[f/e]
```

Rule 539

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[(c*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcT
an[Rt[d/c, 2]*x], 1 - (c*f)/(d*e)])/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c
*(e + f*x^2))/(e*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 526

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^q)/(a*b*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p +
 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 525

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
```

```
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\int \frac{\left(e+fx^2\right)^{3/2}}{\left(a+bx^2\right)\left(c+dx^2\right)^{7/2}}\,dx = -\frac{\int \frac{\sqrt{e+fx^2}\left(2bcde-ad^2e-bc^2f+d^2(be-af)x^2\right)}{\left(c+dx^2\right)^{7/2}}\,dx}{(bc-ad)^2} + \frac{(b(be-af))\int \frac{\sqrt{e+fx^2}}{\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}\,dx}{(bc-ad)^2}$$

$$= -\frac{(de-cf)x\sqrt{e+fx^2}}{5c(bc-ad)\left(c+dx^2\right)^{5/2}} + \frac{\int \frac{-de(bc(9de-4cf)-ad(4de+cf))-df(bc(8de-3cf)-ad(3de+2cf))x^2}{\left(c+dx^2\right)^{5/2}\sqrt{e+fx^2}}\,dx}{5cd(bc-ad)^2} + \cdots$$

$$= -\frac{(de-cf)x\sqrt{e+fx^2}}{5c(bc-ad)\left(c+dx^2\right)^{5/2}} - \frac{(3bc(3de-cf)-2ad(2de+cf))x\sqrt{e+fx^2}}{15c^2(bc-ad)^2\left(c+dx^2\right)^{3/2}} - \frac{b\sqrt{d}(be-af)\cdots}{\sqrt{c}(bc\cdots}$$

$$= -\frac{(de-cf)x\sqrt{e+fx^2}}{5c(bc-ad)\left(c+dx^2\right)^{5/2}} - \frac{(3bc(3de-cf)-2ad(2de+cf))x\sqrt{e+fx^2}}{15c^2(bc-ad)^2\left(c+dx^2\right)^{3/2}} - \frac{b\sqrt{d}(be-af)\cdots}{\sqrt{c}(bc\cdots}$$

$$= -\frac{(de-cf)x\sqrt{e+fx^2}}{5c(bc-ad)\left(c+dx^2\right)^{5/2}} - \frac{(3bc(3de-cf)-2ad(2de+cf))x\sqrt{e+fx^2}}{15c^2(bc-ad)^2\left(c+dx^2\right)^{3/2}} - \frac{b\sqrt{d}(be-af)\cdots}{\sqrt{c}(bc\cdots}$$

Mathematica [C] time = 2.89116, size = 570, normalized size = 0.89

$$\frac{-ax\sqrt{\frac{d}{c}}\left(e+fx^2\right)\left(\left(c+dx^2\right)^2\left(a^2d^2\left(-2c^2f^2-3cdef+8d^2e^2\right)+2abcd\left(7c^2f^2+3cdef-13d^2e^2\right)+3b^2c^2\left(c^2f^2-11cd\cdots\right.\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x^2)^(3/2)/((a + b*x^2)*(c + d*x^2)^(7/2)),x]
```

```
[Out] (-(a*Sqrt[d/c]*x*(e + f*x^2)*(3*c^2*(b*c - a*d)^2*(d*e - c*f)^2 + c*(b*c -
a*d)*(-(d*e) + c*f)*(3*b*c*(-3*d*e + c*f) + 2*a*d*(2*d*e + c*f))*(c + d*x^2
) + (a^2*d^2*(8*d^2*e^2 - 3*c*d*e*f - 2*c^2*f^2) + 3*b^2*c^2*(11*d^2*e^2 -
11*c*d*e*f + c^2*f^2) + 2*a*b*c*d*(-13*d^2*e^2 + 3*c*d*e*f + 7*c^2*f^2))*(c
 + d*x^2)^2)) + I*(c + d*x^2)^2*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*(a*
e*(-3*b^2*c^2*(11*d^2*e^2 - 11*c*d*e*f + c^2*f^2) + a^2*d^2*(-8*d^2*e^2 + 3
*c*d*e*f + 2*c^2*f^2) - 2*a*b*c*d*(-13*d^2*e^2 + 3*c*d*e*f + 7*c^2*f^2))*El
lipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + (d*e - c*f)*(a*(3*b^2*c^2*e*
(11*d*e - 8*c*f) + a^2*d^2*e*(8*d*e + c*f) + a*b*c*(-26*d^2*e^2 - 7*c*d*e*f
 + 15*c^2*f^2))*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - 15*b*c^3*(
b*e - a*f)^2*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]))
)/(15*a*c^3*Sqrt[d/c]*(b*c - a*d)^3*(d*e - c*f)*(c + d*x^2)^(5/2)*Sqrt[e +
f*x^2])
```

Maple [B] time = 0.062, size = 6211, normalized size = 9.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e)^(3/2)/(b*x^2+a)/(d*x^2+c)^(7/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx^2+e)^{\frac{3}{2}}}{(bx^2+a)(dx^2+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e)^(3/2)/(b*x^2+a)/(d*x^2+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x^2 + e)^(3/2)/((b*x^2 + a)*(d*x^2 + c)^(7/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e)^(3/2)/(b*x^2+a)/(d*x^2+c)^(7/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e)**(3/2)/(b*x**2+a)/(d*x**2+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx^2+e)^{\frac{3}{2}}}{(bx^2+a)(dx^2+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e)^(3/2)/(b*x^2+a)/(d*x^2+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((f*x^2 + e)^(3/2)/((b*x^2 + a)*(d*x^2 + c)^(7/2)), x)
```

3.77 $\int \frac{\left(c+dx^2\right)^{5/2}}{\left(a+bx^2\right)\sqrt{e+fx^2}}\,dx$

Optimal. Leaf size=621

$$\frac{d\sqrt{e}\sqrt{c+dx^2}(bc-ad)\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{b^2\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}-\frac{d\sqrt{e}\sqrt{c+dx^2}(de-3cf)\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{3bf^{3/2}\sqrt{e+fx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}+$$

```
[Out] (d*(b*c - a*d)*x*Sqrt[c + d*x^2])/(b^2*Sqrt[e + f*x^2]) - (2*d*(d*e - 2*c*f
)*x*Sqrt[c + d*x^2])/(3*b*f*Sqrt[e + f*x^2]) + (d^2*x*Sqrt[c + d*x^2]*Sqrt[
e + f*x^2])/(3*b*f) - (d*(b*c - a*d)*Sqrt[e]*Sqrt[c + d*x^2]*EllipticE[ArcT
an[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(b^2*Sqrt[f]*Sqrt[(e*(c + d*x^2)
)/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (2*d*Sqrt[e]*(d*e - 2*c*f)*Sqrt[c + d
*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(3*b*f^(3/2)
*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (d*(b*c - a*d)*Sq
rt[e]*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f
)])/(b^2*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) - (
d*Sqrt[e]*(d*e - 3*c*f)*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e
]], 1 - (d*e)/(c*f)])/(3*b*f^(3/2)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sq
rt[e + f*x^2]) + (c^(3/2)*(b*c - a*d)^2*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c
)/(a*d), ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(a*b^2*Sqrt[d]*e*Sq
rt[c + d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))])
```

Rubi [A] time = 0.462658, antiderivative size = 621, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {545, 416, 531, 418, 492, 411, 422, 539}

$$\frac{c^{3/2}\sqrt{e+fx^2}(bc-ad)^2\Pi\left(1-\frac{bc}{ad};\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{ab^2\sqrt{d}e\sqrt{c+dx^2}\sqrt{\frac{c\left(e+fx^2\right)}{e\left(c+dx^2\right)}}}+\frac{dx\sqrt{c+dx^2}(bc-ad)}{b^2\sqrt{e+fx^2}}+\frac{d\sqrt{e}\sqrt{c+dx^2}(bc-ad)F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\right.}{b^2\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^(5/2)/((a + b*x^2)*Sqrt[e + f*x^2]),x]
```


```
[Out] (d*(b*c - a*d)*x*Sqrt[c + d*x^2])/(b^2*Sqrt[e + f*x^2]) - (2*d*(d*e - 2*c*f
)*x*Sqrt[c + d*x^2])/(3*b*f*Sqrt[e + f*x^2]) + (d^2*x*Sqrt[c + d*x^2]*Sqrt[
e + f*x^2])/(3*b*f) - (d*(b*c - a*d)*Sqrt[e]*Sqrt[c + d*x^2]*EllipticE[ArcT
an[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(b^2*Sqrt[f]*Sqrt[(e*(c + d*x^2)
)/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (2*d*Sqrt[e]*(d*e - 2*c*f)*Sqrt[c + d
*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(3*b*f^(3/2)
*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (d*(b*c - a*d)*Sq
rt[e]*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f
)])/(b^2*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) - (
d*Sqrt[e]*(d*e - 3*c*f)*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e
]], 1 - (d*e)/(c*f)])/(3*b*f^(3/2)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sq
rt[e + f*x^2]) + (c^(3/2)*(b*c - a*d)^2*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c
)/(a*d), ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(a*b^2*Sqrt[d]*e*Sq
rt[c + d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))])
```


Rule 545

```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Dist[d/b, Int[(c + d*x^2)^(q - 1)*(e + f*x^2)^r, x], x
] + Dist[(b*c - a*d)/b, Int[((c + d*x^2)^(q - 1)*(e + f*x^2)^r)/(a + b*x^2)
, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && GtQ[q, 1]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 539

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[(c*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcT
an[Rt[d/c, 2]*x], 1 - (c*f)/(d*e)])/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c
*(e + f*x^2))/(e*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rubi steps

$$\int \frac{\left(c+dx^2\right)^{5/2}}{\left(a+bx^2\right)\sqrt{e+fx^2}}\,dx = \frac{d\int \frac{\left(c+dx^2\right)^{3/2}}{\sqrt{e+fx^2}}dx}{b} + \frac{(bc-ad)\int \frac{\left(c+dx^2\right)^{3/2}}{\left(a+bx^2\right)\sqrt{e+fx^2}}dx}{b}$$

$$= \frac{d^2x\sqrt{c+dx^2}\sqrt{e+fx^2}}{3bf} + \frac{(d(bc-ad))\int \frac{\sqrt{c+dx^2}}{\sqrt{e+fx^2}}dx}{b^2} + \frac{(bc-ad)^2\int \frac{\sqrt{c+dx^2}}{\left(a+bx^2\right)\sqrt{e+fx^2}}dx}{b^2} + \frac{d\int \cdots}{\cdots}$$

$$= \frac{d^2x\sqrt{c+dx^2}\sqrt{e+fx^2}}{3bf} + \frac{c^{3/2}(bc-ad)^2\sqrt{e+fx^2}\,\Pi\left(1-\frac{bc}{ad};\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{ab^2\sqrt{d}e\sqrt{c+dx^2}\sqrt{\frac{c\left(e+fx^2\right)}{e\left(c+dx^2\right)}}} + \frac{(cd(bc \cdots}{\cdots}$$

$$= \frac{d(bc-ad)x\sqrt{c+dx^2}}{b^2\sqrt{e+fx^2}} - \frac{2d(de-2cf)x\sqrt{c+dx^2}}{3bf\sqrt{e+fx^2}} + \frac{d^2x\sqrt{c+dx^2}\sqrt{e+fx^2}}{3bf} + \frac{d(bc-ad)\sqrt{e\cdots}}{b^2\cdots}$$

$$= \frac{d(bc-ad)x\sqrt{c+dx^2}}{b^2\sqrt{e+fx^2}} - \frac{2d(de-2cf)x\sqrt{c+dx^2}}{3bf\sqrt{e+fx^2}} + \frac{d^2x\sqrt{c+dx^2}\sqrt{e+fx^2}}{3bf} - \frac{d(bc-ad)\sqrt{e\cdots}}{b^2\cdots}$$

Mathematica [C] time = 1.38856, size = 350, normalized size = 0.56

$$\frac{-iad\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\left(3a^2d^2f^2+3abdf(de-3cf)+b^2\left(9c^2f^2-8cdef+2d^2e^2\right)\right)\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{\frac{d}{c}}\right),\frac{cf}{de}\right)+\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^(5/2)/((a + b*x^2)*Sqrt[e + f*x^2]),x]
```

```
[Out] ((-I)*a*b*d^2*e*(-2*b*d*e + 7*b*c*f - 3*a*d*f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 +
 (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - I*a*d*(3*a^2*d
^2*f^2 + 3*a*b*d*f*(d*e - 3*c*f) + b^2*(2*d^2*e^2 - 8*c*d*e*f + 9*c^2*f^2))
*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x],
(c*f)/(d*e)] + f*(a*b^2*c*d*(d/c)^(3/2)*x*(c + d*x^2)*(e + f*x^2) - (3*I)*(
b*c - a*d)^3*f*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*
d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]))/(3*a*b^3*Sqrt[d/c]*f^2*Sqrt[c +
d*x^2]*Sqrt[e + f*x^2])
```

Maple [A] time = 0.026, size = 988, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(5/2)/(b*x^2+a)/(f*x^2+e)^(1/2),x)
```

```
[Out] 1/3*((-d/c)^(1/2)*x^5*a*b^2*d^3*f^2+(-d/c)^(1/2)*x^3*a*b^2*c*d^2*f^2+(-d/c)
^(1/2)*x^3*a*b^2*d^3*e*f+3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Elliptic
F(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^3*d^3*f^2-9*((d*x^2+c)/c)^(1/2)*((f*x^2
```

```
+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*b*c*d^2*f^2+3*((
d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1
/2))*a^2*b*d^3*e*f+9*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-
d/c)^(1/2),(c*f/d/e)^(1/2))*a*b^2*c^2*d*f^2-8*((d*x^2+c)/c)^(1/2)*((f*x^2+e
)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b^2*c*d^2*e*f+2*((d*
x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2
))*a*b^2*d^3*e^2-3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/
c)^(1/2),(c*f/d/e)^(1/2))*a^2*b*d^3*e*f+7*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)
^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b^2*c*d^2*e*f-2*((d*x^2+
c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a
*b^2*d^3*e^2-3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^
(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a^3*d^3*f^2+9*((d*x^2+c)/c)^(1/2)*
((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(
1/2))*a^2*b*c*d^2*f^2-9*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(
x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a*b^2*c^2*d*f^2+3*((d*x^2
+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(
1/2)/(-d/c)^(1/2))*b^3*c^3*f^2+(-d/c)^(1/2)*x*a*b^2*c*d^2*e*f)*(f*x^2+e)^(1
/2)*(d*x^2+c)^(1/2)/a/(-d/c)^(1/2)/f^2/b^3/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^2+c)^{\frac{5}{2}}}{(bx^2+a)\sqrt{fx^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/(b*x^2+a)/(f*x^2+e)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)*sqrt(f*x^2 + e)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/(b*x^2+a)/(f*x^2+e)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c+dx^2)^{\frac{5}{2}}}{(a+bx^2)\sqrt{e+fx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(5/2)/(b*x**2+a)/(f*x**2+e)**(1/2),x)
```

```
[Out] Integral((c + d*x**2)**(5/2)/((a + b*x**2)*sqrt(e + f*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)^{\frac{5}{2}}}{\left(bx^2+a\right)\sqrt{fx^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/(b*x^2+a)/(f*x^2+e)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)*sqrt(f*x^2 + e)), x)
```

3.78 $\int \frac{\left(c+dx^2\right)^{3/2}}{\left(a+bx^2\right)\sqrt{e+fx^2}}\,dx$

Optimal. Leaf size=319

$$\frac{d\sqrt{e}\sqrt{c+dx^2}\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{b\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}+\frac{c^{3/2}\sqrt{e+fx^2}(bc-ad)\Pi\left(1-\frac{bc}{ad};\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{ab\sqrt{d}e\sqrt{c+dx^2}\sqrt{\frac{c\left(e+fx^2\right)}{e\left(c+dx^2\right)}}}+\frac{dx\sqrt{c+dx^2}}{b\sqrt{e+fx^2}}-\frac{d}{}$$

```
[Out] (d*x*Sqrt[c + d*x^2])/(b*Sqrt[e + f*x^2]) - (d*Sqrt[e]*Sqrt[c + d*x^2]*Elli
pticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(b*Sqrt[f]*Sqrt[(e*(c
+ d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (d*Sqrt[e]*Sqrt[c + d*x^2]*El
lipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(b*Sqrt[f]*Sqrt[(e*(
c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (c^(3/2)*(b*c - a*d)*Sqrt[e
 + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f
)/(d*e)])/(a*b*Sqrt[d]*e*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2
))])
```

Rubi [A] time = 0.175511, antiderivative size = 319, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {545, 422, 418, 492, 411, 539}

$$\frac{c^{3/2}\sqrt{e+fx^2}(bc-ad)\Pi\left(1-\frac{bc}{ad};\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{ab\sqrt{d}e\sqrt{c+dx^2}\sqrt{\frac{c\left(e+fx^2\right)}{e\left(c+dx^2\right)}}}+\frac{dx\sqrt{c+dx^2}}{b\sqrt{e+fx^2}}+\frac{d\sqrt{e}\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{b\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}-\frac{d\sqrt{e}\sqrt{c+}}{b\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^(3/2)/((a + b*x^2)*Sqrt[e + f*x^2]),x]
```


```
[Out] (d*x*Sqrt[c + d*x^2])/(b*Sqrt[e + f*x^2]) - (d*Sqrt[e]*Sqrt[c + d*x^2]*Elli
pticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(b*Sqrt[f]*Sqrt[(e*(c
+ d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (d*Sqrt[e]*Sqrt[c + d*x^2]*El
lipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(b*Sqrt[f]*Sqrt[(e*(
c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (c^(3/2)*(b*c - a*d)*Sqrt[e
 + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f
)/(d*e)])/(a*b*Sqrt[d]*e*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2
))])
```


Rule 545

```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Dist[d/b, Int[(c + d*x^2)^(q - 1)*(e + f*x^2)^r, x], x
] + Dist[(b*c - a*d)/b, Int[((c + d*x^2)^(q - 1)*(e + f*x^2)^r)/(a + b*x^2)
, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && GtQ[q, 1]
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 539

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[(c*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcT
an[Rt[d/c, 2]*x], 1 - (c*f)/(d*e)])/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c
*(e + f*x^2))/(e*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rubi steps

$$\int \frac{\left(c+dx^2\right)^{3/2}}{\left(a+bx^2\right)\sqrt{e+fx^2}}\,dx = \frac{d\int \frac{\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx}{b} + \frac{(bc-ad)\int \frac{\sqrt{c+dx^2}}{\left(a+bx^2\right)\sqrt{e+fx^2}}\,dx}{b}$$

$$= \frac{c^{3/2}(bc-ad)\sqrt{e+fx^2}\,\Pi\left(1-\frac{bc}{ad};\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{ab\sqrt{d}e\sqrt{c+dx^2}\sqrt{\frac{c\left(e+fx^2\right)}{e\left(c+dx^2\right)}}} + \frac{(cd)\int \frac{1}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{b} + \frac{d^2\int \frac{}{\sqrt{c\cdots}}}{}$$

$$= \frac{dx\sqrt{c+dx^2}}{b\sqrt{e+fx^2}} + \frac{d\sqrt{e}\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{b\sqrt{f}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}\sqrt{e+fx^2}} + \frac{c^{3/2}(bc-ad)\sqrt{e+fx^2}\,\Pi\left(1-\frac{bc}{ad};\text{ta}\cdots\right.}{ab\sqrt{d}e\sqrt{c+dx^2}\sqrt{\frac{c(e\cdots}{e(c\cdots}}}$$

$$= \frac{dx\sqrt{c+dx^2}}{b\sqrt{e+fx^2}} - \frac{d\sqrt{e}\sqrt{c+dx^2}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{b\sqrt{f}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}\sqrt{e+fx^2}} + \frac{d\sqrt{e}\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\cdots\right.}{b\sqrt{f}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}\sqrt{e+fx^2}}$$

Mathematica [C] time = 0.370925, size = 197, normalized size = 0.62

$$-\frac{i\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\left(-ad(adf-2bcf+bde)\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{\frac{d}{c}}\right),\frac{cf}{de}\right)+abd^2eE\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)+f(bc-\cdots\right.}{ab^2f\sqrt{\frac{d}{c}}\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^(3/2)/((a + b*x^2)*Sqrt[e + f*x^2]),x]
```

```
[Out] ((-I)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*(a*b*d^2*e*EllipticE[I*ArcSin
h[Sqrt[d/c]*x], (c*f)/(d*e)] - a*d*(b*d*e - 2*b*c*f + a*d*f)*EllipticF[I*Ar
cSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + (b*c - a*d)^2*f*EllipticPi[(b*c)/(a*d),
I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]))/(a*b^2*Sqrt[d/c]*f*Sqrt[c + d*x^2]*S
qrt[e + f*x^2])
```

Maple [A] time = 0.021, size = 341, normalized size = 1.1

$$\frac{1}{fb^2a\left(dfx^4+cfx^2+dex^2+ce\right)}\left(-\text{EllipticF}\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)a^2d^2f+2\,\text{EllipticF}\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)abcdf-\text{EllipticF}\left(x\sqrt{\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(3/2)/(b*x^2+a)/(f*x^2+e)^(1/2),x)
```

```
[Out] (-EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*d^2*f+2*EllipticF(x*(-d/c)^
(1/2),(c*f/d/e)^(1/2))*a*b*c*d*f-EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*
a*b*d^2*e+EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b*d^2*e+EllipticPi(x*
(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a^2*d^2*f-2*EllipticPi(x*(-
d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a*b*c*d*f+EllipticPi(x*(-d/c)
^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*b^2*c^2*f)*((f*x^2+e)/e)^(1/2)*((
d*x^2+c)/c)^(1/2)*(f*x^2+e)^(1/2)*(d*x^2+c)^(1/2)/a/b^2/f/(-d/c)^(1/2)/(d*f
*x^4+c*f*x^2+d*e*x^2+c*e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)^{\frac{3}{2}}}{\left(bx^2+a\right)\sqrt{fx^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/(b*x^2+a)/(f*x^2+e)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)*sqrt(f*x^2 + e)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/(b*x^2+a)/(f*x^2+e)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c + dx^2\right)^{\frac{3}{2}}}{\left(a + bx^2\right)\sqrt{e + fx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(3/2)/(b*x**2+a)/(f*x**2+e)**(1/2),x)
```

```
[Out] Integral((c + d*x**2)**(3/2)/((a + b*x**2)*sqrt(e + f*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2 + c\right)^{\frac{3}{2}}}{\left(bx^2 + a\right)\sqrt{fx^2 + e}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/(b*x^2+a)/(f*x^2+e)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)*sqrt(f*x^2 + e)), x)
```

3.79 $\int \frac{\sqrt{c+dx^2}}{(a+bx^2)\sqrt{e+fx^2}}\,dx$

Optimal. Leaf size=102

$$\frac{c^{3/2}\sqrt{e+fx^2}\,\Pi\left(1-\frac{bc}{ad};\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{a\sqrt{d}e\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$

```
[Out] (c^(3/2)*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcTan[(Sqrt[d]*x)/Sqr
t[c]], 1 - (c*f)/(d*e)])/(a*Sqrt[d]*e*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*x^2))/
(e*(c + d*x^2))])
```

Rubi [A] time = 0.0326949, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.031, Rules used = {539}

$$\frac{c^{3/2}\sqrt{e+fx^2}\,\Pi\left(1-\frac{bc}{ad};\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{a\sqrt{d}e\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^2]/((a + b*x^2)*Sqrt[e + f*x^2]),x]
```

```
[Out] (c^(3/2)*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcTan[(Sqrt[d]*x)/Sqr
t[c]], 1 - (c*f)/(d*e)])/(a*Sqrt[d]*e*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*x^2))/
(e*(c + d*x^2))])
```

Rule 539

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[(c*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcT
an[Rt[d/c, 2]*x], 1 - (c*f)/(d*e)])/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c
*(e + f*x^2))/(e*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^2}}{\left(a+bx^2\right)\sqrt{e+fx^2}}\,dx=\frac{c^{3/2}\sqrt{e+fx^2}\,\Pi\left(1-\frac{bc}{ad};\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{a\sqrt{d}e\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$

Mathematica [C] time = 0.230414, size = 143, normalized size = 1.4

$$-\frac{i\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\left(ad\,\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{\frac{d}{c}}\right),\frac{cf}{de}\right)+(bc-ad)\Pi\left(\frac{bc}{ad};i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)\right)}{ab\sqrt{\frac{d}{c}}\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^2]/((a + b*x^2)*Sqrt[e + f*x^2]),x]
```

```
[Out] ((-I)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*(a*d*EllipticF[I*ArcSinh[Sqrt
[d/c]*x], (c*f)/(d*e)] + (b*c - a*d)*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt
[d/c]*x], (c*f)/(d*e)]))/(a*b*Sqrt[d/c]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Maple [A] time = 0.018, size = 191, normalized size = 1.9

$$\frac{1}{ab\left(dfx^{4}+cfx^{2}+dex^{2}+ce\right)}\left(\text{EllipticF}\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)ad-\text{EllipticPi}\left(x\sqrt{-\frac{d}{c}},\frac{bc}{ad},\sqrt{-\frac{f}{e}}\frac{1}{\sqrt{-\frac{d}{c}}}\right)ad+\text{EllipticPi}\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)/(b*x^2+a)/(f*x^2+e)^(1/2),x)
```

```
[Out] (EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d-EllipticPi(x*(-d/c)^(1/2),b*
c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a*d+EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/
e)^(1/2)/(-d/c)^(1/2))*b*c)/b*((f*x^2+e)/e)^(1/2)*((d*x^2+c)/c)^(1/2)*(f*x^
2+e)^(1/2)*(d*x^2+c)^(1/2)/a/(-d/c)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^{2}+c}}{\left(bx^{2}+a\right)\sqrt{fx^{2}+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)/(f*x^2+e)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)/((b*x^2 + a)*sqrt(f*x^2 + e)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)/(f*x^2+e)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+dx^{2}}}{\left(a+bx^{2}\right)\sqrt{e+fx^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)/(b*x**2+a)/(f*x**2+e)**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*x**2)/((a + b*x**2)*sqrt(e + f*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2+c}}{(bx^2+a)\sqrt{fx^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)/(f*x^2+e)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^2 + c)/((b*x^2 + a)*sqrt(f*x^2 + e)), x)
```

3.80 $\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$

Optimal. Leaf size=100

$$\frac{\sqrt{-c}\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\Pi\left(\frac{bc}{ad};\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{-c}}\right)|\frac{cf}{de}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

```
[Out] (Sqrt[-c]*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), A
rcSin[(Sqrt[d]*x)/Sqrt[-c]], (c*f)/(d*e)])/(a*Sqrt[d]*Sqrt[c + d*x^2]*Sqrt[
e + f*x^2])
```

Rubi [A] time = 0.120808, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {538, 537}

$$\frac{\sqrt{-c}\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\Pi\left(\frac{bc}{ad};\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{-c}}\right)|\frac{cf}{de}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```


```
[Out] (Sqrt[-c]*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), A
rcSin[(Sqrt[d]*x)/Sqrt[-c]], (c*f)/(d*e)])/(a*Sqrt[d]*Sqrt[c + d*x^2]*Sqrt[
e + f*x^2])
```


Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \frac{\sqrt{1+\frac{dx^2}{c}}\int \frac{1}{(a+bx^2)\sqrt{1+\frac{dx^2}{c}}\sqrt{e+fx^2}}\,dx}{\sqrt{c+dx^2}}$$
$$= \frac{\left(\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\right)\int \frac{1}{(a+bx^2)\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}}\,dx}{\sqrt{c+dx^2}\sqrt{e+fx^2}}$$
$$= \frac{\sqrt{-c}\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\Pi\left(\frac{bc}{ad};\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{-c}}\right)|\frac{cf}{de}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

Mathematica [C] time = 0.220757, size = 101, normalized size = 1.01

$$-\frac{i\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\Pi\left(\frac{bc}{ad};i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)}{a\sqrt{\frac{d}{c}}\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

```
[Out] ((-I)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*Arc
Sinh[Sqrt[d/c]*x], (c*f)/(d*e)])/(a*Sqrt[d/c]*Sqrt[c + d*x^2]*Sqrt[e + f*x^
2])
```

Maple [A] time = 0.021, size = 118, normalized size = 1.2

$$\frac{1}{a\left(dfx^4+cfx^2+dex^2+ce\right)}\mathrm{EllipticPi}\left(x\sqrt{-\frac{d}{c}},\frac{bc}{ad},\sqrt{-\frac{f}{e}}\frac{1}{\sqrt{-\frac{d}{c}}}\right)\sqrt{\frac{fx^2+e}{e}}\sqrt{\frac{dx^2+c}{c}}\sqrt{fx^2+e}\sqrt{dx^2+c}\frac{1}{\sqrt{-\frac{d}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

```
[Out] EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*((f*x^2+e)/e)^
(1/2)*((d*x^2+c)/c)^(1/2)*(f*x^2+e)^(1/2)*(d*x^2+c)^(1/2)/a/(-d/c)^(1/2)/(d
*f*x^4+c*f*x^2+d*e*x^2+c*e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)/(d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x**2)*sqrt(c + d*x**2)*sqrt(e + f*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)\sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

3.81 $\int \frac{1}{(a+bx^2)(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx$

Optimal. Leaf size=344

$$-\frac{d\sqrt{e}\sqrt{c+dx^2}(adf-2bcf+bde)\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{e+fx^2}(bc-ad)^2(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{b^2c^{3/2}\sqrt{e+fx^2}\Pi\left(1-\frac{bc}{ad};\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{a\sqrt{d}e\sqrt{c+dx^2}(bc-ad)^2\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}-\frac{a}{\sqrt{c}}$$

```
[Out] -((d^(3/2)*Sqrt[e + f*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)
/(d*e)])/(Sqrt[c]*(b*c - a*d)*(d*e - c*f)*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*x^
2))/(e*(c + d*x^2))])) - (d*Sqrt[e]*(b*d*e - 2*b*c*f + a*d*f)*Sqrt[c + d*x^
2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(c*(b*c - a*d)^
2*Sqrt[f]*(d*e - c*f)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]
) + (b^2*c^(3/2)*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcTan[(Sqrt[d
]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(a*Sqrt[d]*(b*c - a*d)^2*e*Sqrt[c + d*x^2]
*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))])
```

Rubi [A] time = 0.220375, antiderivative size = 344, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {546, 539, 525, 418, 411}

$$\frac{b^2c^{3/2}\sqrt{e+fx^2}\Pi\left(1-\frac{bc}{ad};\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{a\sqrt{d}e\sqrt{c+dx^2}(bc-ad)^2\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}-\frac{d^{3/2}\sqrt{e+fx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\sqrt{c+dx^2}(bc-ad)(de-cf)\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}-\frac{d\sqrt{e}\sqrt{c+dx^2}(adf-2bcf+b}{c\sqrt{f}\sqrt{e+fx^2}(bc-ad}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)*(c + d*x^2)^(3/2)*Sqrt[e + f*x^2]),x]
```



```
[Out] -((d^(3/2)*Sqrt[e + f*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)
/(d*e)])/(Sqrt[c]*(b*c - a*d)*(d*e - c*f)*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*x^
2))/(e*(c + d*x^2))])) - (d*Sqrt[e]*(b*d*e - 2*b*c*f + a*d*f)*Sqrt[c + d*x^
2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(c*(b*c - a*d)^
2*Sqrt[f]*(d*e - c*f)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]
) + (b^2*c^(3/2)*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcTan[(Sqrt[d
]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(a*Sqrt[d]*(b*c - a*d)^2*e*Sqrt[c + d*x^2]
*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))])
```


Rule 546

```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Dist[b^2/(b*c - a*d)^2, Int[((c + d*x^2)^(q + 2)*(e +
f*x^2)^r)/(a + b*x^2), x], x] - Dist[d/(b*c - a*d)^2, Int[(c + d*x^2)^q*(e
+ f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r},
x] && LtQ[q, -1]
```

Rule 539

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[(c*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcT
an[Rt[d/c, 2]*x], 1 - (c*f)/(d*e)])/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c
*(e + f*x^2))/(e*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
```

```
[d/c]
```

Rule 525

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}}\,dx = \frac{b^2\int \frac{\sqrt{c+dx^2}}{\left(a+bx^2\right)\sqrt{e+fx^2}}\,dx}{(bc-ad)^2} - \frac{d\int \frac{2bc-ad+bdx^2}{\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}}\,dx}{(bc-ad)^2}$$

$$= \frac{b^2c^{3/2}\sqrt{e+fx^2}\,\Pi\left(1-\frac{bc}{ad};\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{a\sqrt{d}(bc-ad)^2e\sqrt{c+dx^2}\sqrt{\frac{c\left(e+fx^2\right)}{e\left(c+dx^2\right)}}} - \frac{d^2\int \frac{\sqrt{e+fx^2}}{\left(c+dx^2\right)^{3/2}}\,dx}{(bc-ad)(de-cf)} - \frac{(d(bde}{}$$

$$= -\frac{d^{3/2}\sqrt{e+fx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}(bc-ad)(de-cf)\sqrt{c+dx^2}\sqrt{\frac{c\left(e+fx^2\right)}{e\left(c+dx^2\right)}}} - \frac{d\sqrt{e}(bde-2bcf+adf)\sqrt{c+dx^2}}{c(bc-ad)^2\sqrt{f}(de-cf)\sqrt{}}$$

Mathematica [C] time = 0.787321, size = 365, normalized size = 1.06

$$\frac{\sqrt{\frac{d}{c}}\left(iad\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}(cf-de)\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{\frac{d}{c}}\right),\frac{cf}{de}\right)-ibc^2f\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\Pi\left(\frac{bc}{ad};i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\right.\right.}{ad\sqrt{c+d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)*(c + d*x^2)^(3/2)*Sqrt[e + f*x^2]),x]
```

```
[Out] (Sqrt[d/c]*(a*c*d*(d/c)^(3/2)*e*x + a*c*d*(d/c)^(3/2)*f*x^3 + I*a*d^2*e*Sqr
t[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f
)/(d*e)] + I*a*d*(-(d*e) + c*f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*Ell
ipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + I*b*c*d*e*Sqrt[1 + (d*x^2)/c]
*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/
(d*e)] - I*b*c^2*f*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)
```

```
/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]))/(a*d*(-(b*c) + a*d)*(d*e - c
*f)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Maple [A] time = 0.03, size = 413, normalized size = 1.2

$$\frac{1}{ac\left(ad-bc\right)\left(cf-de\right)\left(dfx^4+cfx^2+dex^2+ce\right)}\left(-x^3ad^2f\sqrt{-\frac{d}{c}}+\mathrm{EllipticF}\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)acdf\sqrt{\frac{dx^2+c}{c}}\sqrt{\frac{fx^2+e}{e}}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)/(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x)
```

```
[Out] (-x^3*a*d^2*f*(-d/c)^(1/2)+EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c*d*
f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-EllipticF(x*(-d/c)^(1/2),(c*f/d/e
)^(1/2))*a*d^2*e*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+EllipticE(x*(-d/c)
^(1/2),(c*f/d/e)^(1/2))*a*d^2*e*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-Ell
ipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*b*c^2*f*((d*x^2+c
)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/
2)/(-d/c)^(1/2))*b*c*d*e*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-x*a*d^2*e*
(-d/c)^(1/2))*(f*x^2+e)^(1/2)*(d*x^2+c)^(1/2)/a/c/(-d/c)^(1/2)/(a*d-b*c)/(c
*f-d*e)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)\left(dx^2+c\right)^{\frac{3}{2}}\sqrt{fx^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((b*x^2 + a)*(d*x^2 + c)^(3/2)*sqrt(f*x^2 + e)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+bx^2\right)\left(c+dx^2\right)^{\frac{3}{2}}\sqrt{e+fx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)/(d*x**2+c)**(3/2)/(f*x**2+e)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x**2)*(c + d*x**2)**(3/2)*sqrt(e + f*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)\left(dx^2+c\right)^{\frac{3}{2}}\sqrt{fx^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)*(d*x^2 + c)^(3/2)*sqrt(f*x^2 + e)), x)
```

3.82 $\int \frac{1}{(a+bx^2)(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx$

Optimal. Leaf size=435

$$-\frac{d\sqrt{e}\sqrt{f}\sqrt{c+dx^2}(ad(de-3cf)-2bc(2de-3cf))\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{3c^2\sqrt{e+fx^2}(bc-ad)^2(de-cf)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{b^2\sqrt{-c}\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\Pi\left(\frac{bc}{ad};\sin\right.}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{e+fx^2}(bc-}$$

```
[Out] -(d^2*x*Sqrt[e + f*x^2])/(3*c*(b*c - a*d)*(d*e - c*f)*(c + d*x^2)^(3/2)) -
(d^(3/2)*(b*c*(5*d*e - 7*c*f) - 2*a*d*(d*e - 2*c*f))*Sqrt[e + f*x^2]*Ellipt
icE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(3*c^(3/2)*(b*c - a*d)^2
*(d*e - c*f)^2*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))]) - (d*
Sqrt[e]*Sqrt[f]*(a*d*(d*e - 3*c*f) - 2*b*c*(2*d*e - 3*c*f))*Sqrt[c + d*x^2]
*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(3*c^2*(b*c - a*d
)^2*(d*e - c*f)^2*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) +
(b^2*Sqrt[-c]*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d
), ArcSin[(Sqrt[d]*x)/Sqrt[-c]], (c*f)/(d*e)])/(a*Sqrt[d]*(b*c - a*d)^2*Sqr
t[c + d*x^2]*Sqrt[e + f*x^2])
```

Rubi [A] time = 0.554213, antiderivative size = 435, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {546, 538, 537, 527, 525, 418, 411}

$$\frac{b^2\sqrt{-c}\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\Pi\left(\frac{bc}{ad};\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{-c}}\right)|\frac{cf}{de}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{e+fx^2}(bc-ad)^2}-\frac{d^{3/2}\sqrt{e+fx^2}(bc(5de-7cf)-2ad(de-2cf))E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{3c^{3/2}\sqrt{c+dx^2}(bc-ad)^2(de-cf)^2\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}-$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)*(c + d*x^2)^(5/2)*Sqrt[e + f*x^2]),x]
```


```
[Out] -(d^2*x*Sqrt[e + f*x^2])/(3*c*(b*c - a*d)*(d*e - c*f)*(c + d*x^2)^(3/2)) -
(d^(3/2)*(b*c*(5*d*e - 7*c*f) - 2*a*d*(d*e - 2*c*f))*Sqrt[e + f*x^2]*Ellipt
icE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(3*c^(3/2)*(b*c - a*d)^2
*(d*e - c*f)^2*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))]) - (d*
Sqrt[e]*Sqrt[f]*(a*d*(d*e - 3*c*f) - 2*b*c*(2*d*e - 3*c*f))*Sqrt[c + d*x^2]
*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(3*c^2*(b*c - a*d
)^2*(d*e - c*f)^2*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) +
(b^2*Sqrt[-c]*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d
), ArcSin[(Sqrt[d]*x)/Sqrt[-c]], (c*f)/(d*e)])/(a*Sqrt[d]*(b*c - a*d)^2*Sqr
t[c + d*x^2]*Sqrt[e + f*x^2])
```


Rule 546

```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Dist[b^2/(b*c - a*d)^2, Int[((c + d*x^2)^(q + 2)*(e +
f*x^2)^r)/(a + b*x^2), x], x] - Dist[d/(b*c - a*d)^2, Int[(c + d*x^2)^q*(e
+ f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r},
x] && LtQ[q, -1]
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 525

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{5/2}\sqrt{e+fx^2}}dx = \frac{b^2\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}\sqrt{e+fx^2}}dx}{(bc-ad)^2} - \frac{d\int \frac{2bc-ad+bdx^2}{(c+dx^2)^{5/2}\sqrt{e+fx^2}}dx}{(bc-ad)^2}$$

$$= -\frac{d^2x\sqrt{e+fx^2}}{3c(bc-ad)(de-cf)(c+dx^2)^{3/2}} + \frac{d\int \frac{-bc(5de-6cf)+ad(2de-3cf)-d(bc-ad)fx^2}{(c+dx^2)^{3/2}\sqrt{e+fx^2}}dx}{3c(bc-ad)^2(de-cf)} + \cdots$$

$$= -\frac{d^2x\sqrt{e+fx^2}}{3c(bc-ad)(de-cf)(c+dx^2)^{3/2}} - \frac{(df(ad(de-3cf)-2bc(2de-3cf)))\int \frac{}{\sqrt{c+d\cdots}}}{3c(bc-ad)^2(de-cf)^2}$$

$$= -\frac{d^2x\sqrt{e+fx^2}}{3c(bc-ad)(de-cf)(c+dx^2)^{3/2}} - \frac{d^{3/2}(bc(5de-7cf)-2ad(de-2cf))\sqrt{e+f\cdots}}{3c^{3/2}(bc-ad)^2(de-cf)^2\sqrt{c+\cdots}}$$

Mathematica [C] time = 3.42181, size = 433, normalized size = 1.

$$\frac{iad(c+dx^2)\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}(cf-de)(ad(2de-3cf)+bc(6cf-5de))\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{\frac{d}{c}}\right),\frac{cf}{de}\right)-3ib^2c^2(c+dx^2\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)*(c + d*x^2)^(5/2)*Sqrt[e + f*x^2]),x]
```

```
[Out] (a*c*d*(d/c)^(3/2)*x*(e + f*x^2)*(b*c*(-6*c*d*e + 8*c^2*f - 5*d^2*e*x^2 + 7
*c*d*f*x^2) + a*d*(-5*c^2*f + 2*d^2*e*x^2 + c*d*(3*e - 4*f*x^2))) + I*a*d^2
*e*(2*a*d*(d*e - 2*c*f) + b*c*(-5*d*e + 7*c*f))*(c + d*x^2)*Sqrt[1 + (d*x^2
)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + I
*a*d*(-(d*e) + c*f)*(a*d*(2*d*e - 3*c*f) + b*c*(-5*d*e + 6*c*f))*(c + d*x^2
)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x],
 (c*f)/(d*e)] - (3*I)*b^2*c^2*(d*e - c*f)^2*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]
*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/
(d*e)])/(3*a*c^2*Sqrt[d/c]*(b*c - a*d)^2*(d*e - c*f)^2*(c + d*x^2)^(3/2)*Sq
rt[e + f*x^2])
```

Maple [B] time = 0.043, size = 2062, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)/(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x)
```

```
[Out] 1/3*(-4*x^5*a^2*c*d^4*f^2*(-d/c)^(1/2)+2*x^3*a^2*d^5*e^2*(-d/c)^(1/2)-2*Ell
ipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*c*d^4*e^2*((d*x^2+c)/c)^(1/2)*((
f*x^2+e)/e)^(1/2)+7*x^5*a*b*c^2*d^3*f^2*(-d/c)^(1/2)-x^3*a^2*c*d^4*e*f*(-d/
c)^(1/2)+2*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a^2*d^5*e^2*((d*x^
2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-2*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2
))*x^2*a^2*d^5*e^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+3*EllipticPi(x*(
```

```
-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*b^2*c^3*d^2*e^2*((d*x^2+c)/c
)^(1/2)*((f*x^2+e)/e)^(1/2)+3*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2
*c^3*d^2*f^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+2*EllipticF(x*(-d/c)^(
1/2),(c*f/d/e)^(1/2))*a^2*c*d^4*e^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)
+8*x^3*a*b*c^3*d^2*f^2*(-d/c)^(1/2)-5*x^3*a*b*c*d^4*e^2*(-d/c)^(1/2)-6*Elli
pticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a*b*c^3*d^2*f^2*((d*x^2+c)/c)^(1/
2)*((f*x^2+e)/e)^(1/2)-5*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a*b*
c*d^4*e^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+4*EllipticE(x*(-d/c)^(1/2
),(c*f/d/e)^(1/2))*x^2*a^2*c*d^4*e*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2
)+5*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a*b*c*d^4*e^2*((d*x^2+c)/
c)^(1/2)*((f*x^2+e)/e)^(1/2)+11*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a
*b*c^3*d^2*e*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-7*EllipticE(x*(-d/c)
^(1/2),(c*f/d/e)^(1/2))*a*b*c^3*d^2*e*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(
1/2)+8*x*a*b*c^3*d^2*e*f*(-d/c)^(1/2)+2*x^5*a^2*d^5*e*f*(-d/c)^(1/2)-5*x^3*
a^2*c^2*d^3*f^2*(-d/c)^(1/2)+3*x*a^2*c*d^4*e^2*(-d/c)^(1/2)+3*EllipticPi(x*
(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*b^2*c^5*f^2*((d*x^2+c)/c)^(
1/2)*((f*x^2+e)/e)^(1/2)-5*x*a^2*c^2*d^3*e*f*(-d/c)^(1/2)-6*x*a*b*c^2*d^3*e
^2*(-d/c)^(1/2)-6*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/
2))*x^2*b^2*c^3*d^2*e*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-5*EllipticF
(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a^2*c*d^4*e*f*((d*x^2+c)/c)^(1/2)*((f*
x^2+e)/e)^(1/2)+11*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a*b*c^2*d^
3*e*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-7*EllipticE(x*(-d/c)^(1/2),(c
*f/d/e)^(1/2))*x^2*a*b*c^2*d^3*e*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+
3*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*x^2*b^2*c^4*
d*f^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+3*EllipticPi(x*(-d/c)^(1/2),b
*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*x^2*b^2*c^2*d^3*e^2*((d*x^2+c)/c)^(1/2)*(
(f*x^2+e)/e)^(1/2)-5*x^5*a*b*c*d^4*e*f*(-d/c)^(1/2)+x^3*a*b*c^2*d^3*e*f*(-d
/c)^(1/2)+3*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a^2*c^2*d^3*f^2*(
(d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-6*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,
(-f/e)^(1/2)/(-d/c)^(1/2))*b^2*c^4*d*e*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^
(1/2)-5*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*c^2*d^3*e*f*((d*x^2+c
)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-6*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*
a*b*c^4*d*f^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-5*EllipticF(x*(-d/c)^
(1/2),(c*f/d/e)^(1/2))*a*b*c^2*d^3*e^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1
/2)+4*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*c^2*d^3*e*f*((d*x^2+c)/
c)^(1/2)*((f*x^2+e)/e)^(1/2)+5*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*
b*c^2*d^3*e^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2))/(f*x^2+e)^(1/2)/(c*f
-d*e)^2/(a*d-b*c)^2/(-d/c)^(1/2)/c^2/a/(d*x^2+c)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)\left(dx^2+c\right)^{\frac{5}{2}}\sqrt{fx^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((b*x^2 + a)*(d*x^2 + c)^(5/2)*sqrt(f*x^2 + e)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{\frac{5}{2}}\sqrt{e+fx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)/(d*x**2+c)**(5/2)/(f*x**2+e)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x**2)*(c + d*x**2)**(5/2)*sqrt(e + f*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+a)(dx^2+c)^{\frac{5}{2}}\sqrt{fx^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)*(d*x^2 + c)^(5/2)*sqrt(f*x^2 + e)), x)
```

3.83 $\int \frac{\left(c+dx^2\right)^{5/2}}{\left(a+bx^2\right)\left(e+fx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=980

$$\frac{e^{3/2}\sqrt{dx^2+c}\,\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)(bc-ad)^3}{abc\sqrt{f}(be-af)^2\sqrt{\frac{e\left(dx^2+c\right)}{c\left(fx^2+e\right)}}\sqrt{fx^2+e}}-\frac{\sqrt{e}(bde+4bcf-3adf)\sqrt{dx^2+c}\,E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)(bc-}{3b\sqrt{f}(be-af)^2\sqrt{\frac{e\left(dx^2+c\right)}{c\left(fx^2+e\right)}}\sqrt{fx^2+e}}$$

```
[Out] ((b*c - a*d)*(b*d*e + 4*b*c*f - 3*a*d*f)*x*Sqrt[c + d*x^2])/(3*b*(b*e - a*f
)^2*Sqrt[e + f*x^2]) + ((b*e*(6*d^2*e^2 - 7*c*d*e*f - c^2*f^2) - a*f*(8*d^2
*e^2 - 13*c*d*e*f + 3*c^2*f^2))*x*Sqrt[c + d*x^2])/(3*e*f*(b*e - a*f)^2*Sqr
t[e + f*x^2]) + ((d*e - c*f)*x*(c + d*x^2)^(3/2))/(e*(b*e - a*f)*Sqrt[e + f
*x^2]) + (d*(b*c - a*d)*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(3*(b*e - a*f)^2
) + (d*(a*f*(4*d*e - 3*c*f) - b*e*(3*d*e - 2*c*f))*x*Sqrt[c + d*x^2]*Sqrt[e
 + f*x^2])/(3*e*f*(b*e - a*f)^2) - ((b*c - a*d)*Sqrt[e]*(b*d*e + 4*b*c*f -
3*a*d*f)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(
c*f)])/(3*b*Sqrt[f]*(b*e - a*f)^2*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqr
t[e + f*x^2]) - ((b*e*(6*d^2*e^2 - 7*c*d*e*f - c^2*f^2) - a*f*(8*d^2*e^2 -
13*c*d*e*f + 3*c^2*f^2))*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[
e]], 1 - (d*e)/(c*f)])/(3*Sqrt[e]*f^(3/2)*(b*e - a*f)^2*Sqrt[(e*(c + d*x^2)
)/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (d*(5*b*c - 3*a*d)*(b*c - a*d)*e^(3/2
)*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/
(3*b*c*Sqrt[f]*(b*e - a*f)^2*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e +
 f*x^2]) - (Sqrt[e]*(2*a*d*f*(2*d*e - 3*c*f) - b*(3*d^2*e^2 - 2*c*d*e*f - 3
*c^2*f^2))*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)
/(c*f)])/(3*f^(3/2)*(b*e - a*f)^2*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqr
t[e + f*x^2]) + ((b*c - a*d)^3*e^(3/2)*Sqrt[c + d*x^2]*EllipticPi[1 - (b*e)
/(a*f), ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(a*b*c*Sqrt[f]*(b*e
- a*f)^2*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rubi [A] time = 1.11334, antiderivative size = 980, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.281, Rules used = {544, 543, 539, 528, 531, 418, 492, 411, 526}

$$\frac{e^{3/2}\sqrt{dx^2+c}\,\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)(bc-ad)^3}{abc\sqrt{f}(be-af)^2\sqrt{\frac{e\left(dx^2+c\right)}{c\left(fx^2+e\right)}}\sqrt{fx^2+e}}-\frac{\sqrt{e}(bde+4bcf-3adf)\sqrt{dx^2+c}\,E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)(bc-}{3b\sqrt{f}(be-af)^2\sqrt{\frac{e\left(dx^2+c\right)}{c\left(fx^2+e\right)}}\sqrt{fx^2+e}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^(5/2)/((a + b*x^2)*(e + f*x^2)^(3/2)),x]
```

```
[Out] ((b*c - a*d)*(b*d*e + 4*b*c*f - 3*a*d*f)*x*Sqrt[c + d*x^2])/(3*b*(b*e - a*f
)^2*Sqrt[e + f*x^2]) + ((b*e*(6*d^2*e^2 - 7*c*d*e*f - c^2*f^2) - a*f*(8*d^2
*e^2 - 13*c*d*e*f + 3*c^2*f^2))*x*Sqrt[c + d*x^2])/(3*e*f*(b*e - a*f)^2*Sqr
t[e + f*x^2]) + ((d*e - c*f)*x*(c + d*x^2)^(3/2))/(e*(b*e - a*f)*Sqrt[e + f
*x^2]) + (d*(b*c - a*d)*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(3*(b*e - a*f)^2
) + (d*(a*f*(4*d*e - 3*c*f) - b*e*(3*d*e - 2*c*f))*x*Sqrt[c + d*x^2]*Sqrt[e
 + f*x^2])/(3*e*f*(b*e - a*f)^2) - ((b*c - a*d)*Sqrt[e]*(b*d*e + 4*b*c*f -
3*a*d*f)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(
c*f)])/(3*b*Sqrt[f]*(b*e - a*f)^2*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqr
t[e + f*x^2]) - ((b*e*(6*d^2*e^2 - 7*c*d*e*f - c^2*f^2) - a*f*(8*d^2*e^2 -
```

```
13*c*d*e*f + 3*c^2*f^2))*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[
e]], 1 - (d*e)/(c*f)])/(3*Sqrt[e]*f^(3/2)*(b*e - a*f)^2*Sqrt[(e*(c + d*x^2)
)/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (d*(5*b*c - 3*a*d)*(b*c - a*d)*e^(3/2
)*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/
(3*b*c*Sqrt[f]*(b*e - a*f)^2*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e +
 f*x^2]) - (Sqrt[e]*(2*a*d*f*(2*d*e - 3*c*f) - b*(3*d^2*e^2 - 2*c*d*e*f - 3
*c^2*f^2))*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)
/(c*f)])/(3*f^(3/2)*(b*e - a*f)^2*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqr
t[e + f*x^2]) + ((b*c - a*d)^3*e^(3/2)*Sqrt[c + d*x^2]*EllipticPi[1 - (b*e)
/(a*f), ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(a*b*c*Sqrt[f]*(b*e
- a*f)^2*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rule 544

```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Dist[(b*(b*e - a*f))/(b*c - a*d)^2, Int[((c + d*x^2)^(
q + 2)*(e + f*x^2)^(r - 1))/(a + b*x^2), x], x] - Dist[1/(b*c - a*d)^2, Int
[(c + d*x^2)^q*(e + f*x^2)^(r - 1)*(2*b*c*d*e - a*d^2*e - b*c^2*f + d^2*(b*
e - a*f)*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[q, -1] && GtQ[
r, 1]
```

Rule 543

```
Int[(((c_) + (d_.)*(x_)^2)^(3/2)*Sqrt[(e_) + (f_.)*(x_)^2])/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Dist[(b*c - a*d)^2/b^2, Int[Sqrt[e + f*x^2]/((a + b*x^
2)*Sqrt[c + d*x^2]), x], x] + Dist[d/b^2, Int[((2*b*c - a*d + b*d*x^2)*Sqrt
[e + f*x^2])/Sqrt[c + d*x^2], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && Pos
Q[d/c] && PosQ[f/e]
```

Rule 539

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[(c*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcT
an[Rt[d/c, 2]*x], 1 - (c*f)/(d*e)])/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c
*(e + f*x^2))/(e*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 526

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^q)/(a*b*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p +
 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rubi steps

$$\int \frac{\left(c+dx^2\right)^{5/2}}{\left(a+bx^2\right)\left(e+fx^2\right)^{3/2}}\,dx = -\frac{\int \frac{\left(c+dx^2\right)^{3/2}\left(-bde^2+2bcef-acf^2+(bc-ad)f^2x^2\right)}{\left(e+fx^2\right)^{3/2}}\,dx}{(be-af)^2} + \frac{(b(bc-ad))\int \frac{\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}}{a+bx^2}\,dx}{(be-af)^2}$$

$$= \frac{(de-cf)x\left(c+dx^2\right)^{3/2}}{e(be-af)\sqrt{e+fx^2}} + \frac{(d(bc-ad))\int \frac{\left(2bc-ad+bdx^2\right)\sqrt{e+fx^2}}{\sqrt{c+dx^2}}\,dx}{b(be-af)^2} + \frac{(bc-ad)^3\int \frac{\sqrt{e+fx^2}}{\left(a+bx^2\right)\sqrt{c+\cdots}}}{b(be-af)^2}$$

$$= \frac{(de-cf)x\left(c+dx^2\right)^{3/2}}{e(be-af)\sqrt{e+fx^2}} + \frac{d(bc-ad)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{3(be-af)^2} + \frac{d(af(4de-3cf)-be(3de-\cdots}{3ef(be-\cdots}$$

$$= \frac{(de-cf)x\left(c+dx^2\right)^{3/2}}{e(be-af)\sqrt{e+fx^2}} + \frac{d(bc-ad)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{3(be-af)^2} + \frac{d(af(4de-3cf)-be(3de-\cdots}{3ef(be-\cdots}$$

$$= \frac{(bc-ad)(bde+4bcf-3adf)x\sqrt{c+dx^2}}{3b(be-af)^2\sqrt{e+fx^2}} + \frac{\left(be\left(6d^2e^2-7cdef-c^2f^2\right)-af\left(8d^2e^2-13\cdots\right.\right.}{3ef(be-af)^2\sqrt{e+\cdots}}$$

$$= \frac{(bc-ad)(bde+4bcf-3adf)x\sqrt{c+dx^2}}{3b(be-af)^2\sqrt{e+fx^2}} + \frac{\left(be\left(6d^2e^2-7cdef-c^2f^2\right)-af\left(8d^2e^2-13\cdots\right.\right.}{3ef(be-af)^2\sqrt{e+\cdots}}$$

Mathematica [C] time = 1.56713, size = 352, normalized size = 0.36

$$\frac{-iad^2e\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}(be-af)(-adf+3bcf-2bde)\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{\frac{d}{c}}\right),\frac{cf}{de}\right)-f\left(ab^2x\sqrt{\frac{d}{c}}\left(c+dx^2\right)(de-cf)^2\right.}{ab^2ef}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^(5/2)/((a + b*x^2)*(e + f*x^2)^(3/2)),x]
```

```
[Out] ((-I)*a*b*d*e*(-(a*d^2*e*f) + b*(2*d^2*e^2 - 2*c*d*e*f + c^2*f^2))*Sqrt[1 +
 (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*
e)] - I*a*d^2*e*(b*e - a*f)*(-2*b*d*e + 3*b*c*f - a*d*f)*Sqrt[1 + (d*x^2)/c
]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - f*(a
*b^2*Sqrt[d/c]*(d*e - c*f)^2*x*(c + d*x^2) + I*(b*c - a*d)^3*e*f*Sqrt[1 + (
d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x
], (c*f)/(d*e)]))/(a*b^2*Sqrt[d/c]*e*f^2*(b*e - a*f)*Sqrt[c + d*x^2]*Sqrt[e
 + f*x^2])
```

Maple [A] time = 0.032, size = 1063, normalized size = 1.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(5/2)/(b*x^2+a)/(f*x^2+e)^(3/2),x)
```

```
[Out] (x^3*a*b^2*c^2*d*f^3*(-d/c)^(1/2)-2*x^3*a*b^2*c*d^2*e*f^2*(-d/c)^(1/2)+x^3*
a*b^2*d^3*e^2*f*(-d/c)^(1/2)+EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*
b*d^3*e^2*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-EllipticE(x*(-d/c)^(1/2
),(c*f/d/e)^(1/2))*a*b^2*c^2*d*e*f^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2
)+2*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b^2*c*d^2*e^2*f*((d*x^2+c)/
c)^(1/2)*((f*x^2+e)/e)^(1/2)-2*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*
b^2*d^3*e^3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+EllipticPi(x*(-d/c)^(1/
2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a^3*d^3*e*f^2*((d*x^2+c)/c)^(1/2)*((f
*x^2+e)/e)^(1/2)-3*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1
/2))*a^2*b*c*d^2*e*f^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+3*EllipticPi
(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a*b^2*c^2*d*e*f^2*((d*x^
2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^
(1/2)/(-d/c)^(1/2))*b^3*c^3*e*f^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-E
llipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^3*d^3*e*f^2*((d*x^2+c)/c)^(1/2)*
((f*x^2+e)/e)^(1/2)+3*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*b*c*d^2
*e*f^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-EllipticF(x*(-d/c)^(1/2),(c*
f/d/e)^(1/2))*a^2*b*d^3*e^2*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-3*Ell
ipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b^2*c*d^2*e^2*f*((d*x^2+c)/c)^(1/2
)*((f*x^2+e)/e)^(1/2)+2*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b^2*d^3
*e^3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+x*a*b^2*c^3*f^3*(-d/c)^(1/2)-2
*x*a*b^2*c^2*d*e*f^2*(-d/c)^(1/2)+x*a*b^2*c*d^2*e^2*f*(-d/c)^(1/2))*(f*x^2+
e)^(1/2)*(d*x^2+c)^(1/2)/e/a/b^2/(-d/c)^(1/2)/f^2/(a*f-b*e)/(d*f*x^4+c*f*x^
2+d*e*x^2+c*e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)^{\frac{5}{2}}}{\left(bx^2+a\right)\left(fx^2+e\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/(b*x^2+a)/(f*x^2+e)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)*(f*x^2 + e)^(3/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/(b*x^2+a)/(f*x^2+e)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(5/2)/(b*x**2+a)/(f*x**2+e)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)^{\frac{5}{2}}}{\left(bx^2+a\right)\left(fx^2+e\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/(b*x^2+a)/(f*x^2+e)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)*(f*x^2 + e)^(3/2)), x)
```

3.84 $\int \frac{\left(c+dx^2\right)^{3/2}}{\left(a+bx^2\right)\left(e+fx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=223

$$\frac{c^{3/2}\sqrt{e+fx^2}(bc-ad)\Pi\left(1-\frac{bc}{ad};\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{a\sqrt{d}e\sqrt{c+dx^2}(be-af)\sqrt{\frac{c\left(e+fx^2\right)}{e\left(c+dx^2\right)}}}+\frac{\sqrt{c+dx^2}(de-cf)E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{\sqrt{e}\sqrt{f}\sqrt{e+fx^2}(be-af)\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}$$

```
[Out] ((d*e - c*f)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*
e)/(c*f)])/(Sqrt[e]*Sqrt[f]*(b*e - a*f)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2)
)]*Sqrt[e + f*x^2]) + (c^(3/2)*(b*c - a*d)*Sqrt[e + f*x^2]*EllipticPi[1 - (
b*c)/(a*d), ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(a*Sqrt[d]*e*(b*
e - a*f)*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))])
```

Rubi [A] time = 0.120364, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {542, 539, 411}

$$\frac{c^{3/2}\sqrt{e+fx^2}(bc-ad)\Pi\left(1-\frac{bc}{ad};\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{a\sqrt{d}e\sqrt{c+dx^2}(be-af)\sqrt{\frac{c\left(e+fx^2\right)}{e\left(c+dx^2\right)}}}+\frac{\sqrt{c+dx^2}(de-cf)E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{\sqrt{e}\sqrt{f}\sqrt{e+fx^2}(be-af)\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^(3/2)/((a + b*x^2)*(e + f*x^2)^(3/2)),x]
```

```
[Out] ((d*e - c*f)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*
e)/(c*f)])/(Sqrt[e]*Sqrt[f]*(b*e - a*f)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2)
)]*Sqrt[e + f*x^2]) + (c^(3/2)*(b*c - a*d)*Sqrt[e + f*x^2]*EllipticPi[1 - (
b*c)/(a*d), ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(a*Sqrt[d]*e*(b*
e - a*f)*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))])
```

Rule 542

```
Int[((e_) + (f_.)*(x_)^2)^(3/2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2
)^(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[Sqrt[e + f*x^2]/((
a + b*x^2)*Sqrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqr
t[e + f*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && P
osQ[d/c] && PosQ[f/e]
```

Rule 539

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[(c*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcT
an[Rt[d/c, 2]*x], 1 - (c*f)/(d*e)])/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c
*(e + f*x^2))/(e*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
```

```
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\left(c+dx^2\right)^{3/2}}{\left(a+bx^2\right)\left(e+fx^2\right)^{3/2}}\,dx = \frac{(bc-ad)\int \frac{\sqrt{c+dx^2}}{\left(a+bx^2\right)\sqrt{e+fx^2}}\,dx}{be-af} + \frac{(de-cf)\int \frac{\sqrt{c+dx^2}}{\left(e+fx^2\right)^{3/2}}\,dx}{be-af}$$

$$= \frac{(de-cf)\sqrt{c+dx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{\sqrt{e}\sqrt{f}(be-af)\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}\sqrt{e+fx^2}} + \frac{c^{3/2}(bc-ad)\sqrt{e+fx^2}\,\Pi\left(1-\frac{bc}{ad};\tan^{-1}\left(\frac{\sqrt{d}\ldots}{\sqrt{\ldots}}\right.\right.}{a\sqrt{d}e(be-af)\sqrt{c+dx^2}\sqrt{\frac{c\left(e+fx^2\right)}{e\left(c+dx^2\right)}}}$$

Mathematica [C] time = 1.04195, size = 304, normalized size = 1.36

$$\frac{-iad^2e\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}(be-af)\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{\frac{d}{c}}\right),\frac{cf}{de}\right)+abfx\sqrt{\frac{d}{c}}\left(c+dx^2\right)(de-cf)-iabde\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx\ldots}{e}}\ldots}{abef\sqrt{\frac{d}{c}}\sqrt{c+dx^2}\sqrt{e+fx^2}(b\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^(3/2)/((a + b*x^2)*(e + f*x^2)^(3/2)),x]
```

```
[Out] (a*b*Sqrt[d/c]*f*(d*e - c*f)*x*(c + d*x^2) - I*a*b*d*e*(-(d*e) + c*f)*Sqrt[
1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/
(d*e)] - I*a*d^2*e*(b*e - a*f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*Elli
pticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - I*(b*c - a*d)^2*e*f*Sqrt[1 + (
d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x
], (c*f)/(d*e)])/(a*b*Sqrt[d/c]*e*f*(b*e - a*f)*Sqrt[c + d*x^2]*Sqrt[e + f*
x^2])
```

Maple [B] time = 0.027, size = 594, normalized size = 2.7

$$\frac{1}{faeb\left(af-be\right)\left(dfx^4+cfx^2+dex^2+ce\right)}\left(x^3abcdf^2\sqrt{-\frac{d}{c}}-x^3abd^2ef\sqrt{-\frac{d}{c}}+\text{EllipticF}\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)a^2d^2ef\sqrt{\frac{dx^2\ldots}{\ldots}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(3/2)/(b*x^2+a)/(f*x^2+e)^(3/2),x)
```

```
[Out] (x^3*a*b*c*d*f^2*(-d/c)^(1/2)-x^3*a*b*d^2*e*f*(-d/c)^(1/2)+EllipticF(x*(-d/
c)^(1/2),(c*f/d/e)^(1/2))*a^2*d^2*e*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/
2)-EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b*d^2*e^2*((d*x^2+c)/c)^(1/2
)*((f*x^2+e)/e)^(1/2)-EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b*c*d*e*f
*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+EllipticE(x*(-d/c)^(1/2),(c*f/d/e)
^(1/2))*a*b*d^2*e^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-EllipticPi(x*(-
d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a^2*d^2*e*f*((d*x^2+c)/c)^(1/
2)*((f*x^2+e)/e)^(1/2)+2*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d
/c)^(1/2))*a*b*c*d*e*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-EllipticPi(x
*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*b^2*c^2*e*f*((d*x^2+c)/c)^
```

```
(1/2)*((f*x^2+e)/e)^(1/2)+x*a*b*c^2*f^2*(-d/c)^(1/2)-x*a*b*c*d*e*f*(-d/c)^(
1/2))*(f*x^2+e)^(1/2)*(d*x^2+c)^(1/2)/b/e/a/f/(-d/c)^(1/2)/(a*f-b*e)/(d*f*x
^4+c*f*x^2+d*e*x^2+c*e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)^{\frac{3}{2}}}{\left(bx^2+a\right)\left(fx^2+e\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/(b*x^2+a)/(f*x^2+e)^(3/2),x, algorithm="maxima")

[Out] integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)*(f*x^2 + e)^(3/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/(b*x^2+a)/(f*x^2+e)^(3/2),x, algorithm="fricas")

[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(3/2)/(b*x**2+a)/(f*x**2+e)**(3/2),x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)^{\frac{3}{2}}}{\left(bx^2+a\right)\left(fx^2+e\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/(b*x^2+a)/(f*x^2+e)^(3/2),x, algorithm="giac")

[Out] integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)*(f*x^2 + e)^(3/2)), x)
```

3.85 $\int \frac{\sqrt{c+dx^2}}{(a+bx^2)(e+fx^2)^{3/2}}\,dx$

Optimal. Leaf size=209

$$\frac{bc^{3/2}\sqrt{e+fx^2}\,\Pi\left(1-\frac{bc}{ad};\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{a\sqrt{d}e\sqrt{c+dx^2}(be-af)\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}-\frac{\sqrt{f}\sqrt{c+dx^2}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{\sqrt{e}\sqrt{e+fx^2}(be-af)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

```
[Out] -((Sqrt[f]*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)
/(c*f)])/(Sqrt[e]*(b*e - a*f)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e
+ f*x^2])) + (b*c^(3/2)*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcTan[
(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(a*Sqrt[d]*e*(b*e - a*f)*Sqrt[c + d
*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))])
```

Rubi [A] time = 0.105284, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {541, 539, 411}

$$\frac{bc^{3/2}\sqrt{e+fx^2}\,\Pi\left(1-\frac{bc}{ad};\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{a\sqrt{d}e\sqrt{c+dx^2}(be-af)\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}-\frac{\sqrt{f}\sqrt{c+dx^2}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{\sqrt{e}\sqrt{e+fx^2}(be-af)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^2]/((a + b*x^2)*(e + f*x^2)^(3/2)),x]
```


```
[Out] -((Sqrt[f]*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)
/(c*f)])/(Sqrt[e]*(b*e - a*f)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e
+ f*x^2])) + (b*c^(3/2)*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcTan[
(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(a*Sqrt[d]*e*(b*e - a*f)*Sqrt[c + d
*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))])
```


Rule 541

```
Int[Sqrt[(e_) + (f_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[b/(b*c - a*d), Int[Sqrt[e + f*x^2]/((a + b*x^2)*S
qrt[c + d*x^2]), x], x] - Dist[d/(b*c - a*d), Int[Sqrt[e + f*x^2]/(c + d*x^
2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[d/c] && PosQ[f/e]
```

Rule 539

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[(c*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcT
an[Rt[d/c, 2]*x], 1 - (c*f)/(d*e)])/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c
*(e + f*x^2))/(e*(c + d*x^2))]]), x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
```

```
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)(e+fx^2)^{3/2}}\,dx = \frac{b\int \frac{\sqrt{c+dx^2}}{(a+bx^2)\sqrt{e+fx^2}}\,dx}{be-af} - \frac{f\int \frac{\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx}{be-af}$$

$$= -\frac{\sqrt{f}\sqrt{c+dx^2}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{\sqrt{e}(be-af)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} + \frac{bc^{3/2}\sqrt{e+fx^2}\Pi\left(1-\frac{bc}{ad};\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{a\sqrt{d}e(be-af)\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$

Mathematica [C] time = 0.466349, size = 207, normalized size = 0.99

$$\frac{-ie\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}(bc-ad)\Pi\left(\frac{bc}{ad};i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)-iade\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}E\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)-afx\sqrt{\frac{d}{c}}\left(c+\right.}{ae\sqrt{\frac{d}{c}}\sqrt{c+dx^2}\sqrt{e+fx^2}(be-af)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^2]/((a + b*x^2)*(e + f*x^2)^(3/2)),x]
```

```
[Out] (-(a*Sqrt[d/c]*f*x*(c + d*x^2)) - I*a*d*e*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x
^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - I*(b*c - a*d)*e*Sqr
t[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt
[d/c]*x], (c*f)/(d*e)])/(a*Sqrt[d/c]*e*(b*e - a*f)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2])
```

Maple [A] time = 0.025, size = 285, normalized size = 1.4

$$\frac{1}{ae(af-be)(dfx^4+cfx^2+dex^2+ce)}\left(x^3adf\sqrt{-\frac{d}{c}}-\mathrm{EllipticE}\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)ade\sqrt{\frac{dx^2+c}{c}}\sqrt{\frac{fx^2+e}{e}}+\mathrm{EllipticPi}\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)/(b*x^2+a)/(f*x^2+e)^(3/2),x)
```

```
[Out] (x^3*a*d*f*(-d/c)^(1/2)-EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d*e*((d
*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/
e)^(1/2)/(-d/c)^(1/2))*a*d*e*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-Ellipt
icPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*b*c*e*((d*x^2+c)/c)^
(1/2)*((f*x^2+e)/e)^(1/2)+x*a*c*f*(-d/c)^(1/2))*(f*x^2+e)^(1/2)*(d*x^2+c)^(
1/2)/e/a/(-d/c)^(1/2)/(a*f-b*e)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2+c}}{(bx^2+a)(fx^2+e)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)/(f*x^2+e)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)/((b*x^2 + a)*(f*x^2 + e)^(3/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)/(f*x^2+e)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+dx^2}}{\left(a+bx^2\right)\left(e+fx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)/(b*x**2+a)/(f*x**2+e)**(3/2),x)
```

```
[Out] Integral(sqrt(c + d*x**2)/((a + b*x**2)*(e + f*x**2)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2+c}}{\left(bx^2+a\right)\left(fx^2+e\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)/(f*x^2+e)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^2 + c)/((b*x^2 + a)*(f*x^2 + e)^(3/2)), x)
```

3.86 $\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}(e+fx^2)^{3/2}}\,dx$

Optimal. Leaf size=344

$$-\frac{\sqrt{e}\sqrt{f}\sqrt{c+dx^2}(-adf-bcf+2bde)\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c\sqrt{e+fx^2}(be-af)^2(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{b^2e^{3/2}\sqrt{c+dx^2}\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{ac\sqrt{f}\sqrt{e+fx^2}(be-af)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+$$

```
[Out] (f^(3/2)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(
c*f)])/(Sqrt[e]*(b*e - a*f)*(d*e - c*f)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2)
)]*Sqrt[e + f*x^2]) - (Sqrt[e]*Sqrt[f]*(2*b*d*e - b*c*f - a*d*f)*Sqrt[c + d
*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(c*(b*e - a*
f)^2*(d*e - c*f)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (
b^2*e^(3/2)*Sqrt[c + d*x^2]*EllipticPi[1 - (b*e)/(a*f), ArcTan[(Sqrt[f]*x)/
Sqrt[e]], 1 - (d*e)/(c*f)])/(a*c*Sqrt[f]*(b*e - a*f)^2*Sqrt[(e*(c + d*x^2))
/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rubi [A] time = 0.222421, antiderivative size = 344, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {546, 539, 525, 418, 411}

$$\frac{b^2e^{3/2}\sqrt{c+dx^2}\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{ac\sqrt{f}\sqrt{e+fx^2}(be-af)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{f^{3/2}\sqrt{c+dx^2}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{\sqrt{e}\sqrt{e+fx^2}(be-af)(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{\sqrt{e}\sqrt{f}\sqrt{c+dx^2}(-adf-bcf}{c\sqrt{e+fx^2}(be-af}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)*Sqrt[c + d*x^2]*(e + f*x^2)^(3/2)),x]
```


```
[Out] (f^(3/2)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(
c*f)])/(Sqrt[e]*(b*e - a*f)*(d*e - c*f)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2)
)]*Sqrt[e + f*x^2]) - (Sqrt[e]*Sqrt[f]*(2*b*d*e - b*c*f - a*d*f)*Sqrt[c + d
*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(c*(b*e - a*
f)^2*(d*e - c*f)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (
b^2*e^(3/2)*Sqrt[c + d*x^2]*EllipticPi[1 - (b*e)/(a*f), ArcTan[(Sqrt[f]*x)/
Sqrt[e]], 1 - (d*e)/(c*f)])/(a*c*Sqrt[f]*(b*e - a*f)^2*Sqrt[(e*(c + d*x^2))
/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```


Rule 546

```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Dist[b^2/(b*c - a*d)^2, Int[((c + d*x^2)^(q + 2)*(e +
f*x^2)^r)/(a + b*x^2), x], x] - Dist[d/(b*c - a*d)^2, Int[(c + d*x^2)^q*(e
+ f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r},
x] && LtQ[q, -1]
```

Rule 539

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[(c*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcT
an[Rt[d/c, 2]*x], 1 - (c*f)/(d*e)])/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c
*(e + f*x^2))/(e*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
```

```
[d/c]
```

Rule 525

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}}\,dx = \frac{b^2\int \frac{\sqrt{e+fx^2}}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{(be-af)^2} - \frac{f\int \frac{2be-af+bfx^2}{\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}}\,dx}{(be-af)^2}$$

$$= \frac{b^2e^{3/2}\sqrt{c+dx^2}\,\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{ac\sqrt{f}(be-af)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} + \frac{f^2\int \frac{\sqrt{c+dx^2}}{\left(e+fx^2\right)^{3/2}}\,dx}{(be-af)(de-cf)} - \frac{(f(2bd$$

$$= \frac{f^{3/2}\sqrt{c+dx^2}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{\sqrt{e}(be-af)(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} - \frac{\sqrt{e}\sqrt{f}(2bde-bcf-adf)\sqrt{c+dx^2}}{c(be-af)^2(de-cf)\sqrt{\frac{e(c}{c(e}}}$$

Mathematica [C] time = 0.619468, size = 221, normalized size = 0.64

$$\frac{-ibe\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}(cf-de)\Pi\left(\frac{bc}{ad};i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)-iadef\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}E\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)-af^2x\sqrt{}}{ae\sqrt{\frac{d}{c}}\sqrt{c+dx^2}\sqrt{e+fx^2}(af-be)(de-cf)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)*Sqrt[c + d*x^2]*(e + f*x^2)^(3/2)),x]
```

```
[Out] (-(a*Sqrt[d/c]*f^2*x*(c + d*x^2)) - I*a*d*e*f*Sqrt[1 + (d*x^2)/c]*Sqrt[1 +
(f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - I*b*e*(-(d*e) +
 c*f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*Arc
Sinh[Sqrt[d/c]*x], (c*f)/(d*e)])/(a*Sqrt[d/c]*e*(-(b*e) + a*f)*(d*e - c*f)*
Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Maple [A] time = 0.026, size = 303, normalized size = 0.9

$$\frac{1}{\left(af-be\right)ae\left(cf-de\right)\left(dfx^4+cfx^2+dex^2+ce\right)}\left(x^3adf^2\sqrt{-\frac{d}{c}}-\sqrt{\frac{dx^2+c}{c}}\sqrt{\frac{fx^2+e}{e}}\mathrm{EllipticE}\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)adef-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

```
[Out] (x^3*a*d*f^2*(-d/c)^(1/2)-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE
(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d*e*f-EllipticPi(x*(-d/c)^(1/2),b*c/a/d,
(-f/e)^(1/2)/(-d/c)^(1/2))*b*c*e*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+
EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*b*d*e^2*((d*x^
2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+x*a*c*f^2*(-d/c)^(1/2))*(f*x^2+e)^(1/2)*(
d*x^2+c)^(1/2)/e/a/(a*f-b*e)/(-d/c)^(1/2)/(c*f-d*e)/(d*f*x^4+c*f*x^2+d*e*x^
2+c*e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)\sqrt{dx^2+c}\left(fx^2+e\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}\left(e+fx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)/(d*x**2+c)**(1/2)/(f*x**2+e)**(3/2),x)
```

```
[Out] Integral(1/((a + b*x**2)*sqrt(c + d*x**2)*(e + f*x**2)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2 + a\right)\sqrt{dx^2 + c}\left(fx^2 + e\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)), x)
```

3.87 $\int \frac{1}{(a+bx^2)(c+dx^2)^{3/2}(e+fx^2)^{3/2}}\,dx$

Optimal. Leaf size=539

$$-\frac{d^2\sqrt{e}\sqrt{c+dx^2}(2adf-3bcf+bde)\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{e+fx^2}(bc-ad)^2(de-cf)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{b^3c^{3/2}\sqrt{e+fx^2}\Pi\left(1-\frac{bc}{ad};\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{a\sqrt{d}e\sqrt{c+dx^2}(bc-ad)^2(be-af)\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}-$$

```
[Out] -((d^2*x)/(c*(b*c - a*d)*(d*e - c*f)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])) - (b
^2*Sqrt[f]*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)
/(c*f)])/((b*c - a*d)^2*Sqrt[e]*(b*e - a*f)*Sqrt[(e*(c + d*x^2))/(c*(e + f*
x^2))]*Sqrt[e + f*x^2]) - (d*Sqrt[f]*(2*b*c^2*f - a*d*(d*e + c*f))*Sqrt[c +
 d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(c*(b*c -
a*d)^2*Sqrt[e]*(d*e - c*f)^2*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e +
 f*x^2]) - (d^2*Sqrt[e]*(b*d*e - 3*b*c*f + 2*a*d*f)*Sqrt[c + d*x^2]*Ellipti
cF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(c*(b*c - a*d)^2*Sqrt[f]*
(d*e - c*f)^2*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (b^3
*c^(3/2)*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcTan[(Sqrt[d]*x)/Sqr
t[c]], 1 - (c*f)/(d*e)])/(a*Sqrt[d]*(b*c - a*d)^2*e*(b*e - a*f)*Sqrt[c + d*
x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))])
```

Rubi [A] time = 0.479277, antiderivative size = 539, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {546, 541, 539, 411, 527, 525, 418}

$$\frac{b^3c^{3/2}\sqrt{e+fx^2}\Pi\left(1-\frac{bc}{ad};\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{a\sqrt{d}e\sqrt{c+dx^2}(bc-ad)^2(be-af)\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}-\frac{b^2\sqrt{f}\sqrt{c+dx^2}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{\sqrt{e}\sqrt{e+fx^2}(bc-ad)^2(be-af)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{d\sqrt{f}\sqrt{c+dx^2}\left(2bc^2f-ad(c\right.}{c\sqrt{e}\sqrt{e+fx^2}(bc-}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)*(c + d*x^2)^(3/2)*(e + f*x^2)^(3/2)),x]
```


```
[Out] -((d^2*x)/(c*(b*c - a*d)*(d*e - c*f)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])) - (b
^2*Sqrt[f]*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)
/(c*f)])/((b*c - a*d)^2*Sqrt[e]*(b*e - a*f)*Sqrt[(e*(c + d*x^2))/(c*(e + f*
x^2))]*Sqrt[e + f*x^2]) - (d*Sqrt[f]*(2*b*c^2*f - a*d*(d*e + c*f))*Sqrt[c +
 d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(c*(b*c -
a*d)^2*Sqrt[e]*(d*e - c*f)^2*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e +
 f*x^2]) - (d^2*Sqrt[e]*(b*d*e - 3*b*c*f + 2*a*d*f)*Sqrt[c + d*x^2]*Ellipti
cF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(c*(b*c - a*d)^2*Sqrt[f]*
(d*e - c*f)^2*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (b^3
*c^(3/2)*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcTan[(Sqrt[d]*x)/Sqr
t[c]], 1 - (c*f)/(d*e)])/(a*Sqrt[d]*(b*c - a*d)^2*e*(b*e - a*f)*Sqrt[c + d*
x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))])
```


Rule 546

```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Dist[b^2/(b*c - a*d)^2, Int[((c + d*x^2)^(q + 2)*(e +
f*x^2)^r)/(a + b*x^2), x], x] - Dist[d/(b*c - a*d)^2, Int[(c + d*x^2)^q*(e
+ f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r},
```

```
x] && LtQ[q, -1]
```

Rule 541

```
Int[Sqrt[(e_) + (f_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[b/(b*c - a*d), Int[Sqrt[e + f*x^2]/((a + b*x^2)*S
qrt[c + d*x^2]), x], x] - Dist[d/(b*c - a*d), Int[Sqrt[e + f*x^2]/(c + d*x^
2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[d/c] && PosQ[f/e]
```

Rule 539

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[(c*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcT
an[Rt[d/c, 2]*x], 1 - (c*f)/(d*e)])/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c
*(e + f*x^2))/(e*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 525

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{3/2}(e+fx^2)^{3/2}}\,dx = \frac{b^2\int \frac{\sqrt{c+dx^2}}{(a+bx^2)(e+fx^2)^{3/2}}\,dx}{(bc-ad)^2} - \frac{d\int \frac{2bc-ad+bdx^2}{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}\,dx}{(bc-ad)^2}$$

$$= -\frac{d^2x}{c(bc-ad)(de-cf)\sqrt{c+dx^2}\sqrt{e+fx^2}} + \frac{b^3\int \frac{\sqrt{c+dx^2}}{(a+bx^2)\sqrt{e+fx^2}}\,dx}{(bc-ad)^2(be-af)} - \frac{(b^2f)\int \frac{\sqrt{c}}{(e+f}}{(bc-ad)^2(b}$$

$$= -\frac{d^2x}{c(bc-ad)(de-cf)\sqrt{c+dx^2}\sqrt{e+fx^2}} - \frac{b^2\sqrt{f}\sqrt{c+dx^2}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\right.}{(bc-ad)^2\sqrt{e}(be-af)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e}}$$

$$= -\frac{d^2x}{c(bc-ad)(de-cf)\sqrt{c+dx^2}\sqrt{e+fx^2}} - \frac{b^2\sqrt{f}\sqrt{c+dx^2}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\right.}{(bc-ad)^2\sqrt{e}(be-af)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e}}$$

Mathematica [C] time = 3.85599, size = 418, normalized size = 0.78

$$\frac{iacde\left(\frac{d}{c}\right)^{3/2}\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}(be-af)(cf-de)\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{\frac{d}{c}}\right),\frac{cf}{de}\right)+ib^2ce\sqrt{\frac{d}{c}}\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}(de-cf)^2\Pi}{ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)*(c + d*x^2)^(3/2)*(e + f*x^2)^(3/2)),x]
```

```
[Out] ((a*d*x*(-(a*d*f*(c^2*f^2 + c*d*f^2*x^2 + d^2*e*(e + f*x^2))) + b*(c^3*f^3
+ c^2*d*f^3*x^2 + d^3*e^2*(e + f*x^2))))/c + I*a*d*Sqrt[d/c]*e*(-(a*d*f*(d*
e + c*f)) + b*(d^2*e^2 + c^2*f^2))*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*
EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + I*a*c*d*(d/c)^(3/2)*e*(b*e
 - a*f)*(-(d*e) + c*f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*
ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + I*b^2*c*Sqrt[d/c]*e*(d*e - c*f)^2*Sqrt
[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[
d/c]*x], (c*f)/(d*e)])/(a*d*(-(b*c) + a*d)*e*(b*e - a*f)*(d*e - c*f)^2*Sqrt
[c + d*x^2]*Sqrt[e + f*x^2])
```

Maple [A] time = 0.035, size = 956, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)/(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x)
```

```
[Out] (x^3*a^2*c*d^2*f^3*(-d/c)^(1/2)+x^3*a^2*d^3*e*f^2*(-d/c)^(1/2)-x^3*a*b*c^2*
d*f^3*(-d/c)^(1/2)-x^3*a*b*d^3*e^2*f*(-d/c)^(1/2)-EllipticF(x*(-d/c)^(1/2),
(c*f/d/e)^(1/2))*a^2*c*d^2*e*f^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+El
lipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*d^3*e^2*f*((d*x^2+c)/c)^(1/2)*(
(f*x^2+e)/e)^(1/2)+EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b*c*d^2*e^2*
f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-EllipticF(x*(-d/c)^(1/2),(c*f/d/e
)^(1/2))*a*b*d^3*e^3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-EllipticE(x*(-
```

```
d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*c*d^2*e*f^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/
e)^(1/2)-EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*d^3*e^2*f*((d*x^2+c)
/c)^(1/2)*((f*x^2+e)/e)^(1/2)+EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b
*c^2*d*e*f^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+EllipticE(x*(-d/c)^(1/
2),(c*f/d/e)^(1/2))*a*b*d^3*e^3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+Ell
ipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*b^2*c^3*e*f^2*((d
*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-2*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-
f/e)^(1/2)/(-d/c)^(1/2))*b^2*c^2*d*e^2*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^
(1/2)+EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*b^2*c*d^
2*e^3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+x*a^2*c^2*d*f^3*(-d/c)^(1/2)+
x*a^2*d^3*e^2*f*(-d/c)^(1/2)-x*a*b*c^3*f^3*(-d/c)^(1/2)-x*a*b*d^3*e^3*(-d/c
)^(1/2))*(f*x^2+e)^(1/2)*(d*x^2+c)^(1/2)/e/a/c/(a*f-b*e)/(-d/c)^(1/2)/(a*d-
b*c)/(c*f-d*e)^2/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)\left(dx^2+c\right)^{\frac{3}{2}}\left(fx^2+e\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((b*x^2 + a)*(d*x^2 + c)^(3/2)*(f*x^2 + e)^(3/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+bx^2\right)\left(c+dx^2\right)^{\frac{3}{2}}\left(e+fx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)/(d*x**2+c)**(3/2)/(f*x**2+e)**(3/2),x)
```

```
[Out] Integral(1/((a + b*x**2)*(c + d*x**2)**(3/2)*(e + f*x**2)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)\left(dx^2+c\right)^{\frac{3}{2}}\left(fx^2+e\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)*(d*x^2 + c)^(3/2)*(f*x^2 + e)^(3/2)), x)
```

3.88 $\int \frac{1}{(a+bx^2)(c+dx^2)^{5/2}(e+fx^2)^{3/2}}\,dx$

Optimal. Leaf size=814

$$\frac{e^{3/2}\sqrt{dx^2+c}\,\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)b^4}{ac(bc-ad)^2\sqrt{f}(be-af)^2\sqrt{\frac{e(dx^2+c)}{c(fx^2+e)}}\sqrt{fx^2+e}}+\frac{f^{3/2}\sqrt{dx^2+c}\,E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)b^2}{(bc-ad)^2\sqrt{e}(be-af)(de-cf)\sqrt{\frac{e(dx^2+c)}{c(fx^2+e)}}\sqrt{fx^2+e}}-\frac{\sqrt{e}\sqrt{f}(2bde-b}{c(bc}$$

```
[Out] -(d^2*x)/(3*c*(b*c - a*d)*(d*e - c*f)*(c + d*x^2)^(3/2)*Sqrt[e + f*x^2]) -
(d^2*(b*c*(5*d*e - 9*c*f) - 2*a*d*(d*e - 3*c*f))*x)/(3*c^2*(b*c - a*d)^2*(d
*e - c*f)^2*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]) + (b^2*f^(3/2)*Sqrt[c + d*x^2]
*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/((b*c - a*d)^2*Sq
rt[e]*(b*e - a*f)*(d*e - c*f)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e
+ f*x^2]) - (d*Sqrt[f]*(b*c*(5*d^2*e^2 - 7*c*d*e*f - 6*c^2*f^2) - a*d*(2*d^
2*e^2 - 7*c*d*e*f - 3*c^2*f^2))*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x
)/Sqrt[e]], 1 - (d*e)/(c*f)])/(3*c^2*(b*c - a*d)^2*Sqrt[e]*(d*e - c*f)^3*Sq
rt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) - (b^2*Sqrt[e]*Sqrt[f]
*(2*b*d*e - b*c*f - a*d*f)*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqr
t[e]], 1 - (d*e)/(c*f)])/(c*(b*c - a*d)^2*(b*e - a*f)^2*(d*e - c*f)*Sqrt[(e
*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (d^2*Sqrt[e]*Sqrt[f]*(b*c
*(7*d*e - 15*c*f) - a*d*(d*e - 9*c*f))*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sq
rt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(3*c^2*(b*c - a*d)^2*(d*e - c*f)^3*Sqr
t[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (b^4*e^(3/2)*Sqrt[c +
 d*x^2]*EllipticPi[1 - (b*e)/(a*f), ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/
(c*f)])/(a*c*(b*c - a*d)^2*Sqrt[f]*(b*e - a*f)^2*Sqrt[(e*(c + d*x^2))/(c*(e
 + f*x^2))]*Sqrt[e + f*x^2])
```

Rubi [A] time = 0.957996, antiderivative size = 814, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {546, 539, 525, 418, 411, 527}

$$\frac{e^{3/2}\sqrt{dx^2+c}\,\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)b^4}{ac(bc-ad)^2\sqrt{f}(be-af)^2\sqrt{\frac{e(dx^2+c)}{c(fx^2+e)}}\sqrt{fx^2+e}}+\frac{f^{3/2}\sqrt{dx^2+c}\,E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)b^2}{(bc-ad)^2\sqrt{e}(be-af)(de-cf)\sqrt{\frac{e(dx^2+c)}{c(fx^2+e)}}\sqrt{fx^2+e}}-\frac{\sqrt{e}\sqrt{f}(2bde-b}{c(bc-ad}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)*(c + d*x^2)^(5/2)*(e + f*x^2)^(3/2)),x]
```

```
[Out] -(d^2*x)/(3*c*(b*c - a*d)*(d*e - c*f)*(c + d*x^2)^(3/2)*Sqrt[e + f*x^2]) -
(d^2*(b*c*(5*d*e - 9*c*f) - 2*a*d*(d*e - 3*c*f))*x)/(3*c^2*(b*c - a*d)^2*(d
*e - c*f)^2*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]) + (b^2*f^(3/2)*Sqrt[c + d*x^2]
*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/((b*c - a*d)^2*Sq
rt[e]*(b*e - a*f)*(d*e - c*f)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e
+ f*x^2]) - (d*Sqrt[f]*(b*c*(5*d^2*e^2 - 7*c*d*e*f - 6*c^2*f^2) - a*d*(2*d^
2*e^2 - 7*c*d*e*f - 3*c^2*f^2))*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x
)/Sqrt[e]], 1 - (d*e)/(c*f)])/(3*c^2*(b*c - a*d)^2*Sqrt[e]*(d*e - c*f)^3*Sq
rt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) - (b^2*Sqrt[e]*Sqrt[f]
*(2*b*d*e - b*c*f - a*d*f)*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqr
t[e]], 1 - (d*e)/(c*f)])/(c*(b*c - a*d)^2*(b*e - a*f)^2*(d*e - c*f)*Sqrt[(e
*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (d^2*Sqrt[e]*Sqrt[f]*(b*c
*(7*d*e - 15*c*f) - a*d*(d*e - 9*c*f))*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sq
rt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(3*c^2*(b*c - a*d)^2*(d*e - c*f)^3*Sqr
```

```
t[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (b^4*e^(3/2)*Sqrt[c +
 d*x^2]*EllipticPi[1 - (b*e)/(a*f), ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/
(c*f)])/(a*c*(b*c - a*d)^2*Sqrt[f]*(b*e - a*f)^2*Sqrt[(e*(c + d*x^2))/(c*(e
 + f*x^2))]*Sqrt[e + f*x^2])
```

Rule 546

```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Dist[b^2/(b*c - a*d)^2, Int[((c + d*x^2)^(q + 2)*(e +
f*x^2)^r)/(a + b*x^2), x], x] - Dist[d/(b*c - a*d)^2, Int[(c + d*x^2)^q*(e
+ f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r},
x] && LtQ[q, -1]
```

Rule 539

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[(c*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcT
an[Rt[d/c, 2]*x], 1 - (c*f)/(d*e)])/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c
*(e + f*x^2))/(e*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 525

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)\left(c+dx^2\right)^{5/2}\left(e+fx^2\right)^{3/2}}\,dx = \frac{b^2\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}}\,dx}{(bc-ad)^2} - \frac{d\int \frac{2bc-ad+bdx^2}{\left(c+dx^2\right)^{5/2}\left(e+fx^2\right)^{3/2}}\,dx}{(bc-ad)^2}$$

$$= -\frac{d^2x}{3c(bc-ad)(de-cf)\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}} + \frac{b^4\int \frac{\sqrt{e+fx^2}}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{(bc-ad)^2(be-af)^2} - \frac{\left(b^2 f \cdots\right.}{(\cdots}$$

$$= -\frac{d^2x}{3c(bc-ad)(de-cf)\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}} - \frac{d^2(bc(5de-9cf)-2ad(\cdots}{3c^2(bc-ad)^2(de-cf)^2\sqrt{c+\cdots}}$$

$$= -\frac{d^2x}{3c(bc-ad)(de-cf)\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}} - \frac{d^2(bc(5de-9cf)-2ad(\cdots}{3c^2(bc-ad)^2(de-cf)^2\sqrt{c+\cdots}}$$

$$= -\frac{d^2x}{3c(bc-ad)(de-cf)\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}} - \frac{d^2(bc(5de-9cf)-2ad(\cdots}{3c^2(bc-ad)^2(de-cf)^2\sqrt{c+\cdots}}$$

<u>Mathematica</u> [C] time = 5.94163, size = 1645, normalized size = 2.02

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)*(c + d*x^2)^(5/2)*(e + f*x^2)^(3/2)),x]
```

```
[Out] ((-I)*a*d*e*(2*a*b*d*(d*e - 3*c*f)*(d*e + c*f)^2 + a^2*d^2*f*(-2*d^2*e^2 +
7*c*d*e*f + 3*c^2*f^2) + b^2*c*(-5*d^3*e^3 + 10*c*d^2*e^2*f + 3*c^3*f^3))*(
c + d*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt
[d/c]*x], (c*f)/(d*e)] + (Sqrt[d/c]*(6*a*b^2*c^2*d^5*e^4*x - 3*a^2*b*c*d^6*
e^4*x - 11*a*b^2*c^3*d^4*e^3*f*x + 2*a^2*b*c^2*d^5*e^3*f*x + 3*a^3*c*d^6*e^
3*f*x + 11*a^2*b*c^3*d^4*e^2*f^2*x - 8*a^3*c^2*d^5*e^2*f^2*x - 3*a*b^2*c^6*
d*f^4*x + 6*a^2*b*c^5*d^2*f^4*x - 3*a^3*c^4*d^3*f^4*x + 5*a*b^2*c*d^6*e^4*x
^3 - 2*a^2*b*d^7*e^4*x^3 - 4*a*b^2*c^2*d^5*e^3*f*x^3 - a^2*b*c*d^6*e^3*f*x^
3 + 2*a^3*d^7*e^3*f*x^3 - 11*a*b^2*c^3*d^4*e^2*f^2*x^3 + 12*a^2*b*c^2*d^5*e
^2*f^2*x^3 - 4*a^3*c*d^6*e^2*f^2*x^3 + 11*a^2*b*c^3*d^4*e*f^3*x^3 - 8*a^3*c
^2*d^5*e*f^3*x^3 - 6*a*b^2*c^5*d^2*f^4*x^3 + 12*a^2*b*c^4*d^3*f^4*x^3 - 6*a
^3*c^3*d^4*f^4*x^3 + 5*a*b^2*c*d^6*e^3*f*x^5 - 2*a^2*b*d^7*e^3*f*x^5 - 10*a
*b^2*c^2*d^5*e^2*f^2*x^5 + 2*a^2*b*c*d^6*e^2*f^2*x^5 + 2*a^3*d^7*e^2*f^2*x^
5 + 10*a^2*b*c^2*d^5*e*f^3*x^5 - 7*a^3*c*d^6*e*f^3*x^5 - 3*a*b^2*c^4*d^3*f^
4*x^5 + 6*a^2*b*c^3*d^4*f^4*x^5 - 3*a^3*c^2*d^5*f^4*x^5 - I*a*c*d^2*Sqrt[d/
c]*e*(b*e - a*f)*(-(d*e) + c*f)*(2*a*d*(d*e - 3*c*f) + b*c*(-5*d*e + 9*c*f)
)*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[S
qrt[d/c]*x], (c*f)/(d*e)] + (3*I)*b^3*c^4*d^3*Sqrt[d/c]*e^4*Sqrt[1 + (d*x^2
)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c
*f)/(d*e)] - (9*I)*b^3*c^7*(d/c)^(5/2)*e^3*f*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (
f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + (9
*I)*b^3*c^7*(d/c)^(3/2)*e^2*f^2*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*Ell
ipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - (3*I)*b^3*c^7*S
qrt[d/c]*e*f^3*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*
d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + (3*I)*b^3*c^3*d^4*Sqrt[d/c]*e^4*
```

```
x^2*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSi
nh[Sqrt[d/c]*x], (c*f)/(d*e)] - (9*I)*b^3*c^4*d^3*Sqrt[d/c]*e^3*f*x^2*Sqrt[
1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d
/c]*x], (c*f)/(d*e)] + (9*I)*b^3*c^7*(d/c)^(5/2)*e^2*f^2*x^2*Sqrt[1 + (d*x^
2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (
c*f)/(d*e)] - (3*I)*b^3*c^7*(d/c)^(3/2)*e*f^3*x^2*Sqrt[1 + (d*x^2)/c]*Sqrt[
1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]
))/d)/(3*a*c^2*Sqrt[d/c]*(b*c - a*d)^2*e*(b*e - a*f)*(-(d*e) + c*f)^3*(c +
d*x^2)^(3/2)*Sqrt[e + f*x^2])
```

Maple [B] time = 0.058, size = 4115, normalized size = 5.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)/(d*x^2+c)^(5/2)/(f*x^2+e)^(3/2),x)
```

```
[Out] -1/3*(-8*x^3*a^3*c^2*d^4*e*f^3*(-d/c)^(1/2)-6*x^3*a^3*c^3*d^3*f^4*(-d/c)^(1
/2)-9*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*b*c^4*d^2*e*f^3*((d*x^2
+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+11*x^3*a^2*b*c^3*d^3*e*f^3*(-d/c)^(1/2)-14
*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b^2*c^3*d^3*e^3*f*((d*x^2+c)/c
)^(1/2)*((f*x^2+e)/e)^(1/2)+3*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2
)/(-d/c)^(1/2))*x^2*b^3*c^5*d*e*f^3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)
+9*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a*b^2*c^3*d^3*e^2*f^2*((d*
x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-14*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(
1/2))*x^2*a*b^2*c^2*d^4*e^3*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+10*El
lipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a*b^2*c^2*d^4*e^3*f*((d*x^2+c)/
c)^(1/2)*((f*x^2+e)/e)^(1/2)-9*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^
2*a^2*b*c^3*d^3*e*f^3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+8*EllipticF(x
*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a^2*b*c^2*d^4*e^2*f^2*((d*x^2+c)/c)^(1/2
)*((f*x^2+e)/e)^(1/2)-6*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a^2*b
*c^3*d^3*e*f^3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-10*EllipticE(x*(-d/c
)^(1/2),(c*f/d/e)^(1/2))*x^2*a^2*b*c^2*d^4*e^2*f^2*((d*x^2+c)/c)^(1/2)*((f*
x^2+e)/e)^(1/2)-2*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a^2*b*c*d^5
*e^3*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+3*EllipticE(x*(-d/c)^(1/2),(
c*f/d/e)^(1/2))*x^2*a*b^2*c^4*d^2*e*f^3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(
1/2)+3*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a^2*b*c*d^5*e^3*f*((d*
x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-9*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f
/e)^(1/2)/(-d/c)^(1/2))*x^2*b^3*c^4*d^2*e^2*f^2*((d*x^2+c)/c)^(1/2)*((f*x^2
+e)/e)^(1/2)+9*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))
*x^2*b^3*c^3*d^3*e^3*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-6*EllipticE(
x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*b*c^4*d^2*e*f^3*((d*x^2+c)/c)^(1/2)*((f
*x^2+e)/e)^(1/2)-10*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*b*c^3*d^3
*e^2*f^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+3*EllipticE(x*(-d/c)^(1/2)
,(c*f/d/e)^(1/2))*x^2*a^3*c^2*d^4*e*f^3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(
1/2)+7*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a^3*c*d^5*e^2*f^2*((d*
x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-5*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1
/2))*x^2*a*b^2*c*d^5*e^4*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+6*Elliptic
F(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a^3*c^2*d^4*e*f^3*((d*x^2+c)/c)^(1/2)
*((f*x^2+e)/e)^(1/2)-8*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a^3*c*
d^5*e^2*f^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+5*EllipticF(x*(-d/c)^(1
/2),(c*f/d/e)^(1/2))*x^2*a*b^2*c*d^5*e^4*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^
(1/2)+8*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*b*c^3*d^3*e^2*f^2*((d
*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+3*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(
1/2))*a^2*b*c^2*d^4*e^3*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+9*Ellipti
cF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b^2*c^4*d^2*e^2*f^2*((d*x^2+c)/c)^(1/2
```

```
)*((f*x^2+e)/e)^(1/2)-2*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*b*c^2
*d^4*e^3*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+3*EllipticE(x*(-d/c)^(1/
2),(c*f/d/e)^(1/2))*a*b^2*c^5*d*e*f^3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/
2)+10*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b^2*c^3*d^3*e^3*f*((d*x^2
+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+2*x^3*a^3*d^6*e^3*f*(-d/c)^(1/2)-2*x^3*a^2
*b*d^6*e^4*(-d/c)^(1/2)-3*x*a^3*c^4*d^2*f^4*(-d/c)^(1/2)-3*x*a*b^2*c^6*f^4*
(-d/c)^(1/2)-3*x^5*a^3*c^2*d^4*f^4*(-d/c)^(1/2)+2*x^5*a^3*d^6*e^2*f^2*(-d/c
)^(1/2)-6*x^3*a*b^2*c^5*d*f^4*(-d/c)^(1/2)+5*x^3*a*b^2*c*d^5*e^4*(-d/c)^(1/
2)-8*x*a^3*c^2*d^4*e^2*f^2*(-d/c)^(1/2)+3*EllipticPi(x*(-d/c)^(1/2),b*c/a/d
,(-f/e)^(1/2)/(-d/c)^(1/2))*b^3*c^6*e*f^3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)
^(1/2)-3*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*b^3*c
^3*d^3*e^4*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+3*x*a^3*c*d^5*e^3*f*(-d/
c)^(1/2)+6*x*a^2*b*c^5*d*f^4*(-d/c)^(1/2)-3*x*a^2*b*c*d^5*e^4*(-d/c)^(1/2)+
6*x*a*b^2*c^2*d^4*e^4*(-d/c)^(1/2)-8*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/
2))*a^3*c^2*d^4*e^2*f^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+2*EllipticF
(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^3*c*d^5*e^3*f*((d*x^2+c)/c)^(1/2)*((f*x^
2+e)/e)^(1/2)-2*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*b*c*d^5*e^4*(
(d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+5*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)
^(1/2))*a*b^2*c^2*d^4*e^4*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-9*Ellipti
cPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*b^3*c^5*d*e^2*f^2*((d
*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+11*x*a^2*b*c^3*d^3*e^2*f^2*(-d/c)^(1/2
)+2*x*a^2*b*c^2*d^4*e^3*f*(-d/c)^(1/2)-11*x*a*b^2*c^3*d^3*e^3*f*(-d/c)^(1/2
)-2*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a^3*d^6*e^3*f*((d*x^2+c)/
c)^(1/2)*((f*x^2+e)/e)^(1/2)+2*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^
2*a^2*b*d^6*e^4*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+2*EllipticF(x*(-d/c
)^(1/2),(c*f/d/e)^(1/2))*x^2*a^3*d^6*e^3*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e
)^(1/2)-2*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a^2*b*d^6*e^4*((d*x
^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-3*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/
e)^(1/2)/(-d/c)^(1/2))*x^2*b^3*c^2*d^4*e^4*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e
)^(1/2)+3*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^3*c^3*d^3*e*f^3*((d*x
^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+7*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/
2))*a^3*c^2*d^4*e^2*f^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-2*EllipticE
(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^3*c*d^5*e^3*f*((d*x^2+c)/c)^(1/2)*((f*x^
2+e)/e)^(1/2)+2*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*b*c*d^5*e^4*(
(d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-5*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)
^(1/2))*a*b^2*c^2*d^4*e^4*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+12*x^3*a^
2*b*c^2*d^4*e^2*f^2*(-d/c)^(1/2)-x^3*a^2*b*c*d^5*e^3*f*(-d/c)^(1/2)-11*x^3*
a*b^2*c^3*d^3*e^2*f^2*(-d/c)^(1/2)-4*x^3*a*b^2*c^2*d^4*e^3*f*(-d/c)^(1/2)+1
0*x^5*a^2*b*c^2*d^4*e*f^3*(-d/c)^(1/2)+2*x^5*a^2*b*c*d^5*e^2*f^2*(-d/c)^(1/
2)-10*x^5*a*b^2*c^2*d^4*e^2*f^2*(-d/c)^(1/2)+5*x^5*a*b^2*c*d^5*e^3*f*(-d/c)
^(1/2)+9*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*b^3*c
^4*d^2*e^3*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+6*EllipticF(x*(-d/c)^(
1/2),(c*f/d/e)^(1/2))*a^3*c^3*d^3*e*f^3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(
1/2)-4*x^3*a^3*c*d^5*e^2*f^2*(-d/c)^(1/2)+12*x^3*a^2*b*c^4*d^2*f^4*(-d/c)^(
1/2)-7*x^5*a^3*c*d^5*e*f^3*(-d/c)^(1/2)+6*x^5*a^2*b*c^3*d^3*f^4*(-d/c)^(1/2
)-2*x^5*a^2*b*d^6*e^3*f*(-d/c)^(1/2)-3*x^5*a*b^2*c^4*d^2*f^4*(-d/c)^(1/2))/
(f*x^2+e)^(1/2)/(c*f-d*e)^3/(a*d-b*c)^2/(-d/c)^(1/2)/(a*f-b*e)/c^2/a/e/(d*x
^2+c)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+a)(dx^2+c)^{\frac{5}{2}}(fx^2+e)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(5/2)/(f*x^2+e)^(3/2),x, algorithm="maxima"
```

```
)
```

```
[Out] integrate(1/((b*x^2 + a)*(d*x^2 + c)^(5/2)*(f*x^2 + e)^(3/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(5/2)/(f*x^2+e)^(3/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)/(d*x**2+c)**(5/2)/(f*x**2+e)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)\left(dx^2+c\right)^{\frac{5}{2}}\left(fx^2+e\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(5/2)/(f*x^2+e)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)*(d*x^2 + c)^(5/2)*(f*x^2 + e)^(3/2)), x)
```

3.89 $\int \frac{(1+x^2)^{3/2}\sqrt{2+x^2}}{a+bx^2}\,dx$

Optimal. Leaf size=242

$$-\frac{\sqrt{x^2+2}(3a-7b)\text{EllipticF}\left(\tan^{-1}(x),\frac{1}{2}\right)}{3\sqrt{2}b^2\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}}-\frac{\sqrt{x^2+2}x(a-2b)}{b^2\sqrt{x^2+1}}+\frac{\sqrt{2}\sqrt{x^2+2}(a-2b)E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{b^2\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}}+\frac{\sqrt{x^2+2}(a-2\cdots}{\sqrt{2}\cdots}$$

```
[Out] -(((a - 2*b)*x*Sqrt[2 + x^2])/(b^2*Sqrt[1 + x^2])) + (x*Sqrt[1 + x^2]*Sqrt[
2 + x^2])/(3*b) + (Sqrt[2]*(a - 2*b)*Sqrt[2 + x^2]*EllipticE[ArcTan[x], 1/2
])/(b^2*Sqrt[1 + x^2]*Sqrt[(2 + x^2)/(1 + x^2)]) - ((3*a - 7*b)*Sqrt[2 + x^
2]*EllipticF[ArcTan[x], 1/2])/(3*Sqrt[2]*b^2*Sqrt[1 + x^2]*Sqrt[(2 + x^2)/(
1 + x^2)]) + ((a - 2*b)*(a - b)*Sqrt[2 + x^2]*EllipticPi[1 - b/a, ArcTan[x]
, 1/2])/(Sqrt[2]*a*b^2*Sqrt[1 + x^2]*Sqrt[(2 + x^2)/(1 + x^2)])
```

Rubi [A] time = 0.147457, antiderivative size = 239, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {543, 539, 528, 531, 418, 492, 411}

$$-\frac{x\sqrt{x^2+2}(a-2b)}{b^2\sqrt{x^2+1}}-\frac{\sqrt{2}\sqrt{x^2+2}(3a-5b)F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{3b^2\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}}+\frac{\sqrt{2}\sqrt{x^2+2}(a-2b)E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{b^2\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}}+\frac{2\sqrt{x^2+1}(a-b)^2\Pi\cdots}{ab^2\sqrt{\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + x^2)^(3/2)*Sqrt[2 + x^2])/(a + b*x^2),x]
```

```
[Out] -(((a - 2*b)*x*Sqrt[2 + x^2])/(b^2*Sqrt[1 + x^2])) + (x*Sqrt[1 + x^2]*Sqrt[
2 + x^2])/(3*b) + (Sqrt[2]*(a - 2*b)*Sqrt[2 + x^2]*EllipticE[ArcTan[x], 1/2
])/(b^2*Sqrt[1 + x^2]*Sqrt[(2 + x^2)/(1 + x^2)]) - (Sqrt[2]*(3*a - 5*b)*Sqr
t[2 + x^2]*EllipticF[ArcTan[x], 1/2])/(3*b^2*Sqrt[1 + x^2]*Sqrt[(2 + x^2)/(
1 + x^2)]) + (2*(a - b)^2*Sqrt[1 + x^2]*EllipticPi[1 - (2*b)/a, ArcTan[x/Sq
rt[2]], -1])/(a*b^2*Sqrt[(1 + x^2)/(2 + x^2)]*Sqrt[2 + x^2])
```

Rule 543

```
Int[(((c_) + (d_.)*(x_)^2)^(3/2)*Sqrt[(e_) + (f_.)*(x_)^2])/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Dist[(b*c - a*d)^2/b^2, Int[Sqrt[e + f*x^2]/((a + b*x^
2)*Sqrt[c + d*x^2]), x], x] + Dist[d/b^2, Int[((2*b*c - a*d + b*d*x^2)*Sqrt
[e + f*x^2])/Sqrt[c + d*x^2], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && Pos
Q[d/c] && PosQ[f/e]
```

Rule 539

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[(c*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcT
an[Rt[d/c, 2]*x], 1 - (c*f)/(d*e)])/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c
*(e + f*x^2))/(e*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
```

```
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(1+x^2\right)^{3/2}\sqrt{2+x^2}}{a+bx^2}\,dx &= \frac{\int \frac{\sqrt{2+x^2}\left(-a+2b+bx^2\right)}{\sqrt{1+x^2}}\,dx}{b^2}+\frac{(a-b)^2\int \frac{\sqrt{2+x^2}}{\sqrt{1+x^2}\left(a+bx^2\right)}\,dx}{b^2}\\
&= \frac{x\sqrt{1+x^2}\sqrt{2+x^2}}{3b}+\frac{2(a-b)^2\sqrt{1+x^2}\,\Pi\left(1-\frac{2b}{a};\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-1\right)}{ab^2\sqrt{\frac{1+x^2}{2+x^2}}\sqrt{2+x^2}}+\frac{\int \frac{-2(3a-5b)-3(a-2b)x^2}{\sqrt{1+x^2}\sqrt{2+x^2}}\,dx}{3b^2}\\
&= \frac{x\sqrt{1+x^2}\sqrt{2+x^2}}{3b}+\frac{2(a-b)^2\sqrt{1+x^2}\,\Pi\left(1-\frac{2b}{a};\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-1\right)}{ab^2\sqrt{\frac{1+x^2}{2+x^2}}\sqrt{2+x^2}}-\frac{(2(3a-5b))\int \frac{1}{\sqrt{1+x^2}\sqrt{2+\ldots}}}{3b^2}\\
&= -\frac{(a-2b)x\sqrt{2+x^2}}{b^2\sqrt{1+x^2}}+\frac{x\sqrt{1+x^2}\sqrt{2+x^2}}{3b}-\frac{\sqrt{2}(3a-5b)\sqrt{2+x^2}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{3b^2\sqrt{1+x^2}\sqrt{\frac{2+x^2}{1+x^2}}}+\frac{2(a-b)^2\sqrt{\ldots}}{\ldots}\\
&= -\frac{(a-2b)x\sqrt{2+x^2}}{b^2\sqrt{1+x^2}}+\frac{x\sqrt{1+x^2}\sqrt{2+x^2}}{3b}+\frac{\sqrt{2}(a-2b)\sqrt{2+x^2}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{b^2\sqrt{1+x^2}\sqrt{\frac{2+x^2}{1+x^2}}}-\frac{\sqrt{2}(3a-5b)\ldots}{3b^2\ldots}
\end{aligned}$$

Mathematica [C] time = 0.355601, size = 204, normalized size = 0.84

$$\frac{-ia\left(3a^2-9ab+7b^2\right)\text{EllipticF}\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right),2\right)+3ia^3\Pi\left(\frac{2b}{a};i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-12ia^2b\Pi\left(\frac{2b}{a};i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)+ab^2x\ldots}{3ab^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + x^2)^(3/2)*Sqrt[2 + x^2])/(a + b*x^2),x]
```

```
[Out] (a*b^2*x*Sqrt[1 + x^2]*Sqrt[2 + x^2] + (3*I)*a*(a - 2*b)*b*EllipticE[I*ArcS
inh[x/Sqrt[2]], 2] - I*a*(3*a^2 - 9*a*b + 7*b^2)*EllipticF[I*ArcSinh[x/Sqrt
[2]], 2] + (3*I)*a^3*EllipticPi[(2*b)/a, I*ArcSinh[x/Sqrt[2]], 2] - (12*I)*
a^2*b*EllipticPi[(2*b)/a, I*ArcSinh[x/Sqrt[2]], 2] + (15*I)*a*b^2*EllipticP
i[(2*b)/a, I*ArcSinh[x/Sqrt[2]], 2] - (6*I)*b^3*EllipticPi[(2*b)/a, I*ArcSi
nh[x/Sqrt[2]], 2])/(3*a*b^3)
```

Maple [C] time = 0.035, size = 370, normalized size = 1.5

$$-\frac{1}{\left(3x^4+9x^2+6\right)b^3a}\sqrt{x^2+1}\sqrt{x^2+2}\left(-x^5ab^2+3i\sqrt{x^2+1}\sqrt{x^2+2}\mathrm{EllipticF}\left(\frac{i}{2}x\sqrt{2},\sqrt{2}\right)a^3-9i\sqrt{x^2+1}\sqrt{x^2+2}\mathrm{El}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)^(3/2)*(x^2+2)^(1/2)/(b*x^2+a),x)
```

```
[Out] -1/3*(x^2+1)^(1/2)*(x^2+2)^(1/2)*(-x^5*a*b^2+3*I*(x^2+1)^(1/2)*(x^2+2)^(1/2
)*EllipticF(1/2*I*x*2^(1/2),2^(1/2))*a^3-9*I*(x^2+1)^(1/2)*(x^2+2)^(1/2)*El
lipticF(1/2*I*x*2^(1/2),2^(1/2))*a^2*b+7*I*(x^2+1)^(1/2)*(x^2+2)^(1/2)*Elli
pticF(1/2*I*x*2^(1/2),2^(1/2))*a*b^2-3*I*(x^2+1)^(1/2)*(x^2+2)^(1/2)*Ellipt
icE(1/2*I*x*2^(1/2),2^(1/2))*a^2*b+6*I*(x^2+1)^(1/2)*(x^2+2)^(1/2)*Elliptic
E(1/2*I*x*2^(1/2),2^(1/2))*a*b^2-3*I*(x^2+1)^(1/2)*(x^2+2)^(1/2)*EllipticPi
(1/2*I*x*2^(1/2),2*b/a,2^(1/2))*a^3+12*I*(x^2+1)^(1/2)*(x^2+2)^(1/2)*Ellipt
icPi(1/2*I*x*2^(1/2),2*b/a,2^(1/2))*a^2*b-15*I*(x^2+1)^(1/2)*(x^2+2)^(1/2)*
EllipticPi(1/2*I*x*2^(1/2),2*b/a,2^(1/2))*a*b^2+6*I*(x^2+1)^(1/2)*(x^2+2)^(
1/2)*EllipticPi(1/2*I*x*2^(1/2),2*b/a,2^(1/2))*b^3-3*x^3*a*b^2-2*x*a*b^2)/(
x^4+3*x^2+2)/b^3/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2+2}\left(x^2+1\right)^{\frac{3}{2}}}{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(3/2)*(x^2+2)^(1/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2 + 2)*(x^2 + 1)^(3/2)/(b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{x^2+2}\left(x^2+1\right)^{\frac{3}{2}}}{bx^2+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(3/2)*(x^2+2)^(1/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2 + 2)*(x^2 + 1)^(3/2)/(b*x^2 + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)**(3/2)*(x**2+2)**(1/2)/(b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2+2}\left(x^2+1\right)^{\frac{3}{2}}}{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(3/2)*(x^2+2)^(1/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 + 2)*(x^2 + 1)^(3/2)/(b*x^2 + a), x)
```

3.90 $\int \frac{\sqrt{1+x^2}\sqrt{2+x^2}}{a+bx^2}\,dx$

Optimal. Leaf size=192

$$\frac{\sqrt{x^2+2}\,\text{EllipticF}\left(\tan^{-1}(x),\frac{1}{2}\right)}{\sqrt{2}b\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}}-\frac{\sqrt{x^2+2}(a-2b)\Pi\left(1-\frac{b}{a};\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}ab\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}}+\frac{\sqrt{x^2+2}x}{b\sqrt{x^2+1}}-\frac{\sqrt{2}\sqrt{x^2+2}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{b\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}}$$

```
[Out] (x*Sqrt[2 + x^2])/(b*Sqrt[1 + x^2]) - (Sqrt[2]*Sqrt[2 + x^2]*EllipticE[ArcT
an[x], 1/2])/(b*Sqrt[1 + x^2]*Sqrt[(2 + x^2)/(1 + x^2)]) + (Sqrt[2 + x^2]*E
llipticF[ArcTan[x], 1/2])/(Sqrt[2]*b*Sqrt[1 + x^2]*Sqrt[(2 + x^2)/(1 + x^2)
]) - ((a - 2*b)*Sqrt[2 + x^2]*EllipticPi[1 - b/a, ArcTan[x], 1/2])/(Sqrt[2]
*a*b*Sqrt[1 + x^2]*Sqrt[(2 + x^2)/(1 + x^2)])
```

Rubi [A] time = 0.0918102, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {534, 422, 418, 492, 411, 539}

$$-\frac{\sqrt{x^2+2}(a-2b)\Pi\left(1-\frac{b}{a};\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}ab\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}}+\frac{\sqrt{x^2+2}x}{b\sqrt{x^2+1}}+\frac{\sqrt{x^2+2}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}b\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}}-\frac{\sqrt{2}\sqrt{x^2+2}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{b\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[1 + x^2]*Sqrt[2 + x^2])/(a + b*x^2),x]
```

```
[Out] (x*Sqrt[2 + x^2])/(b*Sqrt[1 + x^2]) - (Sqrt[2]*Sqrt[2 + x^2]*EllipticE[ArcT
an[x], 1/2])/(b*Sqrt[1 + x^2]*Sqrt[(2 + x^2)/(1 + x^2)]) + (Sqrt[2 + x^2]*E
llipticF[ArcTan[x], 1/2])/(Sqrt[2]*b*Sqrt[1 + x^2]*Sqrt[(2 + x^2)/(1 + x^2)
]) - ((a - 2*b)*Sqrt[2 + x^2]*EllipticPi[1 - b/a, ArcTan[x], 1/2])/(Sqrt[2]
*a*b*Sqrt[1 + x^2]*Sqrt[(2 + x^2)/(1 + x^2)])
```

Rule 534

```
Int[(Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)^2])/((a_) + (b_.)*(x_
)^2), x_Symbol] :> Dist[d/b, Int[Sqrt[e + f*x^2]/Sqrt[c + d*x^2], x], x] +
Dist[(b*c - a*d)/b, Int[Sqrt[e + f*x^2]/((a + b*x^2)*Sqrt[c + d*x^2]), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[d/c, 0] && GtQ[f/e, 0] &&  !Simpl
erSqrtQ[d/c, f/e]
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 539

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[(c*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcT
an[Rt[d/c, 2]*x], 1 - (c*f)/(d*e)])/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c
*(e + f*x^2))/(e*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rubi steps

$$\int \frac{\sqrt{1+x^2}\sqrt{2+x^2}}{a+bx^2}\,dx = \frac{\int \frac{\sqrt{1+x^2}}{\sqrt{2+x^2}}\,dx}{b} + \frac{(-a+2b)\int \frac{\sqrt{1+x^2}}{\sqrt{2+x^2}\left(a+bx^2\right)}\,dx}{b}$$

$$= -\frac{(a-2b)\sqrt{2+x^2}\,\Pi\left(1-\frac{b}{a};\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}ab\sqrt{1+x^2}\sqrt{\frac{2+x^2}{1+x^2}}} + \frac{\int \frac{1}{\sqrt{1+x^2}\sqrt{2+x^2}}\,dx}{b} + \frac{\int \frac{x^2}{\sqrt{1+x^2}\sqrt{2+x^2}}\,dx}{b}$$

$$= \frac{x\sqrt{2+x^2}}{b\sqrt{1+x^2}} + \frac{\sqrt{2+x^2}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}b\sqrt{1+x^2}\sqrt{\frac{2+x^2}{1+x^2}}} - \frac{(a-2b)\sqrt{2+x^2}\,\Pi\left(1-\frac{b}{a};\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}ab\sqrt{1+x^2}\sqrt{\frac{2+x^2}{1+x^2}}} - \frac{\int \frac{\sqrt{2+x^2}}{\left(1+x^2\right)^{3/2}}\,dx}{b}$$

$$= \frac{x\sqrt{2+x^2}}{b\sqrt{1+x^2}} - \frac{\sqrt{2}\sqrt{2+x^2}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{b\sqrt{1+x^2}\sqrt{\frac{2+x^2}{1+x^2}}} + \frac{\sqrt{2+x^2}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}b\sqrt{1+x^2}\sqrt{\frac{2+x^2}{1+x^2}}} - \frac{(a-2b)\sqrt{2+x^2}\,\Pi\left(1-\frac{b}{a};\mathrm{t}\right.}{\sqrt{2}ab\sqrt{1+x^2}\sqrt{\frac{2+}{1+}}}$$

Mathematica [C] time = 0.191546, size = 71, normalized size = 0.37

$$\frac{i\left((a-b)\left(a\,\text{EllipticF}\left(i\sinh^{-1}(x),\frac{1}{2}\right)-(a-2b)\Pi\left(\frac{b}{a};i\sinh^{-1}(x)|\frac{1}{2}\right)\right)-2abE\left(i\sinh^{-1}(x)|\frac{1}{2}\right)\right)}{\sqrt{2}ab^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[1 + x^2]*Sqrt[2 + x^2])/(a + b*x^2),x]
```

```
[Out] (I*(-2*a*b*EllipticE[I*ArcSinh[x], 1/2] + (a - b)*(a*EllipticF[I*ArcSinh[x]
, 1/2] - (a - 2*b)*EllipticPi[b/a, I*ArcSinh[x], 1/2])))/(Sqrt[2]*a*b^2)
```

Maple [C] time = 0.008, size = 120, normalized size = 0.6

$$\frac{-i}{ab^2}\left(\text{EllipticE}\left(\frac{i}{2}x\sqrt{2},\sqrt{2}\right)ba-\text{EllipticF}\left(\frac{i}{2}x\sqrt{2},\sqrt{2}\right)a^2+2\,\text{EllipticF}\left(i/2x\sqrt{2},\sqrt{2}\right)ba+a^2\text{EllipticPi}\left(\frac{i}{2}x\sqrt{2},2\frac{b}{a},\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)^(1/2)*(x^2+2)^(1/2)/(b*x^2+a),x)
```

```
[Out] -I*(EllipticE(1/2*I*x*2^(1/2),2^(1/2))*b*a-EllipticF(1/2*I*x*2^(1/2),2^(1/2
))*a^2+2*EllipticF(1/2*I*x*2^(1/2),2^(1/2))*b*a+a^2*EllipticPi(1/2*I*x*2^(1
/2),2*b/a,2^(1/2))-3*EllipticPi(1/2*I*x*2^(1/2),2*b/a,2^(1/2))*b*a+2*Ellipt
icPi(1/2*I*x*2^(1/2),2*b/a,2^(1/2))*b^2)/a/b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2+2}\sqrt{x^2+1}}{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2)*(x^2+2)^(1/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2 + 2)*sqrt(x^2 + 1)/(b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^2+2}\sqrt{x^2+1}}{bx^2+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2)*(x^2+2)^(1/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2 + 2)*sqrt(x^2 + 1)/(b*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2+1}\sqrt{x^2+2}}{a+bx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)**(1/2)*(x**2+2)**(1/2)/(b*x**2+a),x)
```

```
[Out] Integral(sqrt(x**2 + 1)*sqrt(x**2 + 2)/(a + b*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2+2}\sqrt{x^2+1}}{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2)*(x^2+2)^(1/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 + 2)*sqrt(x^2 + 1)/(b*x^2 + a), x)
```

3.91 $\int \frac{\sqrt{2+x^2}}{\sqrt{1+x^2}(a+bx^2)}\,dx$

Optimal. Leaf size=58

$$\frac{2\sqrt{x^2+1}\,\Pi\left(1-\frac{2b}{a};\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-1\right)}{a\sqrt{\frac{x^2+1}{x^2+2}}\sqrt{x^2+2}}$$

```
[Out] (2*Sqrt[1 + x^2]*EllipticPi[1 - (2*b)/a, ArcTan[x/Sqrt[2]], -1])/(a*Sqrt[(1
 + x^2)/(2 + x^2)]*Sqrt[2 + x^2])
```

Rubi [A] time = 0.0217733, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {539}

$$\frac{2\sqrt{x^2+1}\,\Pi\left(1-\frac{2b}{a};\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-1\right)}{a\sqrt{\frac{x^2+1}{x^2+2}}\sqrt{x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + x^2]/(Sqrt[1 + x^2]*(a + b*x^2)),x]
```

```
[Out] (2*Sqrt[1 + x^2]*EllipticPi[1 - (2*b)/a, ArcTan[x/Sqrt[2]], -1])/(a*Sqrt[(1
 + x^2)/(2 + x^2)]*Sqrt[2 + x^2])
```

Rule 539

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[(c*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcT
an[Rt[d/c, 2]*x], 1 - (c*f)/(d*e)])/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c
*(e + f*x^2))/(e*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rubi steps

$$\int \frac{\sqrt{2+x^2}}{\sqrt{1+x^2}\left(a+bx^2\right)}\,dx = \frac{2\sqrt{1+x^2}\,\Pi\left(1-\frac{2b}{a};\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-1\right)}{a\sqrt{\frac{1+x^2}{2+x^2}}\sqrt{2+x^2}}$$

Mathematica [C] time = 0.172834, size = 50, normalized size = 0.86

$$-\frac{i\left(a\text{EllipticF}\left(i\sinh^{-1}(x),\frac{1}{2}\right)-(a-2b)\Pi\left(\frac{b}{a};i\sinh^{-1}(x)\middle|\frac{1}{2}\right)\right)}{\sqrt{2}ab}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + x^2]/(Sqrt[1 + x^2]*(a + b*x^2)),x]
```

```
[Out] ((-I)*(a*EllipticF[I*ArcSinh[x], 1/2] - (a - 2*b)*EllipticPi[b/a, I*ArcSinh
[x], 1/2]))/(Sqrt[2]*a*b)
```

Maple [A] time = 0.013, size = 64, normalized size = 1.1

$$\frac{-i}{ab}\left(a\text{EllipticF}\left(\frac{i}{2}x\sqrt{2},\sqrt{2}\right)-a\text{EllipticPi}\left(\frac{i}{2}x\sqrt{2},2\frac{b}{a},\sqrt{2}\right)+2\,b\text{EllipticPi}\left(i/2x\sqrt{2},2\frac{b}{a},\sqrt{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+2)^(1/2)/(x^2+1)^(1/2)/(b*x^2+a),x)
```

```
[Out] -I*(a*EllipticF(1/2*I*x*2^(1/2),2^(1/2))-a*EllipticPi(1/2*I*x*2^(1/2),2*b/a
,2^(1/2))+2*b*EllipticPi(1/2*I*x*2^(1/2),2*b/a,2^(1/2)))/a/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{x^2+2}}{\left(bx^2+a\right)\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2)^(1/2)/(x^2+1)^(1/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2 + 2)/((b*x^2 + a)*sqrt(x^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^2+2}\sqrt{x^2+1}}{bx^4+(a+b)x^2+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2)^(1/2)/(x^2+1)^(1/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2 + 2)*sqrt(x^2 + 1)/(b*x^4 + (a + b)*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{x^2+2}}{\left(a+bx^2\right)\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+2)**(1/2)/(x**2+1)**(1/2)/(b*x**2+a),x)
```

```
[Out] Integral(sqrt(x**2 + 2)/((a + b*x**2)*sqrt(x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2+2}}{\left(bx^2+a\right)\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2)^(1/2)/(x^2+1)^(1/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 + 2)/((b*x^2 + a)*sqrt(x^2 + 1)), x)
```

3.92 $\int \frac{\sqrt{2+x^2}}{\left(1+x^2\right)^{3/2}\left(a+bx^2\right)}\,dx$

Optimal. Leaf size=121

$$\frac{\sqrt{2}\sqrt{x^2+2}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}(a-b)}-\frac{2b\sqrt{x^2+1}\Pi\left(1-\frac{2b}{a};\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|-1\right)}{a\sqrt{\frac{x^2+1}{x^2+2}}\sqrt{x^2+2}(a-b)}$$

```
[Out] (Sqrt[2]*Sqrt[2 + x^2]*EllipticE[ArcTan[x], 1/2])/((a - b)*Sqrt[1 + x^2]*Sq
rt[(2 + x^2)/(1 + x^2)]) - (2*b*Sqrt[1 + x^2]*EllipticPi[1 - (2*b)/a, ArcTa
n[x/Sqrt[2]], -1])/(a*(a - b)*Sqrt[(1 + x^2)/(2 + x^2)]*Sqrt[2 + x^2])
```

Rubi [A] time = 0.0597335, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {541, 539, 411}

$$\frac{\sqrt{2}\sqrt{x^2+2}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}(a-b)}-\frac{2b\sqrt{x^2+1}\Pi\left(1-\frac{2b}{a};\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|-1\right)}{a\sqrt{\frac{x^2+1}{x^2+2}}\sqrt{x^2+2}(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + x^2]/((1 + x^2)^(3/2)*(a + b*x^2)),x]
```

```
[Out] (Sqrt[2]*Sqrt[2 + x^2]*EllipticE[ArcTan[x], 1/2])/((a - b)*Sqrt[1 + x^2]*Sq
rt[(2 + x^2)/(1 + x^2)]) - (2*b*Sqrt[1 + x^2]*EllipticPi[1 - (2*b)/a, ArcTa
n[x/Sqrt[2]], -1])/(a*(a - b)*Sqrt[(1 + x^2)/(2 + x^2)]*Sqrt[2 + x^2])
```

Rule 541

```
Int[Sqrt[(e_) + (f_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[b/(b*c - a*d), Int[Sqrt[e + f*x^2]/((a + b*x^2)*S
qrt[c + d*x^2]), x], x] - Dist[d/(b*c - a*d), Int[Sqrt[e + f*x^2]/(c + d*x^
2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[d/c] && PosQ[f/e]
```

Rule 539

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[(c*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcT
an[Rt[d/c, 2]*x], 1 - (c*f)/(d*e)])/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c
*(e + f*x^2))/(e*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\sqrt{2+x^2}}{\left(1+x^2\right)^{3/2}\left(a+bx^2\right)}\,dx = -\frac{b\int \frac{\sqrt{2+x^2}}{\sqrt{1+x^2}\left(a+bx^2\right)}\,dx}{a-b} - \frac{\int \frac{\sqrt{2+x^2}}{\left(1+x^2\right)^{3/2}}\,dx}{-a+b}$$

$$= \frac{\sqrt{2}\sqrt{2+x^2}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{(a-b)\sqrt{1+x^2}\sqrt{\frac{2+x^2}{1+x^2}}} - \frac{2b\sqrt{1+x^2}\Pi\left(1-\frac{2b}{a};\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-1\right)}{a(a-b)\sqrt{\frac{1+x^2}{2+x^2}}\sqrt{2+x^2}}$$

Mathematica [C] time = 0.346171, size = 122, normalized size = 1.01

$$\frac{-i\sqrt{2}\,\text{EllipticF}\left(i\sinh^{-1}(x),\frac{1}{2}\right)+\frac{2i\sqrt{2}b\Pi\left(\frac{b}{a};i\sinh^{-1}(x)\middle|\frac{1}{2}\right)}{a}-i\sqrt{2}\Pi\left(\frac{b}{a};i\sinh^{-1}(x)\middle|\frac{1}{2}\right)+\frac{2\sqrt{x^2+2}x}{\sqrt{x^2+1}}+2i\sqrt{2}E\left(i\sinh^{-1}(x)\middle|\frac{1}{2}\right)}{2a-2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + x^2]/((1 + x^2)^(3/2)*(a + b*x^2)),x]
```

```
[Out] ((2*x*Sqrt[2 + x^2])/Sqrt[1 + x^2] + (2*I)*Sqrt[2]*EllipticE[I*ArcSinh[x],
1/2] - I*Sqrt[2]*EllipticF[I*ArcSinh[x], 1/2] - I*Sqrt[2]*EllipticPi[b/a, I
*ArcSinh[x], 1/2] + ((2*I)*Sqrt[2]*b*EllipticPi[b/a, I*ArcSinh[x], 1/2])/a)
/(2*a - 2*b)
```

Maple [A] time = 0.033, size = 147, normalized size = 1.2

$$\frac{1}{a\left(x^4+3x^2+2\right)(a-b)}\left(i\text{EllipticE}\left(\frac{i}{2}x\sqrt{2},\sqrt{2}\right)a\sqrt{x^2+1}\sqrt{x^2+2}-i\text{EllipticPi}\left(\frac{i}{2}x\sqrt{2},2\frac{b}{a},\sqrt{2}\right)a\sqrt{x^2+1}\sqrt{x^2+2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+2)^(1/2)/(x^2+1)^(3/2)/(b*x^2+a),x)
```

```
[Out] (I*EllipticE(1/2*I*x*2^(1/2),2^(1/2))*a*(x^2+1)^(1/2)*(x^2+2)^(1/2)-I*Ellip
ticPi(1/2*I*x*2^(1/2),2*b/a,2^(1/2))*a*(x^2+1)^(1/2)*(x^2+2)^(1/2)+2*I*Elli
pticPi(1/2*I*x*2^(1/2),2*b/a,2^(1/2))*b*(x^2+1)^(1/2)*(x^2+2)^(1/2)+a*x^3+2
*a*x)*(x^2+1)^(1/2)*(x^2+2)^(1/2)/a/(x^4+3*x^2+2)/(a-b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2+2}}{\left(bx^2+a\right)\left(x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2)^(1/2)/(x^2+1)^(3/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2 + 2)/((b*x^2 + a)*(x^2 + 1)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^2+2}\sqrt{x^2+1}}{bx^6+(a+2\,b)x^4+(2\,a+b)x^2+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2)^(1/2)/(x^2+1)^(3/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2 + 2)*sqrt(x^2 + 1)/(b*x^6 + (a + 2*b)*x^4 + (2*a + b)*x^2
 + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+2)**(1/2)/(x**2+1)**(3/2)/(b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2+2}}{\left(bx^2+a\right)\left(x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2)^(1/2)/(x^2+1)^(3/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 + 2)/((b*x^2 + a)*(x^2 + 1)^(3/2)), x)
```

3.93 $\int \frac{\sqrt{2+x^2}}{\left(1+x^2\right)^{5/2}\left(a+bx^2\right)}\,dx$

Optimal. Leaf size=215

$$-\frac{\sqrt{2}\sqrt{x^2+2}\text{EllipticF}\left(\tan^{-1}(x),\frac{1}{2}\right)}{3\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}(a-b)}+\frac{2b^2\sqrt{x^2+1}\Pi\left(1-\frac{2b}{a};\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-1\right)}{a\sqrt{\frac{x^2+1}{x^2+2}}\sqrt{x^2+2}(a-b)^2}+\frac{x\sqrt{x^2+2}}{3\left(x^2+1\right)^{3/2}(a-b)}+\frac{\sqrt{2}\sqrt{x^2+2}(a}{\sqrt{x^2+}}$$

```
[Out] (x*Sqrt[2 + x^2])/(3*(a - b)*(1 + x^2)^(3/2)) + (Sqrt[2]*(a - 2*b)*Sqrt[2 +
 x^2]*EllipticE[ArcTan[x], 1/2])/((a - b)^2*Sqrt[1 + x^2]*Sqrt[(2 + x^2)/(1
 + x^2)]) - (Sqrt[2]*Sqrt[2 + x^2]*EllipticF[ArcTan[x], 1/2])/(3*(a - b)*Sq
rt[1 + x^2]*Sqrt[(2 + x^2)/(1 + x^2)]) + (2*b^2*Sqrt[1 + x^2]*EllipticPi[1
- (2*b)/a, ArcTan[x/Sqrt[2]], -1])/(a*(a - b)^2*Sqrt[(1 + x^2)/(2 + x^2)]*S
qrt[2 + x^2])
```

Rubi [A] time = 0.144995, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {546, 539, 526, 525, 418, 411}

$$\frac{2b^2\sqrt{x^2+1}\Pi\left(1-\frac{2b}{a};\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-1\right)}{a\sqrt{\frac{x^2+1}{x^2+2}}\sqrt{x^2+2}(a-b)^2}+\frac{x\sqrt{x^2+2}}{3\left(x^2+1\right)^{3/2}(a-b)}-\frac{\sqrt{2}\sqrt{x^2+2}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{3\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}(a-b)}+\frac{\sqrt{2}\sqrt{x^2+2}(a-2b)E\left(\text{t}\right.}{\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}(a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + x^2]/((1 + x^2)^(5/2)*(a + b*x^2)),x]
```

```
[Out] (x*Sqrt[2 + x^2])/(3*(a - b)*(1 + x^2)^(3/2)) + (Sqrt[2]*(a - 2*b)*Sqrt[2 +
 x^2]*EllipticE[ArcTan[x], 1/2])/((a - b)^2*Sqrt[1 + x^2]*Sqrt[(2 + x^2)/(1
 + x^2)]) - (Sqrt[2]*Sqrt[2 + x^2]*EllipticF[ArcTan[x], 1/2])/(3*(a - b)*Sq
rt[1 + x^2]*Sqrt[(2 + x^2)/(1 + x^2)]) + (2*b^2*Sqrt[1 + x^2]*EllipticPi[1
- (2*b)/a, ArcTan[x/Sqrt[2]], -1])/(a*(a - b)^2*Sqrt[(1 + x^2)/(2 + x^2)]*S
qrt[2 + x^2])
```

Rule 546

```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Dist[b^2/(b*c - a*d)^2, Int[((c + d*x^2)^(q + 2)*(e +
f*x^2)^r)/(a + b*x^2), x], x] - Dist[d/(b*c - a*d)^2, Int[(c + d*x^2)^q*(e
+ f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r},
x] && LtQ[q, -1]
```

Rule 539

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[(c*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcT
an[Rt[d/c, 2]*x], 1 - (c*f)/(d*e)])/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c
*(e + f*x^2))/(e*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 526

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
```

```
d*x^n)^q)/(a*b*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p +
 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 525

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{2+x^2}}{\left(1+x^2\right)^{5/2}\left(a+bx^2\right)}\,dx &= -\frac{\int \frac{\sqrt{2+x^2}\left(-a+2b+bx^2\right)}{\left(1+x^2\right)^{5/2}}\,dx}{(a-b)^2}+\frac{b^2\int \frac{\sqrt{2+x^2}}{\sqrt{1+x^2}\left(a+bx^2\right)}\,dx}{(a-b)^2}\\
&= \frac{x\sqrt{2+x^2}}{3(a-b)\left(1+x^2\right)^{3/2}}+\frac{2b^2\sqrt{1+x^2}\,\Pi\left(1-\frac{2b}{a};\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-1\right)}{a(a-b)^2\sqrt{\frac{1+x^2}{2+x^2}}\sqrt{2+x^2}}+\frac{\int \frac{2(2a-5b)+(a-4b)x^2}{\left(1+x^2\right)^{3/2}\sqrt{2+x^2}}\,dx}{3(a-b)^2}\\
&= \frac{x\sqrt{2+x^2}}{3(a-b)\left(1+x^2\right)^{3/2}}+\frac{2b^2\sqrt{1+x^2}\,\Pi\left(1-\frac{2b}{a};\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-1\right)}{a(a-b)^2\sqrt{\frac{1+x^2}{2+x^2}}\sqrt{2+x^2}}+\frac{(a-2b)\int \frac{\sqrt{2+x^2}}{\left(1+x^2\right)^{3/2}}\,dx}{(a-b)^2}-\frac{2}{\cdots}\\
&= \frac{x\sqrt{2+x^2}}{3(a-b)\left(1+x^2\right)^{3/2}}+\frac{\sqrt{2}(a-2b)\sqrt{2+x^2}\,E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{(a-b)^2\sqrt{1+x^2}\sqrt{\frac{2+x^2}{1+x^2}}}-\frac{\sqrt{2}\sqrt{2+x^2}\,F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{3(a-b)\sqrt{1+x^2}\sqrt{\frac{2+x^2}{1+x^2}}}+\frac{2b\cdots}{\cdots}
\end{aligned}
$$

Mathematica [C] time = 0.377982, size = 357, normalized size = 1.66

$$
\frac{-i\sqrt{2}a\left(x^2+1\right)^2(4a-7b)\text{EllipticF}\left(i\sinh^{-1}(x),\frac{1}{2}\right)+6a^2\sqrt{x^2+1}\sqrt{x^2+2}x^3+8a^2\sqrt{x^2+1}\sqrt{x^2+2}x-6i\sqrt{2}b^2x^4\Pi\left(\frac{b}{a};i\sinh\cdots\right.}{\cdots}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + x^2]/((1 + x^2)^(5/2)*(a + b*x^2)),x]
```

```
[Out] (8*a^2*x*Sqrt[1 + x^2]*Sqrt[2 + x^2] - 14*a*b*x*Sqrt[1 + x^2]*Sqrt[2 + x^2]
 + 6*a^2*x^3*Sqrt[1 + x^2]*Sqrt[2 + x^2] - 12*a*b*x^3*Sqrt[1 + x^2]*Sqrt[2
+ x^2] + (6*I)*Sqrt[2]*a*(a - 2*b)*(1 + x^2)^2*EllipticE[I*ArcSinh[x], 1/2]
 - I*Sqrt[2]*a*(4*a - 7*b)*(1 + x^2)^2*EllipticF[I*ArcSinh[x], 1/2] + (3*I)
*Sqrt[2]*a*b*EllipticPi[b/a, I*ArcSinh[x], 1/2] - (6*I)*Sqrt[2]*b^2*Ellipti
cPi[b/a, I*ArcSinh[x], 1/2] + (6*I)*Sqrt[2]*a*b*x^2*EllipticPi[b/a, I*ArcSi
nh[x], 1/2] - (12*I)*Sqrt[2]*b^2*x^2*EllipticPi[b/a, I*ArcSinh[x], 1/2] + (
3*I)*Sqrt[2]*a*b*x^4*EllipticPi[b/a, I*ArcSinh[x], 1/2] - (6*I)*Sqrt[2]*b^2
*x^4*EllipticPi[b/a, I*ArcSinh[x], 1/2])/(6*a*(a - b)^2*(1 + x^2)^2)
```

Maple [B] time = 0.033, size = 477, normalized size = 2.2

$$-\frac{1}{3\,(a-b)^2\,a}\left(6\,i\mathrm{EllipticE}\left(\frac{i}{2}x\sqrt{2},\sqrt{2}\right)x^2ab\sqrt{x^2+2}\sqrt{x^2+1}-i\mathrm{EllipticF}\left(\frac{i}{2}x\sqrt{2},\sqrt{2}\right)ab\sqrt{x^2+2}\sqrt{x^2+1}-3\,i\mathrm{Ellipt}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+2)^(1/2)/(x^2+1)^(5/2)/(b*x^2+a),x)
```

```
[Out] -1/3*(6*I*EllipticE(1/2*I*x*2^(1/2),2^(1/2))*x^2*a*b*(x^2+2)^(1/2)*(x^2+1)^
(1/2)-I*EllipticF(1/2*I*x*2^(1/2),2^(1/2))*a*b*(x^2+2)^(1/2)*(x^2+1)^(1/2)-
3*I*EllipticE(1/2*I*x*2^(1/2),2^(1/2))*a^2*(x^2+2)^(1/2)*(x^2+1)^(1/2)+I*El
lipticF(1/2*I*x*2^(1/2),2^(1/2))*x^2*a^2*(x^2+2)^(1/2)*(x^2+1)^(1/2)-3*I*El
lipticE(1/2*I*x*2^(1/2),2^(1/2))*x^2*a^2*(x^2+2)^(1/2)*(x^2+1)^(1/2)-3*I*El
lipticPi(1/2*I*x*2^(1/2),2*b/a,2^(1/2))*a*b*(x^2+2)^(1/2)*(x^2+1)^(1/2)-3*x
^5*a^2+6*x^5*a*b+6*I*EllipticE(1/2*I*x*2^(1/2),2^(1/2))*a*b*(x^2+2)^(1/2)*(
x^2+1)^(1/2)-I*EllipticF(1/2*I*x*2^(1/2),2^(1/2))*x^2*a*b*(x^2+2)^(1/2)*(x^
2+1)^(1/2)-3*I*EllipticPi(1/2*I*x*2^(1/2),2*b/a,2^(1/2))*x^2*a*b*(x^2+2)^(1
/2)*(x^2+1)^(1/2)+6*I*EllipticPi(1/2*I*x*2^(1/2),2*b/a,2^(1/2))*x^2*b^2*(x^
2+2)^(1/2)*(x^2+1)^(1/2)+I*EllipticF(1/2*I*x*2^(1/2),2^(1/2))*a^2*(x^2+2)^(
1/2)*(x^2+1)^(1/2)+6*I*EllipticPi(1/2*I*x*2^(1/2),2*b/a,2^(1/2))*b^2*(x^2+2
)^(1/2)*(x^2+1)^(1/2)-10*x^3*a^2+19*x^3*a*b-8*a^2*x+14*x*a*b)/(x^2+2)^(1/2)
/(a-b)^2/a/(x^2+1)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2+2}}{(bx^2+a)(x^2+1)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2)^(1/2)/(x^2+1)^(5/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2 + 2)/((b*x^2 + a)*(x^2 + 1)^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{x^2+2}\sqrt{x^2+1}}{bx^8+(a+3\,b)x^6+3\,(a+b)x^4+(3\,a+b)x^2+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2)^(1/2)/(x^2+1)^(5/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2 + 2)*sqrt(x^2 + 1)/(b*x^8 + (a + 3*b)*x^6 + 3*(a + b)*x^4
 + (3*a + b)*x^2 + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+2)**(1/2)/(x**2+1)**(5/2)/(b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2+2}}{(bx^2+a)(x^2+1)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2)^(1/2)/(x^2+1)^(5/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 + 2)/((b*x^2 + a)*(x^2 + 1)^(5/2)), x)
```

3.94 $\int \frac{\sqrt{2+dx^2}\sqrt{3+fx^2}}{a+bx^2}\,dx$

Optimal. Leaf size=298

$$\frac{3d\sqrt{dx^2+2}\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{3}}\right),1-\frac{3d}{2f}\right)}{\sqrt{2}b\sqrt{f}\sqrt{fx^2+3}\sqrt{\frac{dx^2+2}{fx^2+3}}}+\frac{3\sqrt{dx^2+2}(2b-ad)\Pi\left(1-\frac{3b}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{3}}\right)|1-\frac{3d}{2f}\right)}{\sqrt{2}ab\sqrt{f}\sqrt{fx^2+3}\sqrt{\frac{dx^2+2}{fx^2+3}}}+\frac{fx\sqrt{dx^2+2}}{b\sqrt{fx^2+3}}-$$

```
[Out] (f*x*Sqrt[2 + d*x^2])/(b*Sqrt[3 + f*x^2]) - (Sqrt[2]*Sqrt[f]*Sqrt[2 + d*x^2
]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[3]], 1 - (3*d)/(2*f)])/(b*Sqrt[(2 + d*x
^2)/(3 + f*x^2)]*Sqrt[3 + f*x^2]) + (3*d*Sqrt[2 + d*x^2]*EllipticF[ArcTan[(
Sqrt[f]*x)/Sqrt[3]], 1 - (3*d)/(2*f)])/(Sqrt[2]*b*Sqrt[f]*Sqrt[(2 + d*x^2)/
(3 + f*x^2)]*Sqrt[3 + f*x^2]) + (3*(2*b - a*d)*Sqrt[2 + d*x^2]*EllipticPi[1
 - (3*b)/(a*f), ArcTan[(Sqrt[f]*x)/Sqrt[3]], 1 - (3*d)/(2*f)])/(Sqrt[2]*a*b
*Sqrt[f]*Sqrt[(2 + d*x^2)/(3 + f*x^2)]*Sqrt[3 + f*x^2])
```

Rubi [A] time = 0.179037, antiderivative size = 298, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {535, 422, 418, 492, 411, 539}

$$\frac{3\sqrt{dx^2+2}(2b-ad)\Pi\left(1-\frac{3b}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{3}}\right)|1-\frac{3d}{2f}\right)}{\sqrt{2}ab\sqrt{f}\sqrt{fx^2+3}\sqrt{\frac{dx^2+2}{fx^2+3}}}+\frac{fx\sqrt{dx^2+2}}{b\sqrt{fx^2+3}}+\frac{3d\sqrt{dx^2+2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{3}}\right)|1-\frac{3d}{2f}\right)}{\sqrt{2}b\sqrt{f}\sqrt{fx^2+3}\sqrt{\frac{dx^2+2}{fx^2+3}}}-\frac{\sqrt{2}\sqrt{f}\sqrt{d}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 + d*x^2]*Sqrt[3 + f*x^2])/(a + b*x^2),x]
```

```
[Out] (f*x*Sqrt[2 + d*x^2])/(b*Sqrt[3 + f*x^2]) - (Sqrt[2]*Sqrt[f]*Sqrt[2 + d*x^2
]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[3]], 1 - (3*d)/(2*f)])/(b*Sqrt[(2 + d*x
^2)/(3 + f*x^2)]*Sqrt[3 + f*x^2]) + (3*d*Sqrt[2 + d*x^2]*EllipticF[ArcTan[(
Sqrt[f]*x)/Sqrt[3]], 1 - (3*d)/(2*f)])/(Sqrt[2]*b*Sqrt[f]*Sqrt[(2 + d*x^2)/
(3 + f*x^2)]*Sqrt[3 + f*x^2]) + (3*(2*b - a*d)*Sqrt[2 + d*x^2]*EllipticPi[1
 - (3*b)/(a*f), ArcTan[(Sqrt[f]*x)/Sqrt[3]], 1 - (3*d)/(2*f)])/(Sqrt[2]*a*b
*Sqrt[f]*Sqrt[(2 + d*x^2)/(3 + f*x^2)]*Sqrt[3 + f*x^2])
```

Rule 535

```
Int[(Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)^2])/((a_) + (b_.)*(x_
)^2), x_Symbol] :> Dist[d/b, Int[Sqrt[e + f*x^2]/Sqrt[c + d*x^2], x], x] +
Dist[(b*c - a*d)/b, Int[Sqrt[e + f*x^2]/((a + b*x^2)*Sqrt[c + d*x^2]), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !SimplerSqrtQ[-(f/e), -(d/c)]
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
```

```
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 539

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[(c*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcT
an[Rt[d/c, 2]*x], 1 - (c*f)/(d*e)])/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c
*(e + f*x^2))/(e*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{2+dx^2}\sqrt{3+fx^2}}{a+bx^2}\,dx &= \frac{d\int \frac{\sqrt{3+fx^2}}{\sqrt{2+dx^2}}\,dx}{b} + \frac{(2b-ad)\int \frac{\sqrt{3+fx^2}}{\left(a+bx^2\right)\sqrt{2+dx^2}}\,dx}{b} \\
&= \frac{3(2b-ad)\sqrt{2+dx^2}\,\Pi\left(1-\frac{3b}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{3}}\right)|1-\frac{3d}{2f}\right)}{\sqrt{2}ab\sqrt{f}\sqrt{\frac{2+dx^2}{3+fx^2}}\sqrt{3+fx^2}} + \frac{(3d)\int \frac{1}{\sqrt{2+dx^2}\sqrt{3+fx^2}}\,dx}{b} + \frac{(df)\int \frac{}{\sqrt{2+d}}}{b} \\
&= \frac{fx\sqrt{2+dx^2}}{b\sqrt{3+fx^2}} + \frac{3d\sqrt{2+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{3}}\right)|1-\frac{3d}{2f}\right)}{\sqrt{2}b\sqrt{f}\sqrt{\frac{2+dx^2}{3+fx^2}}\sqrt{3+fx^2}} + \frac{3(2b-ad)\sqrt{2+dx^2}\,\Pi\left(1-\frac{3b}{af};\tan^{-1}\left(\right.\right.}{\sqrt{2}ab\sqrt{f}\sqrt{\frac{2+dx^2}{3+fx^2}}\sqrt{3+fx^2}} \\
&= \frac{fx\sqrt{2+dx^2}}{b\sqrt{3+fx^2}} - \frac{\sqrt{2}\sqrt{f}\sqrt{2+dx^2}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{3}}\right)|1-\frac{3d}{2f}\right)}{b\sqrt{\frac{2+dx^2}{3+fx^2}}\sqrt{3+fx^2}} + \frac{3d\sqrt{2+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{3}}\right)|1-\frac{3d}{2f}\right.}{\sqrt{2}b\sqrt{f}\sqrt{\frac{2+dx^2}{3+fx^2}}\sqrt{3+fx^2}}
\end{aligned}$$

Mathematica [C] time = 0.302216, size = 134, normalized size = 0.45

$$\frac{i\left((ad-2b)\left(af\,\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{2}}\right),\frac{2f}{3d}\right)+(3b-af)\Pi\left(\frac{2b}{ad};i\sinh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{2}}\right)|\frac{2f}{3d}\right)\right)-3abdE\left(i\sinh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{2}}\right)|\frac{2f}{3d}\right)\right)}{\sqrt{3}ab^2\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2 + d*x^2]*Sqrt[3 + f*x^2])/(a + b*x^2),x]
```

```
[Out] (I*(-3*a*b*d*EllipticE[I*ArcSinh[(Sqrt[d]*x)/Sqrt[2]], (2*f)/(3*d)] + (-2*b
 + a*d)*(a*f*EllipticF[I*ArcSinh[(Sqrt[d]*x)/Sqrt[2]], (2*f)/(3*d)] + (3*b
- a*f)*EllipticPi[(2*b)/(a*d), I*ArcSinh[(Sqrt[d]*x)/Sqrt[2]], (2*f)/(3*d)]
```

```
)))/(Sqrt[3]*a*b^2*Sqrt[d])
```

Maple [A] time = 0.027, size = 293, normalized size = 1.

$$\frac{\sqrt{2}}{2\,ab^{2}}\left(a^{2}\text{EllipticPi}\left(\frac{x\sqrt{3}}{3}\sqrt{-f},3\,\frac{b}{af},\frac{\sqrt{2}\sqrt{3}}{2}\sqrt{-d}\frac{1}{\sqrt{-f}}\right)df-\text{EllipticF}\left(\frac{x\sqrt{3}}{3}\sqrt{-f},\frac{\sqrt{2}\sqrt{3}}{2}\sqrt{\frac{d}{f}}\right)a^{2}df+2\,f\text{EllipticE}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+2)^(1/2)*(f*x^2+3)^(1/2)/(b*x^2+a),x)
```

```
[Out] 1/2*(a^2*EllipticPi(1/3*x*3^(1/2)*(-f)^(1/2),3*b/a/f,1/2*2^(1/2)*(-d)^(1/2)
*3^(1/2)/(-f)^(1/2))*d*f-EllipticF(1/3*x*3^(1/2)*(-f)^(1/2),1/2*2^(1/2)*3^(
1/2)*(1/f*d)^(1/2))*a^2*d*f+2*f*EllipticE(1/3*x*3^(1/2)*(-f)^(1/2),1/2*2^(1
/2)*3^(1/2)*(1/f*d)^(1/2))*b*a-3*EllipticPi(1/3*x*3^(1/2)*(-f)^(1/2),3*b/a/
f,1/2*2^(1/2)*(-d)^(1/2)*3^(1/2)/(-f)^(1/2))*d*b*a-2*EllipticPi(1/3*x*3^(1/
2)*(-f)^(1/2),3*b/a/f,1/2*2^(1/2)*(-d)^(1/2)*3^(1/2)/(-f)^(1/2))*f*b*a+3*El
lipticF(1/3*x*3^(1/2)*(-f)^(1/2),1/2*2^(1/2)*3^(1/2)*(1/f*d)^(1/2))*d*b*a+6
*EllipticPi(1/3*x*3^(1/2)*(-f)^(1/2),3*b/a/f,1/2*2^(1/2)*(-d)^(1/2)*3^(1/2)
/(-f)^(1/2))*b^2)*2^(1/2)/a/(-f)^(1/2)/b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^{2}+2}\sqrt{fx^{2}+3}}{bx^{2}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+2)^(1/2)*(f*x^2+3)^(1/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + 2)*sqrt(f*x^2 + 3)/(b*x^2 + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+2)^(1/2)*(f*x^2+3)^(1/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^{2}+2}\sqrt{fx^{2}+3}}{a+bx^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+2)**(1/2)*(f*x**2+3)**(1/2)/(b*x**2+a),x)
```

```
[Out] Integral(sqrt(d*x**2 + 2)*sqrt(f*x**2 + 3)/(a + b*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2+2}\sqrt{fx^2+3}}{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+2)^(1/2)*(f*x^2+3)^(1/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^2 + 2)*sqrt(f*x^2 + 3)/(b*x^2 + a), x)
```

3.95 $\int \frac{\sqrt{2+dx^2}}{\left(a+bx^2\right)\sqrt{3+fx^2}}\,dx$

Optimal. Leaf size=93

$$\frac{2\sqrt{fx^2+3}\,\Pi\left(1-\frac{2b}{ad};\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{2}}\right)|1-\frac{2f}{3d}\right)}{\sqrt{3}a\sqrt{d}\sqrt{dx^2+2}\sqrt{\frac{fx^2+3}{dx^2+2}}}$$

```
[Out] (2*Sqrt[3 + f*x^2]*EllipticPi[1 - (2*b)/(a*d), ArcTan[(Sqrt[d]*x)/Sqrt[2]],
 1 - (2*f)/(3*d)])/(Sqrt[3]*a*Sqrt[d]*Sqrt[2 + d*x^2]*Sqrt[(3 + f*x^2)/(2 +
 d*x^2)])
```

Rubi [A] time = 0.0361127, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.031, Rules used = {539}

$$\frac{2\sqrt{fx^2+3}\,\Pi\left(1-\frac{2b}{ad};\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{2}}\right)|1-\frac{2f}{3d}\right)}{\sqrt{3}a\sqrt{d}\sqrt{dx^2+2}\sqrt{\frac{fx^2+3}{dx^2+2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + d*x^2]/((a + b*x^2)*Sqrt[3 + f*x^2]),x]
```

```
[Out] (2*Sqrt[3 + f*x^2]*EllipticPi[1 - (2*b)/(a*d), ArcTan[(Sqrt[d]*x)/Sqrt[2]],
 1 - (2*f)/(3*d)])/(Sqrt[3]*a*Sqrt[d]*Sqrt[2 + d*x^2]*Sqrt[(3 + f*x^2)/(2 +
 d*x^2)])
```

Rule 539

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[(c*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcT
an[Rt[d/c, 2]*x], 1 - (c*f)/(d*e)])/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c
*(e + f*x^2))/(e*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rubi steps

$$\int \frac{\sqrt{2+dx^2}}{\left(a+bx^2\right)\sqrt{3+fx^2}}\,dx=\frac{2\sqrt{3+fx^2}\,\Pi\left(1-\frac{2b}{ad};\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{2}}\right)|1-\frac{2f}{3d}\right)}{\sqrt{3}a\sqrt{d}\sqrt{2+dx^2}\sqrt{\frac{3+fx^2}{2+dx^2}}}$$

Mathematica [C] time = 0.198352, size = 94, normalized size = 1.01

$$-\frac{i\left(ad\,\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{2}}\right),\frac{2f}{3d}\right)+(2b-ad)\Pi\left(\frac{2b}{ad};i\sinh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{2}}\right)|\frac{2f}{3d}\right)\right)}{\sqrt{3}ab\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + d*x^2]/((a + b*x^2)*Sqrt[3 + f*x^2]),x]
```

```
[Out] ((-I)*(a*d*EllipticF[I*ArcSinh[(Sqrt[d]*x)/Sqrt[2]], (2*f)/(3*d)] + (2*b -
a*d)*EllipticPi[(2*b)/(a*d), I*ArcSinh[(Sqrt[d]*x)/Sqrt[2]], (2*f)/(3*d)]))
/(Sqrt[3]*a*b*Sqrt[d])
```

Maple [A] time = 0.02, size = 133, normalized size = 1.4

$$\frac{\sqrt{2}}{2\,ab}\left(\mathrm{EllipticF}\left(\frac{x\sqrt{3}}{3}\sqrt{-f},\frac{\sqrt{2}\sqrt{3}}{2}\sqrt{\frac{d}{f}}\right)ad-\mathrm{EllipticPi}\left(\frac{x\sqrt{3}}{3}\sqrt{-f},3\,\frac{b}{af},\frac{\sqrt{2}\sqrt{3}}{2}\sqrt{-d}\frac{1}{\sqrt{-f}}\right)ad+2\,\mathrm{EllipticPi}\left(1/3\,x\sqrt{3}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+2)^(1/2)/(b*x^2+a)/(f*x^2+3)^(1/2),x)
```

```
[Out] 1/2*2^(1/2)*(EllipticF(1/3*x*3^(1/2)*(-f)^(1/2),1/2*2^(1/2)*3^(1/2)*(1/f*d)
^(1/2))*a*d-EllipticPi(1/3*x*3^(1/2)*(-f)^(1/2),3*b/a/f,1/2*2^(1/2)*(-d)^(1
/2)*3^(1/2)/(-f)^(1/2))*a*d+2*EllipticPi(1/3*x*3^(1/2)*(-f)^(1/2),3*b/a/f,1
/2*2^(1/2)*(-d)^(1/2)*3^(1/2)/(-f)^(1/2))*b)/(-f)^(1/2)/a/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2+2}}{\left(bx^2+a\right)\sqrt{fx^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+2)^(1/2)/(b*x^2+a)/(f*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + 2)/((b*x^2 + a)*sqrt(f*x^2 + 3)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+2)^(1/2)/(b*x^2+a)/(f*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2+2}}{\left(a+bx^2\right)\sqrt{fx^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+2)**(1/2)/(b*x**2+a)/(f*x**2+3)**(1/2),x)
```

```
[Out] Integral(sqrt(d*x**2 + 2)/((a + b*x**2)*sqrt(f*x**2 + 3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2+2}}{(bx^2+a)\sqrt{fx^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+2)^(1/2)/(b*x^2+a)/(f*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^2 + 2)/((b*x^2 + a)*sqrt(f*x^2 + 3)), x)
```

3.96 $\int \frac{1}{(a+bx^2)\sqrt{2+dx^2}\sqrt{3+fx^2}}\,dx$

Optimal. Leaf size=49

$$\frac{\Pi\left(\frac{2b}{ad};\sin^{-1}\left(\frac{\sqrt{-d}x}{\sqrt{2}}\right)|\frac{2f}{3d}\right)}{\sqrt{3}a\sqrt{-d}}$$

```
[Out] EllipticPi[(2*b)/(a*d), ArcSin[(Sqrt[-d]*x)/Sqrt[2]], (2*f)/(3*d)]/(Sqrt[3]
*a*Sqrt[-d])
```

Rubi [A] time = 0.0415932, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.031, Rules used = {537}

$$\frac{\Pi\left(\frac{2b}{ad};\sin^{-1}\left(\frac{\sqrt{-d}x}{\sqrt{2}}\right)|\frac{2f}{3d}\right)}{\sqrt{3}a\sqrt{-d}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)*Sqrt[2 + d*x^2]*Sqrt[3 + f*x^2]),x]
```

```
[Out] EllipticPi[(2*b)/(a*d), ArcSin[(Sqrt[-d]*x)/Sqrt[2]], (2*f)/(3*d)]/(Sqrt[3]
*a*Sqrt[-d])
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)\sqrt{2+dx^2}\sqrt{3+fx^2}}\,dx=\frac{\Pi\left(\frac{2b}{ad};\sin^{-1}\left(\frac{\sqrt{-d}x}{\sqrt{2}}\right)|\frac{2f}{3d}\right)}{\sqrt{3}a\sqrt{-d}}$$

Mathematica [A] time = 0.01666, size = 49, normalized size = 1.

$$\frac{\Pi\left(\frac{2b}{ad};\sin^{-1}\left(\frac{\sqrt{-d}x}{\sqrt{2}}\right)|\frac{2f}{3d}\right)}{\sqrt{3}a\sqrt{-d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)*Sqrt[2 + d*x^2]*Sqrt[3 + f*x^2]),x]
```

```
[Out] EllipticPi[(2*b)/(a*d), ArcSin[(Sqrt[-d]*x)/Sqrt[2]], (2*f)/(3*d)]/(Sqrt[3]
*a*Sqrt[-d])
```

Maple [A] time = 0.019, size = 53, normalized size = 1.1

$$\frac{\sqrt{2}}{2a}\text{EllipticPi}\left(\frac{x\sqrt{3}}{3}\sqrt{-f}, 3\frac{b}{af}, \frac{\sqrt{2}\sqrt{3}}{2}\sqrt{-d}\frac{1}{\sqrt{-f}}\right)\frac{1}{\sqrt{-f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)/(d*x^2+2)^(1/2)/(f*x^2+3)^(1/2),x)
```

```
[Out] 1/2*2^(1/2)*EllipticPi(1/3*x*3^(1/2)*(-f)^(1/2),3*b/a/f,1/2*2^(1/2)*(-d)^(1
/2)*3^(1/2)/(-f)^(1/2))/(-f)^(1/2)/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)\sqrt{dx^2 + 2}\sqrt{fx^2 + 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+2)^(1/2)/(f*x^2+3)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((b*x^2 + a)*sqrt(d*x^2 + 2)*sqrt(f*x^2 + 3)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+2)^(1/2)/(f*x^2+3)^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a + bx^2)\sqrt{dx^2 + 2}\sqrt{fx^2 + 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)/(d*x**2+2)**(1/2)/(f*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x**2)*sqrt(d*x**2 + 2)*sqrt(f*x**2 + 3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+a)\sqrt{dx^2+2}\sqrt{fx^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+2)^(1/2)/(f*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)*sqrt(d*x^2 + 2)*sqrt(f*x^2 + 3)), x)
```

3.97 $\int \frac{\sqrt{1-x^2}}{\left(-1+x^2\right)\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=36

$$-\frac{\sqrt{\frac{bx^2}{a}+1}\text{EllipticF}\left(\sin^{-1}(x),-\frac{b}{a}\right)}{\sqrt{a+bx^2}}$$

```
[Out] -((Sqrt[1 + (b*x^2)/a]*EllipticF[ArcSin[x], -(b/a)])/Sqrt[a + b*x^2])
```

Rubi [A] time = 0.0248665, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {21, 421, 419}

$$-\frac{\sqrt{\frac{bx^2}{a}+1}F\left(\sin^{-1}(x)|-\frac{b}{a}\right)}{\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x^2]/((-1 + x^2)*Sqrt[a + b*x^2]),x]
```

```
[Out] -((Sqrt[1 + (b*x^2)/a]*EllipticF[ArcSin[x], -(b/a)])/Sqrt[a + b*x^2])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\sqrt{1-x^2}}{(-1+x^2)\sqrt{a+bx^2}}\,dx = -\int \frac{1}{\sqrt{1-x^2}\sqrt{a+bx^2}}\,dx$$
$$= -\frac{\sqrt{1+\frac{bx^2}{a}}\int \frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{bx^2}{a}}}\,dx}{\sqrt{a+bx^2}}$$
$$= -\frac{\sqrt{1+\frac{bx^2}{a}}F\left(\sin^{-1}(x)|-\frac{b}{a}\right)}{\sqrt{a+bx^2}}$$

Mathematica [A] time = 0.0420609, size = 37, normalized size = 1.03

$$-\frac{\sqrt{\frac{a+bx^2}{a}}\text{EllipticF}\left(\sin^{-1}(x),-\frac{b}{a}\right)}{\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x^2]/((-1 + x^2)*Sqrt[a + b*x^2]),x]
```

```
[Out] -((Sqrt[(a + b*x^2)/a]*EllipticF[ArcSin[x], -(b/a)])/Sqrt[a + b*x^2])
```

Maple [A] time = 0.026, size = 35, normalized size = 1.

$$-\sqrt{\frac{bx^2+a}{a}}\text{EllipticF}\left(x,\sqrt{-\frac{b}{a}}\right)\frac{1}{\sqrt{bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)^(1/2)/(x^2-1)/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/(b*x^2+a)^(1/2)*((b*x^2+a)/a)^(1/2)*EllipticF(x,(-b/a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^2+1}}{\sqrt{bx^2+a}(x^2-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(x^2-1)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2 + 1)/(sqrt(b*x^2 + a)*(x^2 - 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^2+a}\sqrt{-x^2+1}}{bx^4+(a-b)x^2-a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(x^2-1)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(-x^2 + 1)/(b*x^4 + (a - b)*x^2 - a), x)
```

Sympy [A] time = 5.7869, size = 19, normalized size = 0.53

$$\begin{cases} -\frac{F\left(\operatorname{asin}{\left(x \right)}\middle| - \frac{b}{a}\right)}{\sqrt{a}} & \text{for}\: x > -1 \wedge x < 1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)**(1/2)/(x**2-1)/(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((-elliptic_f(asin(x), -b/a)/sqrt(a), (x > -1) & (x < 1)))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^{2} + 1}}{\sqrt{b x^{2} + a}\left(x^{2} - 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(x^2-1)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^2 + 1)/(sqrt(b*x^2 + a)*(x^2 - 1)), x)
```

3.98 $\int \frac{a+bx^2}{\sqrt{c+dx^2}\left(e+fx^2\right)^2} dx$

Optimal. Leaf size=113

$$\frac{x\sqrt{c+dx^2}(be-af)}{2e\left(e+fx^2\right)(de-cf)}-\frac{(acf-2ade+bce)\tanh^{-1}\left(\frac{x\sqrt{de-cf}}{\sqrt{e}\sqrt{c+dx^2}}\right)}{2e^{3/2}(de-cf)^{3/2}}$$

```
[Out] ((b*e - a*f)*x*Sqrt[c + d*x^2])/(2*e*(d*e - c*f)*(e + f*x^2)) - ((b*c*e - 2
*a*d*e + a*c*f)*ArcTanh[(Sqrt[d*e - c*f]*x)/(Sqrt[e]*Sqrt[c + d*x^2])])/(2*
e^(3/2)*(d*e - c*f)^(3/2))
```

Rubi [A] time = 0.118809, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {527, 12, 377, 208}

$$\frac{x\sqrt{c+dx^2}(be-af)}{2e\left(e+fx^2\right)(de-cf)}-\frac{(acf-2ade+bce)\tanh^{-1}\left(\frac{x\sqrt{de-cf}}{\sqrt{e}\sqrt{c+dx^2}}\right)}{2e^{3/2}(de-cf)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
```

```
[Out] ((b*e - a*f)*x*Sqrt[c + d*x^2])/(2*e*(d*e - c*f)*(e + f*x^2)) - ((b*c*e - 2
*a*d*e + a*c*f)*ArcTanh[(Sqrt[d*e - c*f]*x)/(Sqrt[e]*Sqrt[c + d*x^2])])/(2*
e^(3/2)*(d*e - c*f)^(3/2))
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{a+bx^2}{\sqrt{c+dx^2}\left(e+fx^2\right)^2}\,dx = \frac{(be-af)x\sqrt{c+dx^2}}{2e(de-cf)\left(e+fx^2\right)} + \frac{\int \frac{-bce+2ade-acf}{\sqrt{c+dx^2}\left(e+fx^2\right)}\,dx}{2e(de-cf)}$$

$$= \frac{(be-af)x\sqrt{c+dx^2}}{2e(de-cf)\left(e+fx^2\right)} - \frac{(bce-2ade+acf)\int \frac{1}{\sqrt{c+dx^2}\left(e+fx^2\right)}\,dx}{2e(de-cf)}$$

$$= \frac{(be-af)x\sqrt{c+dx^2}}{2e(de-cf)\left(e+fx^2\right)} - \frac{(bce-2ade+acf)\,\text{Subst}\left(\int \frac{1}{e-(de-cf)x^2}\,dx, x, \frac{x}{\sqrt{c+dx^2}}\right)}{2e(de-cf)}$$

$$= \frac{(be-af)x\sqrt{c+dx^2}}{2e(de-cf)\left(e+fx^2\right)} - \frac{(bce-2ade+acf)\tanh^{-1}\left(\frac{\sqrt{de-cf}\,x}{\sqrt{e}\sqrt{c+dx^2}}\right)}{2e^{3/2}(de-cf)^{3/2}}$$

Mathematica [A] time = 0.578427, size = 191, normalized size = 1.69

$$\frac{x(be-af)\left(f\left(c+dx^2\right) - \frac{\left(e+fx^2\right)(2de-cf)\tanh^{-1}\left(\sqrt{\frac{x^2(de-cf)}{e\left(c+dx^2\right)}}\right)}{e\sqrt{\frac{x^2(de-cf)}{e\left(c+dx^2\right)}}}\right)}{2ef\sqrt{c+dx^2}\left(e+fx^2\right)(de-cf)} + \frac{b\tanh^{-1}\left(\frac{x\sqrt{de-cf}}{\sqrt{e}\sqrt{c+dx^2}}\right)}{\sqrt{e}f\sqrt{de-cf}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
```

```
[Out] ((b*e - a*f)*x*(f*(c + d*x^2) - ((2*d*e - c*f)*(e + f*x^2)*ArcTanh[Sqrt[((d
*e - c*f)*x^2)/(e*(c + d*x^2))]])/(e*Sqrt[((d*e - c*f)*x^2)/(e*(c + d*x^2))
])))/(2*e*f*(d*e - c*f)*Sqrt[c + d*x^2]*(e + f*x^2)) + (b*ArcTanh[(Sqrt[d*e
 - c*f]*x)/(Sqrt[e]*Sqrt[c + d*x^2])])/(Sqrt[e]*f*Sqrt[d*e - c*f])
```

Maple [B] time = 0.03, size = 1622, normalized size = 14.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/(f*x^2+e)^2/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/4/e/(-e*f)^(1/2)/((c*f-d*e)/f)^(1/2)*ln((2*(c*f-d*e)/f+2*d*(-e*f)^(1/2)/
f*(x-(-e*f)^(1/2)/f)+2*((c*f-d*e)/f)^(1/2)*((x-(-e*f)^(1/2)/f)^2*d+2*d*(-e*
f)^(1/2)/f*(x-(-e*f)^(1/2)/f)+(c*f-d*e)/f)^(1/2))/(x-(-e*f)^(1/2)/f))*a-1/4
/(-e*f)^(1/2)/f/((c*f-d*e)/f)^(1/2)*ln((2*(c*f-d*e)/f+2*d*(-e*f)^(1/2)/f*(x
-(-e*f)^(1/2)/f)+2*((c*f-d*e)/f)^(1/2)*((x-(-e*f)^(1/2)/f)^2*d+2*d*(-e*f)^(
1/2)/f*(x-(-e*f)^(1/2)/f)+(c*f-d*e)/f)^(1/2))/(x-(-e*f)^(1/2)/f))*b+1/4/e/(
c*f-d*e)/(x-(-e*f)^(1/2)/f)*((x-(-e*f)^(1/2)/f)^2*d+2*d*(-e*f)^(1/2)/f*(x-(
-e*f)^(1/2)/f)+(c*f-d*e)/f)^(1/2)*a-1/4/f/(c*f-d*e)/(x-(-e*f)^(1/2)/f)*((x-
(-e*f)^(1/2)/f)^2*d+2*d*(-e*f)^(1/2)/f*(x-(-e*f)^(1/2)/f)+(c*f-d*e)/f)^(1/2
)*b-1/4/e/f*d*(-e*f)^(1/2)/(c*f-d*e)/((c*f-d*e)/f)^(1/2)*ln((2*(c*f-d*e)/f+
2*d*(-e*f)^(1/2)/f*(x-(-e*f)^(1/2)/f)+2*((c*f-d*e)/f)^(1/2)*((x-(-e*f)^(1/2
)/f)^2*d+2*d*(-e*f)^(1/2)/f*(x-(-e*f)^(1/2)/f)+(c*f-d*e)/f)^(1/2))/(x-(-e*f
)^(1/2)/f))*a+1/4/f^2*d*(-e*f)^(1/2)/(c*f-d*e)/((c*f-d*e)/f)^(1/2)*ln((2*(c
*f-d*e)/f+2*d*(-e*f)^(1/2)/f*(x-(-e*f)^(1/2)/f)+2*((c*f-d*e)/f)^(1/2)*((x-(
```

```
-e*f)^(1/2)/f)^2*d+2*d*(-e*f)^(1/2)/f*(x-(-e*f)^(1/2)/f)+(c*f-d*e)/f)^(1/2)
)/(x-(-e*f)^(1/2)/f))*b+1/4/e/(-e*f)^(1/2)/((c*f-d*e)/f)^(1/2)*ln((2*(c*f-d
*e)/f-2*d*(-e*f)^(1/2)/f*(x+(-e*f)^(1/2)/f)+2*((c*f-d*e)/f)^(1/2)*((x+(-e*f
)^(1/2)/f)^2*d-2*d*(-e*f)^(1/2)/f*(x+(-e*f)^(1/2)/f)+(c*f-d*e)/f)^(1/2))/(x
+(-e*f)^(1/2)/f))*a+1/4/(-e*f)^(1/2)/f/((c*f-d*e)/f)^(1/2)*ln((2*(c*f-d*e)/
f-2*d*(-e*f)^(1/2)/f*(x+(-e*f)^(1/2)/f)+2*((c*f-d*e)/f)^(1/2)*((x+(-e*f)^(1
/2)/f)^2*d-2*d*(-e*f)^(1/2)/f*(x+(-e*f)^(1/2)/f)+(c*f-d*e)/f)^(1/2))/(x+(-e
*f)^(1/2)/f))*b+1/4/e/(c*f-d*e)/(x+(-e*f)^(1/2)/f)*((x+(-e*f)^(1/2)/f)^2*d-
2*d*(-e*f)^(1/2)/f*(x+(-e*f)^(1/2)/f)+(c*f-d*e)/f)^(1/2)*a-1/4/f/(c*f-d*e)/
(x+(-e*f)^(1/2)/f)*((x+(-e*f)^(1/2)/f)^2*d-2*d*(-e*f)^(1/2)/f*(x+(-e*f)^(1/
2)/f)+(c*f-d*e)/f)^(1/2)*b+1/4/e/f*d*(-e*f)^(1/2)/(c*f-d*e)/((c*f-d*e)/f)^(
1/2)*ln((2*(c*f-d*e)/f-2*d*(-e*f)^(1/2)/f*(x+(-e*f)^(1/2)/f)+2*((c*f-d*e)/f
)^(1/2)*((x+(-e*f)^(1/2)/f)^2*d-2*d*(-e*f)^(1/2)/f*(x+(-e*f)^(1/2)/f)+(c*f-
d*e)/f)^(1/2))/(x+(-e*f)^(1/2)/f))*a-1/4/f^2*d*(-e*f)^(1/2)/(c*f-d*e)/((c*f
-d*e)/f)^(1/2)*ln((2*(c*f-d*e)/f-2*d*(-e*f)^(1/2)/f*(x+(-e*f)^(1/2)/f)+2*((
c*f-d*e)/f)^(1/2)*((x+(-e*f)^(1/2)/f)^2*d-2*d*(-e*f)^(1/2)/f*(x+(-e*f)^(1/2
)/f)+(c*f-d*e)/f)^(1/2))/(x+(-e*f)^(1/2)/f))*b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx^2+a}{\sqrt{dx^2+c}\left(fx^2+e\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(f*x^2+e)^2/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)/(sqrt(d*x^2 + c)*(f*x^2 + e)^2), x)
```

Fricas [B] time = 13.9268, size = 1079, normalized size = 9.55

$$\left[\frac{4\left(bde^3+acef^2-(bc+ad)e^2f\right)\sqrt{dx^2+c}\,x-\left(acef+(bc-2\,ad)e^2+\left(acf^2+(bc-2\,ad)ef\right)x^2\right)\sqrt{de^2-cef}\log\left(\frac{\left(8\,d^2e^2-8\,cd\right.}{}\right.}{8\left(d^2e^5-2\,cde^4f+c^2e^3f^2+\left(d^2e^4f-2\,cde^3f^2+c^2e^2f^3\right)x^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(f*x^2+e)^2/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(4*(b*d*e^3 + a*c*e*f^2 - (b*c + a*d)*e^2*f)*sqrt(d*x^2 + c)*x - (a*c*
e*f + (b*c - 2*a*d)*e^2 + (a*c*f^2 + (b*c - 2*a*d)*e*f)*x^2)*sqrt(d*e^2 - c
*e*f)*log(((8*d^2*e^2 - 8*c*d*e*f + c^2*f^2)*x^4 + c^2*e^2 + 2*(4*c*d*e^2 -
 3*c^2*e*f)*x^2 + 4*((2*d*e - c*f)*x^3 + c*e*x)*sqrt(d*e^2 - c*e*f)*sqrt(d*
x^2 + c))/(f^2*x^4 + 2*e*f*x^2 + e^2)))/(d^2*e^5 - 2*c*d*e^4*f + c^2*e^3*f^
2 + (d^2*e^4*f - 2*c*d*e^3*f^2 + c^2*e^2*f^3)*x^2), 1/4*(2*(b*d*e^3 + a*c*e
*f^2 - (b*c + a*d)*e^2*f)*sqrt(d*x^2 + c)*x + (a*c*e*f + (b*c - 2*a*d)*e^2
+ (a*c*f^2 + (b*c - 2*a*d)*e*f)*x^2)*sqrt(-d*e^2 + c*e*f)*arctan(1/2*sqrt(-
d*e^2 + c*e*f)*((2*d*e - c*f)*x^2 + c*e)*sqrt(d*x^2 + c)/((d^2*e^2 - c*d*e*
f)*x^3 + (c*d*e^2 - c^2*e*f)*x)))/(d^2*e^5 - 2*c*d*e^4*f + c^2*e^3*f^2 + (d
^2*e^4*f - 2*c*d*e^3*f^2 + c^2*e^2*f^3)*x^2)]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/(f*x**2+e)**2/(d*x**2+c)**(1/2),x)
```

```
[Out] Exception raised: ValueError
```

Giac [B] time = 3.94619, size = 454, normalized size = 4.02

$$-\frac{\left(ac\sqrt{d}f+bc\sqrt{d}e-2\,ad^{\frac{3}{2}}e\right)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2f-cf+2\,de}{2\,\sqrt{cdfe-d^2e^2}}\right)}{2\,\sqrt{cdfe-d^2e^2}\left(cfe-de^2\right)}-\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2ac\sqrt{d}f^2-\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2bc\sqrt{d}}{\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4f-2\left(\sqrt{d}x\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(f*x^2+e)^2/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(a*c*sqrt(d)*f + b*c*sqrt(d)*e - 2*a*d^(3/2)*e)*arctan(1/2*((sqrt(d)*x
 - sqrt(d*x^2 + c))^2*f - c*f + 2*d*e)/sqrt(c*d*f*e - d^2*e^2))/(sqrt(c*d*f
*e - d^2*e^2)*(c*f*e - d*e^2)) - ((sqrt(d)*x - sqrt(d*x^2 + c))^2*a*c*sqrt(
d)*f^2 - (sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c*sqrt(d)*f*e - 2*(sqrt(d)*x - s
qrt(d*x^2 + c))^2*a*d^(3/2)*f*e - a*c^2*sqrt(d)*f^2 + 2*(sqrt(d)*x - sqrt(d
*x^2 + c))^2*b*d^(3/2)*e^2 + b*c^2*sqrt(d)*f*e)/(((sqrt(d)*x - sqrt(d*x^2 +
 c))^4*f - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*c*f + 4*(sqrt(d)*x - sqrt(d*x^
2 + c))^2*d*e + c^2*f)*(c*f^2*e - d*f*e^2))
```

3.99 $\int \frac{\sqrt{c-dx^2}\sqrt{e+fx^2}}{\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=359

$$-\frac{\sqrt{c}\sqrt{d}\sqrt{1-\frac{dx^2}{c}}\sqrt{\frac{fx^2}{e}+1}(af+be)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{cf}{de}\right)}{2ab^2\sqrt{c-dx^2}\sqrt{e+fx^2}}+\frac{\sqrt{c}\sqrt{1-\frac{dx^2}{c}}\sqrt{\frac{fx^2}{e}+1}\left(a^2df+b^2ce\right)\Pi\left(-\frac{bc}{ad};\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\ldots\right)}{2a^2b^2\sqrt{d}\sqrt{c-dx^2}\sqrt{e+fx^2}}$$

```
[Out] (x*Sqrt[c - d*x^2]*Sqrt[e + f*x^2])/(2*a*(a + b*x^2)) + (Sqrt[c]*Sqrt[d]*Sq
rt[1 - (d*x^2)/c]*Sqrt[e + f*x^2]*EllipticE[ArcSin[(Sqrt[d]*x)/Sqrt[c]], -(
(c*f)/(d*e))])/(2*a*b*Sqrt[c - d*x^2]*Sqrt[1 + (f*x^2)/e]) - (Sqrt[c]*Sqrt[
d]*(b*e + a*f)*Sqrt[1 - (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[ArcSin[(Sq
rt[d]*x)/Sqrt[c]], -((c*f)/(d*e))])/(2*a*b^2*Sqrt[c - d*x^2]*Sqrt[e + f*x^2
]) + (Sqrt[c]*(b^2*c*e + a^2*d*f)*Sqrt[1 - (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*E
llipticPi[-((b*c)/(a*d)), ArcSin[(Sqrt[d]*x)/Sqrt[c]], -((c*f)/(d*e))])/(2*
a^2*b^2*Sqrt[d]*Sqrt[c - d*x^2]*Sqrt[e + f*x^2])
```

Rubi [A] time = 0.323645, antiderivative size = 359, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {548, 524, 427, 426, 424, 421, 419, 538, 537}

$$\frac{\sqrt{c}\sqrt{1-\frac{dx^2}{c}}\sqrt{\frac{fx^2}{e}+1}\left(a^2df+b^2ce\right)\Pi\left(-\frac{bc}{ad};\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{cf}{de}\right)}{2a^2b^2\sqrt{d}\sqrt{c-dx^2}\sqrt{e+fx^2}}-\frac{\sqrt{c}\sqrt{d}\sqrt{1-\frac{dx^2}{c}}\sqrt{\frac{fx^2}{e}+1}(af+be)F\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{cf}{de}\right)}{2ab^2\sqrt{c-dx^2}\sqrt{e+fx^2}}\ldots$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - d*x^2]*Sqrt[e + f*x^2])/(a + b*x^2)^2,x]
```

```
[Out] (x*Sqrt[c - d*x^2]*Sqrt[e + f*x^2])/(2*a*(a + b*x^2)) + (Sqrt[c]*Sqrt[d]*Sq
rt[1 - (d*x^2)/c]*Sqrt[e + f*x^2]*EllipticE[ArcSin[(Sqrt[d]*x)/Sqrt[c]], -(
(c*f)/(d*e))])/(2*a*b*Sqrt[c - d*x^2]*Sqrt[1 + (f*x^2)/e]) - (Sqrt[c]*Sqrt[
d]*(b*e + a*f)*Sqrt[1 - (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[ArcSin[(Sq
rt[d]*x)/Sqrt[c]], -((c*f)/(d*e))])/(2*a*b^2*Sqrt[c - d*x^2]*Sqrt[e + f*x^2
]) + (Sqrt[c]*(b^2*c*e + a^2*d*f)*Sqrt[1 - (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*E
llipticPi[-((b*c)/(a*d)), ArcSin[(Sqrt[d]*x)/Sqrt[c]], -((c*f)/(d*e))])/(2*
a^2*b^2*Sqrt[d]*Sqrt[c - d*x^2]*Sqrt[e + f*x^2])
```

Rule 548

```
Int[(Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)^2])/((a_) + (b_.)*(x_
)^2)^2, x_Symbol] :> Simp[(x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(2*a*(a + b*x
^2)), x] + (Dist[(d*f)/(2*a*b^2), Int[(a - b*x^2)/(Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2]), x], x] + Dist[(b^2*c*e - a^2*d*f)/(2*a*b^2), Int[1/((a + b*x^2)*S
qrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 427

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d*x^2)/c
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int \frac{\sqrt{c-dx^2}\sqrt{e+fx^2}}{\left(a+bx^2\right)^2}\,dx = \frac{x\sqrt{c-dx^2}\sqrt{e+fx^2}}{2a\left(a+bx^2\right)} - \frac{(df)\int \frac{a-bx^2}{\sqrt{c-dx^2}\sqrt{e+fx^2}}\,dx}{2ab^2} + \frac{1}{2}\left(\frac{ce}{a}+\frac{adf}{b^2}\right)\int \frac{1}{\left(a+bx^2\right)\sqrt{c-dx^2}\sqrt{e+fx}}$$

$$= \frac{x\sqrt{c-dx^2}\sqrt{e+fx^2}}{2a\left(a+bx^2\right)} + \frac{d\int \frac{\sqrt{e+fx^2}}{\sqrt{c-dx^2}}\,dx}{2ab} - \frac{(d(be+af))\int \frac{1}{\sqrt{c-dx^2}\sqrt{e+fx^2}}\,dx}{2ab^2} + \frac{\left(\left(\frac{ce}{a}+\frac{adf}{b^2}\right)\sqrt{1-\frac{dx^2}{c}}\right.}{2}$$

$$= \frac{x\sqrt{c-dx^2}\sqrt{e+fx^2}}{2a\left(a+bx^2\right)} + \frac{\left(d\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{\sqrt{e+fx^2}}{\sqrt{1-\frac{dx^2}{c}}}\,dx}{2ab\sqrt{c-dx^2}} - \frac{\left(d(be+af)\sqrt{1+\frac{fx^2}{e}}\right)\int \frac{1}{\sqrt{c-dx^2}\sqrt{1+\frac{fx^2}{e}}}\,dx}{2ab^2\sqrt{e+fx^2}}$$

$$= \frac{x\sqrt{c-dx^2}\sqrt{e+fx^2}}{2a\left(a+bx^2\right)} + \frac{\sqrt{c}\left(\frac{ce}{a}+\frac{adf}{b^2}\right)\sqrt{1-\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\,\Pi\left(-\frac{bc}{ad};\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\mid -\frac{cf}{de}\right)}{2a\sqrt{d}\sqrt{c-dx^2}\sqrt{e+fx^2}} + \frac{\left(d\sqrt{1-\right.}{2}$$

$$= \frac{x\sqrt{c-dx^2}\sqrt{e+fx^2}}{2a\left(a+bx^2\right)} + \frac{\sqrt{c}\sqrt{d}\sqrt{1-\frac{dx^2}{c}}\sqrt{e+fx^2}\,E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\mid -\frac{cf}{de}\right)}{2ab\sqrt{c-dx^2}\sqrt{1+\frac{fx^2}{e}}} - \frac{\sqrt{c}\sqrt{d}(be+af)\sqrt{1-}{2ab^2}$$

Mathematica [C] time = 2.41177, size = 422, normalized size = 1.18

$$\frac{-\frac{ic\sqrt{-\frac{d}{c}}\sqrt{1-\frac{dx^2}{c}}\sqrt{\frac{fx^2}{e}+1}(af+be)\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{-\frac{d}{c}}\right),-\frac{cf}{de}\right)}{b^2} + \frac{iacf\sqrt{-\frac{d}{c}}\sqrt{1-\frac{dx^2}{c}}\sqrt{\frac{fx^2}{e}+1}\,\Pi\left(-\frac{bc}{ad};i\sinh^{-1}\left(\sqrt{-\frac{d}{c}}x\right)\mid -\frac{cf}{de}\right)}{b^2} + \frac{ide\sqrt{1-\frac{dx^2}{c}}\sqrt{\frac{fx^2}{e}+1}\,\Pi\left(-\right.}{a\left(-\frac{d}{c}\right.}}{2a\sqrt{c-dx^2}\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c - d*x^2]*Sqrt[e + f*x^2])/(a + b*x^2)^2,x]
```

```
[Out] ((c*e*x)/(a + b*x^2) - (d*e*x^3)/(a + b*x^2) + (c*f*x^3)/(a + b*x^2) - (d*f
*x^5)/(a + b*x^2) + (I*c*Sqrt[-(d/c)]*e*Sqrt[1 - (d*x^2)/c]*Sqrt[1 + (f*x^2
)/e]*EllipticE[I*ArcSinh[Sqrt[-(d/c)]*x], -((c*f)/(d*e))])/b - (I*c*Sqrt[-(
d/c)]*(b*e + a*f)*Sqrt[1 - (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSi
nh[Sqrt[-(d/c)]*x], -((c*f)/(d*e))])/b^2 + (I*d*e*Sqrt[1 - (d*x^2)/c]*Sqrt[
1 + (f*x^2)/e]*EllipticPi[-((b*c)/(a*d)), I*ArcSinh[Sqrt[-(d/c)]*x], -((c*f
)/(d*e))])/(a*(-(d/c))^(3/2)) + (I*a*c*Sqrt[-(d/c)]*f*Sqrt[1 - (d*x^2)/c]*S
qrt[1 + (f*x^2)/e]*EllipticPi[-((b*c)/(a*d)), I*ArcSinh[Sqrt[-(d/c)]*x], -(
(c*f)/(d*e))])/b^2)/(2*a*Sqrt[c - d*x^2]*Sqrt[e + f*x^2])
```

Maple [B] time = 0.043, size = 793, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^2,x)
```

```
[Out] 1/2*(-d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)*((d/c)^(1/2)*x^5*a*b^2*d*f+(-(d*x^2-c)
/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(d/c)^(1/2),(-c*f/d/e)^(1/2))*x^2
```

```
*a^2*b*d*f+(-(d*x^2-c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(d/c)^(1/2)
,(-c*f/d/e)^(1/2))*x^2*a*b^2*d*e-(-(d*x^2-c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*E
llipticE(x*(d/c)^(1/2),(-c*f/d/e)^(1/2))*x^2*a*b^2*d*e-(-(d*x^2-c)/c)^(1/2)
*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(d/c)^(1/2),-b*c/a/d,(-f/e)^(1/2)/(d/c)^(
1/2))*x^2*a^2*b*d*f-(-(d*x^2-c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(
d/c)^(1/2),-b*c/a/d,(-f/e)^(1/2)/(d/c)^(1/2))*x^2*b^3*c*e-(d/c)^(1/2)*x^3*a
*b^2*c*f+(d/c)^(1/2)*x^3*a*b^2*d*e+(-(d*x^2-c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)
*EllipticF(x*(d/c)^(1/2),(-c*f/d/e)^(1/2))*a^3*d*f+(-(d*x^2-c)/c)^(1/2)*((f
*x^2+e)/e)^(1/2)*EllipticF(x*(d/c)^(1/2),(-c*f/d/e)^(1/2))*a^2*b*d*e-(-(d*x
^2-c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(d/c)^(1/2),(-c*f/d/e)^(1/2)
)*a^2*b*d*e-(-(d*x^2-c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(d/c)^(1/
2),-b*c/a/d,(-f/e)^(1/2)/(d/c)^(1/2))*a^3*d*f-(-(d*x^2-c)/c)^(1/2)*((f*x^2+
e)/e)^(1/2)*EllipticPi(x*(d/c)^(1/2),-b*c/a/d,(-f/e)^(1/2)/(d/c)^(1/2))*a*b
^2*c*e-(d/c)^(1/2)*x*a*b^2*c*e)/(d*f*x^4-c*f*x^2+d*e*x^2-c*e)/a^2/(b*x^2+a)
/b^2/(d/c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-dx^2+c}\sqrt{fx^2+e}}{\left(bx^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^2,x, algorithm="maxima
")
```

```
[Out] integrate(sqrt(-d*x^2 + c)*sqrt(f*x^2 + e)/(b*x^2 + a)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^2,x, algorithm="fricas
")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c-dx^2}\sqrt{e+fx^2}}{\left(a+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x**2+c)**(1/2)*(f*x**2+e)**(1/2)/(b*x**2+a)**2,x)
```

```
[Out] Integral(sqrt(c - d*x**2)*sqrt(e + f*x**2)/(a + b*x**2)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-dx^2+c}\sqrt{fx^2+e}}{\left(bx^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-d*x^2 + c)*sqrt(f*x^2 + e)/(b*x^2 + a)^2, x)
```

3.100 $\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=381

$$\frac{d\sqrt{e}\sqrt{f}\sqrt{c+dx^2}\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{2b^2c\sqrt{e+fx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}+\frac{\sqrt{-c}\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\left(b^2ce-a^2df\right)\Pi\left(\frac{bc}{ad};\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{-c}}\right)|\frac{cf}{de}\right)}{2a^2b^2\sqrt{d}\sqrt{c+dx^2}\sqrt{e+fx^2}}+x\cdots$$

```
[Out] -(f*x*Sqrt[c + d*x^2])/(2*a*b*Sqrt[e + f*x^2]) + (x*Sqrt[c + d*x^2]*Sqrt[e
+ f*x^2])/(2*a*(a + b*x^2)) + (Sqrt[e]*Sqrt[f]*Sqrt[c + d*x^2]*EllipticE[Ar
cTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(2*a*b*Sqrt[(e*(c + d*x^2))/(c
*(e + f*x^2))]*Sqrt[e + f*x^2]) + (d*Sqrt[e]*Sqrt[f]*Sqrt[c + d*x^2]*Ellipt
icF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(2*b^2*c*Sqrt[(e*(c + d*
x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (Sqrt[-c]*(b^2*c*e - a^2*d*f)*Sqr
t[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), ArcSin[(Sqrt[d
]*x)/Sqrt[-c]], (c*f)/(d*e)])/(2*a^2*b^2*Sqrt[d]*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2])
```

Rubi [A] time = 0.291334, antiderivative size = 381, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {548, 531, 418, 492, 411, 538, 537}

$$\frac{\sqrt{-c}\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\left(b^2ce-a^2df\right)\Pi\left(\frac{bc}{ad};\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{-c}}\right)|\frac{cf}{de}\right)}{2a^2b^2\sqrt{d}\sqrt{c+dx^2}\sqrt{e+fx^2}}+\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2a\left(a+bx^2\right)}-\frac{fx\sqrt{c+dx^2}}{2ab\sqrt{e+fx^2}}+\frac{\sqrt{e}\sqrt{f}\sqrt{c+dx^2}\cdots}{2ab\sqrt{e}\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(a + b*x^2)^2,x]
```

```
[Out] -(f*x*Sqrt[c + d*x^2])/(2*a*b*Sqrt[e + f*x^2]) + (x*Sqrt[c + d*x^2]*Sqrt[e
+ f*x^2])/(2*a*(a + b*x^2)) + (Sqrt[e]*Sqrt[f]*Sqrt[c + d*x^2]*EllipticE[Ar
cTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(2*a*b*Sqrt[(e*(c + d*x^2))/(c
*(e + f*x^2))]*Sqrt[e + f*x^2]) + (d*Sqrt[e]*Sqrt[f]*Sqrt[c + d*x^2]*Ellipt
icF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(2*b^2*c*Sqrt[(e*(c + d*
x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (Sqrt[-c]*(b^2*c*e - a^2*d*f)*Sqr
t[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), ArcSin[(Sqrt[d
]*x)/Sqrt[-c]], (c*f)/(d*e)])/(2*a^2*b^2*Sqrt[d]*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2])
```

Rule 548

```
Int[(Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)^2])/((a_) + (b_.)*(x_
)^2)^2, x_Symbol] :> Simp[(x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(2*a*(a + b*x
^2)), x] + (Dist[(d*f)/(2*a*b^2), Int[(a - b*x^2)/(Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2]), x], x] + Dist[(b^2*c*e - a^2*d*f)/(2*a*b^2), Int[1/((a + b*x^2)*S
qrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
```

```
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{\left(a+bx^2\right)^2}\,dx = \frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2a\left(a+bx^2\right)} + \frac{(df)\int \frac{a-bx^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{2ab^2} + \frac{1}{2}\left(\frac{ce}{a}-\frac{adf}{b^2}\right)\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}\sqrt{e+\cdots}}$$

$$= \frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2a\left(a+bx^2\right)} + \frac{(df)\int \frac{1}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{2b^2} - \frac{(df)\int \frac{x^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{2ab} + \frac{\left(\left(\frac{ce}{a}-\frac{adf}{b^2}\right)\sqrt{1\cdots}\right.}{\cdots}$$

$$= -\frac{fx\sqrt{c+dx^2}}{2ab\sqrt{e+fx^2}} + \frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2a\left(a+bx^2\right)} + \frac{d\sqrt{e}\sqrt{f}\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{2b^2c\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}\sqrt{e+fx^2}} + \frac{(ef)\int\cdots}{\cdots}$$

$$= -\frac{fx\sqrt{c+dx^2}}{2ab\sqrt{e+fx^2}} + \frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2a\left(a+bx^2\right)} + \frac{\sqrt{e}\sqrt{f}\sqrt{c+dx^2}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{2ab\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}\sqrt{e+fx^2}} + \frac{d\sqrt{e}\sqrt{\cdots}}{\cdots}$$

Mathematica [C] time = 1.99279, size = 401, normalized size = 1.05

$$\frac{-\frac{ic\sqrt{\frac{d}{c}}\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}(af+be)\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{\frac{d}{c}}\right),\frac{cf}{de}\right)}{b^2} + \frac{iacf\sqrt{\frac{d}{c}}\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\Pi\left(\frac{bc}{ad};i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)}{b^2} - \frac{ice\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\Pi\left(\frac{bc}{ad};i\sinh\cdots\right.}{a\sqrt{\frac{d}{c}}}\cdots}{2a\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(a + b*x^2)^2,x]
```

```
[Out] ((c*e*x)/(a + b*x^2) + (d*e*x^3)/(a + b*x^2) + (c*f*x^3)/(a + b*x^2) + (d*f
*x^5)/(a + b*x^2) + (I*c*Sqrt[d/c]*e*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e
]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/b - (I*c*Sqrt[d/c]*(b*e +
 a*f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]
*x], (c*f)/(d*e)])/b^2 - (I*c*e*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*Ell
ipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/(a*Sqrt[d/c]) +
(I*a*c*Sqrt[d/c]*f*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)
/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/b^2)/(2*a*Sqrt[c + d*x^2]*Sqr
t[e + f*x^2])
```

Maple [A] time = 0.036, size = 765, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^2,x)
```

```
[Out] 1/2*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)*((-d/c)^(1/2)*x^5*a*b^2*d*f+((d*x^2+c)/
c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*
a^2*b*d*f+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),
(c*f/d/e)^(1/2))*x^2*a*b^2*d*e-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Elli
pticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a*b^2*d*e-((d*x^2+c)/c)^(1/2)*((f
```

```
*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2
))*x^2*a^2*b*d*f+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c
)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*x^2*b^3*c*e+(-d/c)^(1/2)*x^3*a*b
^2*c*f+(-d/c)^(1/2)*x^3*a*b^2*d*e+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*E
llipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^3*d*f+((d*x^2+c)/c)^(1/2)*((f*x^
2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*b*d*e-((d*x^2+c
)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^
2*b*d*e-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b
*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a^3*d*f+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)
^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a*b^2*c
*e+(-d/c)^(1/2)*x*a*b^2*c*e)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)/a^2/(b*x^2+a)/b^
2/(-d/c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2+c}\sqrt{fx^2+e}}{(bx^2+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^2,x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b*x^2 + a)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^2,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{(a+bx^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)*(f*x**2+e)**(1/2)/(b*x**2+a)**2,x)
```

```
[Out] Integral(sqrt(c + d*x**2)*sqrt(e + f*x**2)/(a + b*x**2)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2 + c}\sqrt{fx^2 + e}}{\left(bx^2 + a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b*x^2 + a)^2, x)
```

3.101 $\int \frac{1}{(a+bx^2)^2\sqrt{c-dx^2}\sqrt{e+fx^2}}\,dx$

Optimal. Leaf size=426

$$-\frac{\sqrt{c}\sqrt{d}\sqrt{1-\frac{dx^2}{c}}\sqrt{\frac{fx^2}{e}+1}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{cf}{de}\right)}{2a\sqrt{c-dx^2}\sqrt{e+fx^2}(ad+bc)}+\frac{\sqrt{c}\sqrt{1-\frac{dx^2}{c}}\sqrt{\frac{fx^2}{e}+1}\left(-3a^2df+ab(2de-2cf)+b^2ce\right)\Pi\left(-\frac{bc}{ad}\right.}{2a^2\sqrt{d}\sqrt{c-dx^2}\sqrt{e+fx^2}(ad+bc)(be-af)}$$

```
[Out] (b^2*x*Sqrt[c - d*x^2]*Sqrt[e + f*x^2])/(2*a*(b*c + a*d)*(b*e - a*f)*(a + b
*x^2)) + (b*Sqrt[c]*Sqrt[d]*Sqrt[1 - (d*x^2)/c]*Sqrt[e + f*x^2]*EllipticE[A
rcSin[(Sqrt[d]*x)/Sqrt[c]], -((c*f)/(d*e))])/(2*a*(b*c + a*d)*(b*e - a*f)*S
qrt[c - d*x^2]*Sqrt[1 + (f*x^2)/e]) - (Sqrt[c]*Sqrt[d]*Sqrt[1 - (d*x^2)/c]*
Sqrt[1 + (f*x^2)/e]*EllipticF[ArcSin[(Sqrt[d]*x)/Sqrt[c]], -((c*f)/(d*e))])
/(2*a*(b*c + a*d)*Sqrt[c - d*x^2]*Sqrt[e + f*x^2]) + (Sqrt[c]*(b^2*c*e - 3*
a^2*d*f + a*b*(2*d*e - 2*c*f))*Sqrt[1 - (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*Elli
pticPi[-((b*c)/(a*d)), ArcSin[(Sqrt[d]*x)/Sqrt[c]], -((c*f)/(d*e))])/(2*a^2
*Sqrt[d]*(b*c + a*d)*(b*e - a*f)*Sqrt[c - d*x^2]*Sqrt[e + f*x^2])
```

Rubi [A] time = 0.373077, antiderivative size = 426, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {549, 524, 427, 426, 424, 421, 419, 538, 537}

$$\frac{\sqrt{c}\sqrt{1-\frac{dx^2}{c}}\sqrt{\frac{fx^2}{e}+1}\left(-3a^2df+ab(2de-2cf)+b^2ce\right)\Pi\left(-\frac{bc}{ad};\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{cf}{de}\right)}{2a^2\sqrt{d}\sqrt{c-dx^2}\sqrt{e+fx^2}(ad+bc)(be-af)}+\frac{b^2x\sqrt{c-dx^2}\sqrt{e+fx^2}}{2a\left(a+bx^2\right)(ad+bc)(be-af)}-$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^2*Sqrt[c - d*x^2]*Sqrt[e + f*x^2]),x]
```


```
[Out] (b^2*x*Sqrt[c - d*x^2]*Sqrt[e + f*x^2])/(2*a*(b*c + a*d)*(b*e - a*f)*(a + b
*x^2)) + (b*Sqrt[c]*Sqrt[d]*Sqrt[1 - (d*x^2)/c]*Sqrt[e + f*x^2]*EllipticE[A
rcSin[(Sqrt[d]*x)/Sqrt[c]], -((c*f)/(d*e))])/(2*a*(b*c + a*d)*(b*e - a*f)*S
qrt[c - d*x^2]*Sqrt[1 + (f*x^2)/e]) - (Sqrt[c]*Sqrt[d]*Sqrt[1 - (d*x^2)/c]*
Sqrt[1 + (f*x^2)/e]*EllipticF[ArcSin[(Sqrt[d]*x)/Sqrt[c]], -((c*f)/(d*e))])
/(2*a*(b*c + a*d)*Sqrt[c - d*x^2]*Sqrt[e + f*x^2]) + (Sqrt[c]*(b^2*c*e - 3*
a^2*d*f + a*b*(2*d*e - 2*c*f))*Sqrt[1 - (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*Elli
pticPi[-((b*c)/(a*d)), ArcSin[(Sqrt[d]*x)/Sqrt[c]], -((c*f)/(d*e))])/(2*a^2
*Sqrt[d]*(b*c + a*d)*(b*e - a*f)*Sqrt[c - d*x^2]*Sqrt[e + f*x^2])
```


Rule 549

```
Int[1/(((a_) + (b_.)*(x_)^2)^2*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Simp[(b^2*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(2*a*(b
*c - a*d)*(b*e - a*f)*(a + b*x^2)), x] + (-Dist[(d*f)/(2*a*(b*c - a*d)*(b*e
 - a*f)), Int[(a + b*x^2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x] + Dist[
(b^2*c*e + 3*a^2*d*f - 2*a*b*(d*e + c*f))/(2*a*(b*c - a*d)*(b*e - a*f)), In
t[1/((a + b*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x]) /; FreeQ[{a, b,
c, d, e, f}, x]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
```

```
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 427

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d*x^2)/c
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)^2\sqrt{c-dx^2}\sqrt{e+fx^2}}\,dx = \frac{b^2x\sqrt{c-dx^2}\sqrt{e+fx^2}}{2a(bc+ad)(be-af)\left(a+bx^2\right)} + \frac{(df)\int \frac{a+bx^2}{\sqrt{c-dx^2}\sqrt{e+fx^2}}\,dx}{2a(bc+ad)(be-af)} + \frac{\left(b^2ce-3a^2df-2ab(-\cdots\right.}{2a(bc\cdots}$$

$$= \frac{b^2x\sqrt{c-dx^2}\sqrt{e+fx^2}}{2a(bc+ad)(be-af)\left(a+bx^2\right)} - \frac{d\int \frac{1}{\sqrt{c-dx^2}\sqrt{e+fx^2}}\,dx}{2a(bc+ad)} + \frac{(bd)\int \frac{\sqrt{e+fx^2}}{\sqrt{c-dx^2}}\,dx}{2a(bc+ad)(be-af)} + \cdots$$

$$= \frac{b^2x\sqrt{c-dx^2}\sqrt{e+fx^2}}{2a(bc+ad)(be-af)\left(a+bx^2\right)} + \frac{\left(bd\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{\sqrt{e+fx^2}}{\sqrt{1-\frac{dx^2}{c}}}\,dx}{2a(bc+ad)(be-af)\sqrt{c-dx^2}} - \frac{\left(d\sqrt{1+\frac{fx^2}{e}}\right)\int \cdots}{2a(bc+ad\cdots}$$

$$= \frac{b^2x\sqrt{c-dx^2}\sqrt{e+fx^2}}{2a(bc+ad)(be-af)\left(a+bx^2\right)} + \frac{\sqrt{c}\left(b^2ce-3a^2df+ab(2de-2cf)\right)\sqrt{1-\frac{dx^2}{c}}\sqrt{1\cdots}}{2a^2\sqrt{d}(bc+ad)(be-af)\sqrt{c-\cdots}}$$

$$= \frac{b^2x\sqrt{c-dx^2}\sqrt{e+fx^2}}{2a(bc+ad)(be-af)\left(a+bx^2\right)} + \frac{b\sqrt{c}\sqrt{d}\sqrt{1-\frac{dx^2}{c}}\sqrt{e+fx^2}E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{cf}{de}\right)}{2a(bc+ad)(be-af)\sqrt{c-dx^2}\sqrt{1+\frac{fx^2}{e}}} \cdots$$

<u>Mathematica</u> [C] time = 5.69896, size = 617, normalized size = 1.45

$$\frac{ic\sqrt{-\frac{d}{c}}\sqrt{1-\frac{dx^2}{c}}\sqrt{\frac{fx^2}{e}+1}(be-af)\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{-\frac{d}{c}}\right),-\frac{cf}{de}\right)+\frac{ib^2ce\sqrt{1-\frac{dx^2}{c}}\sqrt{\frac{fx^2}{e}+1}\Pi\left(-\frac{bc}{ad};i\sinh^{-1}\left(\sqrt{-\frac{d}{c}}x\right)|-\frac{cf}{de}\right)}{a\sqrt{-\frac{d}{c}}}-\frac{b^2cex}{a+bx^2}\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)^2*Sqrt[c - d*x^2]*Sqrt[e + f*x^2]),x]
```

```
[Out] (-((b^2*c*e*x)/(a + b*x^2)) + (b^2*d*e*x^3)/(a + b*x^2) - (b^2*c*f*x^3)/(a
+ b*x^2) + (b^2*d*f*x^5)/(a + b*x^2) - I*b*c*Sqrt[-(d/c)]*e*Sqrt[1 - (d*x^2
)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[-(d/c)]*x], -((c*f)/(d*e)
)] + I*c*Sqrt[-(d/c)]*(b*e - a*f)*Sqrt[1 - (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*E
llipticF[I*ArcSinh[Sqrt[-(d/c)]*x], -((c*f)/(d*e))] + (I*b^2*c*e*Sqrt[1 - (
d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[-((b*c)/(a*d)), I*ArcSinh[Sqrt[-(d
/c)]*x], -((c*f)/(d*e))])/(a*Sqrt[-(d/c)]) - (2*I)*b*c*Sqrt[-(d/c)]*e*Sqrt[
1 - (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[-((b*c)/(a*d)), I*ArcSinh[Sqr
t[-(d/c)]*x], -((c*f)/(d*e))] + ((2*I)*b*d*f*Sqrt[1 - (d*x^2)/c]*Sqrt[1 + (
f*x^2)/e]*EllipticPi[-((b*c)/(a*d)), I*ArcSinh[Sqrt[-(d/c)]*x], -((c*f)/(d*
e))])/(-(d/c))^(3/2) + (3*I)*a*c*Sqrt[-(d/c)]*f*Sqrt[1 - (d*x^2)/c]*Sqrt[1
+ (f*x^2)/e]*EllipticPi[-((b*c)/(a*d)), I*ArcSinh[Sqrt[-(d/c)]*x], -((c*f)/
(d*e))])/(2*a*(b*c + a*d)*(-(b*e) + a*f)*Sqrt[c - d*x^2]*Sqrt[e + f*x^2])
```

<u>Maple</u> [B] time = 0.046, size = 1105, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^2/(-d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

```
[Out] 1/2*(-(d/c)^(1/2)*x^5*a*b^2*d*f+(-(d*x^2-c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*El
lipticF(x*(d/c)^(1/2),(-c*f/d/e)^(1/2))*x^2*a^2*b*d*f-(-(d*x^2-c)/c)^(1/2)*
((f*x^2+e)/e)^(1/2)*EllipticF(x*(d/c)^(1/2),(-c*f/d/e)^(1/2))*x^2*a*b^2*d*e
+(-(d*x^2-c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(d/c)^(1/2),(-c*f/d/e
)^(1/2))*x^2*a*b^2*d*e-3*(-(d*x^2-c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticP
i(x*(d/c)^(1/2),-b*c/a/d,(-f/e)^(1/2)/(d/c)^(1/2))*x^2*a^2*b*d*f-2*(-(d*x^2
-c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(d/c)^(1/2),-b*c/a/d,(-f/e)^(
1/2)/(d/c)^(1/2))*x^2*a*b^2*c*f+2*(-(d*x^2-c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*
EllipticPi(x*(d/c)^(1/2),-b*c/a/d,(-f/e)^(1/2)/(d/c)^(1/2))*x^2*a*b^2*d*e+(
-(d*x^2-c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(d/c)^(1/2),-b*c/a/d,(
-f/e)^(1/2)/(d/c)^(1/2))*x^2*b^3*c*e+(d/c)^(1/2)*x^3*a*b^2*c*f-(d/c)^(1/2)*
x^3*a*b^2*d*e+(-(d*x^2-c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(d/c)^(1
/2),(-c*f/d/e)^(1/2))*a^3*d*f-(-(d*x^2-c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Elli
pticF(x*(d/c)^(1/2),(-c*f/d/e)^(1/2))*a^2*b*d*e+(-(d*x^2-c)/c)^(1/2)*((f*x^
2+e)/e)^(1/2)*EllipticE(x*(d/c)^(1/2),(-c*f/d/e)^(1/2))*a^2*b*d*e-3*(-(d*x^
2-c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(d/c)^(1/2),-b*c/a/d,(-f/e)^
(1/2)/(d/c)^(1/2))*a^3*d*f-2*(-(d*x^2-c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Ellip
ticPi(x*(d/c)^(1/2),-b*c/a/d,(-f/e)^(1/2)/(d/c)^(1/2))*a^2*b*c*f+2*(-(d*x^2
-c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(d/c)^(1/2),-b*c/a/d,(-f/e)^(
1/2)/(d/c)^(1/2))*a^2*b*d*e+(-(d*x^2-c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Ellipt
icPi(x*(d/c)^(1/2),-b*c/a/d,(-f/e)^(1/2)/(d/c)^(1/2))*a*b^2*c*e+(d/c)^(1/2)
*x*a*b^2*c*e)*(f*x^2+e)^(1/2)*(-d*x^2+c)^(1/2)/(d/c)^(1/2)/(b*x^2+a)/a^2/(a
*f-b*e)/(a*d+b*c)/(d*f*x^4-c*f*x^2+d*e*x^2-c*e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^2\sqrt{-dx^2+c}\sqrt{fx^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(-d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(1/((b*x^2 + a)^2*sqrt(-d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(-d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="fric
as")
```

```
[Out] Timed out
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**2/(-d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
```

```
[Out] Exception raised: ValueError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^2\sqrt{-dx^2+c}\sqrt{fx^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(-d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="giac
")
```

```
[Out] integrate(1/((b*x^2 + a)^2*sqrt(-d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

3.102 $\int \frac{1}{\left(a+bx^2\right)^2\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$

Optimal. Leaf size=485

$$-\frac{d\sqrt{e}\sqrt{f}\sqrt{c+dx^2}\,\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{2c\sqrt{e+fx^2}(bc-ad)(be-af)\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}+\frac{\sqrt{-c}\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\left(3a^2df-2ab(cf+de)+b^2ce\right)\Pi\left(\frac{bc}{ad};\sin\right.}{2a^2\sqrt{d}\sqrt{c+dx^2}\sqrt{e+fx^2}(bc-ad)(be-af)}$$

```
[Out] -(b*f*x*Sqrt[c + d*x^2])/(2*a*(b*c - a*d)*(b*e - a*f)*Sqrt[e + f*x^2]) + (b
^2*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(2*a*(b*c - a*d)*(b*e - a*f)*(a + b*x
^2)) + (b*Sqrt[e]*Sqrt[f]*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt
[e]], 1 - (d*e)/(c*f)])/(2*a*(b*c - a*d)*(b*e - a*f)*Sqrt[(e*(c + d*x^2))/(
c*(e + f*x^2))]*Sqrt[e + f*x^2]) - (d*Sqrt[e]*Sqrt[f]*Sqrt[c + d*x^2]*Ellip
ticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(2*c*(b*c - a*d)*(b*e -
 a*f)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (Sqrt[-c]*(b
^2*c*e + 3*a^2*d*f - 2*a*b*(d*e + c*f))*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2
)/e]*EllipticPi[(b*c)/(a*d), ArcSin[(Sqrt[d]*x)/Sqrt[-c]], (c*f)/(d*e)])/(2
*a^2*Sqrt[d]*(b*c - a*d)*(b*e - a*f)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Rubi [A] time = 0.347122, antiderivative size = 485, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {549, 531, 418, 492, 411, 538, 537}

$$\frac{\sqrt{-c}\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\left(3a^2df-2ab(cf+de)+b^2ce\right)\Pi\left(\frac{bc}{ad};\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{-c}}\right)|\frac{cf}{de}\right)}{2a^2\sqrt{d}\sqrt{c+dx^2}\sqrt{e+fx^2}(bc-ad)(be-af)}+\frac{b^2x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2a\left(a+bx^2\right)(bc-ad)(be-af)}-\frac{\quad}{2a\sqrt{\quad}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^2*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```


```
[Out] -(b*f*x*Sqrt[c + d*x^2])/(2*a*(b*c - a*d)*(b*e - a*f)*Sqrt[e + f*x^2]) + (b
^2*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(2*a*(b*c - a*d)*(b*e - a*f)*(a + b*x
^2)) + (b*Sqrt[e]*Sqrt[f]*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt
[e]], 1 - (d*e)/(c*f)])/(2*a*(b*c - a*d)*(b*e - a*f)*Sqrt[(e*(c + d*x^2))/(
c*(e + f*x^2))]*Sqrt[e + f*x^2]) - (d*Sqrt[e]*Sqrt[f]*Sqrt[c + d*x^2]*Ellip
ticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(2*c*(b*c - a*d)*(b*e -
 a*f)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (Sqrt[-c]*(b
^2*c*e + 3*a^2*d*f - 2*a*b*(d*e + c*f))*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2
)/e]*EllipticPi[(b*c)/(a*d), ArcSin[(Sqrt[d]*x)/Sqrt[-c]], (c*f)/(d*e)])/(2
*a^2*Sqrt[d]*(b*c - a*d)*(b*e - a*f)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```


Rule 549

```
Int[1/(((a_) + (b_.)*(x_)^2)^2*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Simp[(b^2*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(2*a*(b
*c - a*d)*(b*e - a*f)*(a + b*x^2)), x] + (-Dist[(d*f)/(2*a*(b*c - a*d)*(b*e
 - a*f)), Int[(a + b*x^2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x] + Dist[
(b^2*c*e + 3*a^2*d*f - 2*a*b*(d*e + c*f))/(2*a*(b*c - a*d)*(b*e - a*f)), In
t[1/((a + b*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x]) /; FreeQ[{a, b,
c, d, e, f}, x]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)^2\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \frac{b^2x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2a(bc-ad)(be-af)\left(a+bx^2\right)} - \frac{(df)\int \frac{a+bx^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{2a(bc-ad)(be-af)} + \frac{\left(b^2ce+3a^2df-2ab\right.}{2a}$$

$$= \frac{b^2x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2a(bc-ad)(be-af)\left(a+bx^2\right)} - \frac{(df)\int \frac{1}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{2(bc-ad)(be-af)} - \frac{(bdf)\int \frac{x^2}{\sqrt{c+dx^2}\sqrt{e+fx}}}{2a(bc-ad)(be-a}$$

$$= -\frac{bfx\sqrt{c+dx^2}}{2a(bc-ad)(be-af)\sqrt{e+fx^2}} + \frac{b^2x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2a(bc-ad)(be-af)\left(a+bx^2\right)} - \frac{d\sqrt{e}\sqrt{f}\sqrt{c}}{2c(bc-ad}$$

$$= -\frac{bfx\sqrt{c+dx^2}}{2a(bc-ad)(be-af)\sqrt{e+fx^2}} + \frac{b^2x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2a(bc-ad)(be-af)\left(a+bx^2\right)} + \frac{b\sqrt{e}\sqrt{f}\sqrt{c}}{2a(bc-ad}$$

Mathematica [C] time = 2.79509, size = 587, normalized size = 1.21

$$\frac{-ic\sqrt{\frac{d}{c}}\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}(be-af)\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{\frac{d}{c}}\right),\frac{cf}{de}\right) - \frac{ib^2ce\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\Pi\left(\frac{bc}{ad};i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)}{a\sqrt{\frac{d}{c}}} + \frac{b^2cex}{a+bx^2} + \frac{b}{a}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)^2*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

```
[Out] ((b^2*c*e*x)/(a + b*x^2) + (b^2*d*e*x^3)/(a + b*x^2) + (b^2*c*f*x^3)/(a + b
*x^2) + (b^2*d*f*x^5)/(a + b*x^2) + I*b*c*Sqrt[d/c]*e*Sqrt[1 + (d*x^2)/c]*S
qrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - I*c*Sqr
t[d/c]*(b*e - a*f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcS
inh[Sqrt[d/c]*x], (c*f)/(d*e)] - (I*b^2*c*e*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f
*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/(a*S
qrt[d/c]) + (2*I)*b*c*Sqrt[d/c]*e*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*E
llipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + ((2*I)*b*c*f*
Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[S
qrt[d/c]*x], (c*f)/(d*e)])/Sqrt[d/c] - (3*I)*a*c*Sqrt[d/c]*f*Sqrt[1 + (d*x^
2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (
c*f)/(d*e)])/(2*a*(-(b*c) + a*d)*(-(b*e) + a*f)*Sqrt[c + d*x^2]*Sqrt[e + f*
x^2])
```

Maple [B] time = 0.028, size = 1078, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^2/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

```
[Out] -1/2*(-(-d/c)^(1/2)*x^5*a*b^2*d*f+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*E
llipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a^2*b*d*f-((d*x^2+c)/c)^(1/2)*
```

```
((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a*b^2*d*e
+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)
^(1/2))*x^2*a*b^2*d*e-3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(
x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*x^2*a^2*b*d*f+2*((d*x^2+c
)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/
2)/(-d/c)^(1/2))*x^2*a*b^2*c*f+2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*El
lipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*x^2*a*b^2*d*e-((
d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f
/e)^(1/2)/(-d/c)^(1/2))*x^2*b^3*c*e-(-d/c)^(1/2)*x^3*a*b^2*c*f-(-d/c)^(1/2)
*x^3*a*b^2*d*e+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(
1/2),(c*f/d/e)^(1/2))*a^3*d*f-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Ellip
ticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*b*d*e+((d*x^2+c)/c)^(1/2)*((f*x^2+
e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*b*d*e-3*((d*x^2+c
)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/
2)/(-d/c)^(1/2))*a^3*d*f+2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Elliptic
Pi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a^2*b*c*f+2*((d*x^2+c)
/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2
)/(-d/c)^(1/2))*a^2*b*d*e-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticP
i(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a*b^2*c*e-(-d/c)^(1/2)*
x*a*b^2*c*e)*(f*x^2+e)^(1/2)*(d*x^2+c)^(1/2)/(b*x^2+a)/a^2/(a*f-b*e)/(-d/c)
^(1/2)/(a*d-b*c)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2 + a\right)^2 \sqrt{dx^2 + c}\sqrt{fx^2 + e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(1/((b*x^2 + a)^2*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**2/(d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
```

```
[Out] Exception raised: ValueError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^2\sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(1/((b*x^2 + a)^2*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

3.103 $\int \frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx$

Optimal. Leaf size=36

$$\text{Unintegrable}\left(\frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}},x\right)$$

```
[Out] Defer[Int][((a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/Sqrt[e + f*x^2], x]
```

Rubi [A] time = 0.0571239, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/Sqrt[e + f*x^2],x]
```

```
[Out] Defer[Int][((a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/Sqrt[e + f*x^2], x]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx=\int \frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx$$

Mathematica [A] time = 0.741494, size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/Sqrt[e + f*x^2],x]
```

```
[Out] Integrate[((a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/Sqrt[e + f*x^2], x]
```

Maple [A] time = 0.071, size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^{\frac{3}{2}}\sqrt{dx^2+c}\,\frac{1}{\sqrt{fx^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

```
[Out] int((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{3}{2}}\sqrt{dx^2+c}}{\sqrt{fx^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)/sqrt(f*x^2 + e), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="fri
cas")
```

```
[Out] Timed out
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^2\right)^{\frac{3}{2}}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)*(d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)**(3/2)*sqrt(c + d*x**2)/sqrt(e + f*x**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{3}{2}}\sqrt{dx^2+c}}{\sqrt{fx^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)/sqrt(f*x^2 + e), x)
```

3.104 $\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx$

Optimal. Leaf size=545

$$\frac{b\sqrt{e}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right),\frac{e(bc-ad)}{c(be-af)}\right)}{2df\sqrt{e+fx^2}\sqrt{be-af}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}+\frac{dx\sqrt{a+bx^2}\sqrt{e+fx^2}}{2f\sqrt{c+dx^2}}-\frac{\sqrt{e}\sqrt{a+bx^2}\sqrt{de-cf}\sqrt{\cdots}}{2f\cdots}$$

```
[Out] (d*x*Sqrt[a + b*x^2]*Sqrt[e + f*x^2])/(2*f*Sqrt[c + d*x^2]) - (Sqrt[e]*Sqrt
[d*e - c*f]*Sqrt[a + b*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))]*EllipticE
[ArcSin[(Sqrt[d*e - c*f]*x)/(Sqrt[e]*Sqrt[c + d*x^2])], -(((b*c - a*d)*e)/(
a*(d*e - c*f)))])/(2*f*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[e + f*x^2
]) + (b*Sqrt[e]*(d*e - c*f)*Sqrt[c + d*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a + b*
x^2))]*EllipticF[ArcSin[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])], ((b
*c - a*d)*e)/(c*(b*e - a*f))])/(2*d*f*Sqrt[b*e - a*f]*Sqrt[(a*(c + d*x^2))/
(c*(a + b*x^2))]*Sqrt[e + f*x^2]) - (c*Sqrt[e]*(b*d*e - b*c*f - a*d*f)*Sqrt
[a + b*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))]*EllipticPi[(d*e)/(d*e - c
*f), ArcSin[(Sqrt[d*e - c*f]*x)/(Sqrt[e]*Sqrt[c + d*x^2])], -(((b*c - a*d)*
e)/(a*(d*e - c*f)))])/(2*a*d*f*Sqrt[d*e - c*f]*Sqrt[(c*(a + b*x^2))/(a*(c +
 d*x^2))]*Sqrt[e + f*x^2])
```

Rubi [A] time = 0.486577, antiderivative size = 545, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.206, Rules used = {555, 554, 424, 552, 419, 553, 537}

$$\frac{dx\sqrt{a+bx^2}\sqrt{e+fx^2}}{2f\sqrt{c+dx^2}}+\frac{b\sqrt{e}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,F\left(\sin^{-1}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)|\frac{(bc-ad)e}{c(be-af)}\right)}{2df\sqrt{e+fx^2}\sqrt{be-af}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{\sqrt{e}\sqrt{a+bx^2}\sqrt{de-cf}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}E\cdots}{2f\sqrt{e+fx^2}\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/Sqrt[e + f*x^2],x]
```


```
[Out] (d*x*Sqrt[a + b*x^2]*Sqrt[e + f*x^2])/(2*f*Sqrt[c + d*x^2]) - (Sqrt[e]*Sqrt
[d*e - c*f]*Sqrt[a + b*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))]*EllipticE
[ArcSin[(Sqrt[d*e - c*f]*x)/(Sqrt[e]*Sqrt[c + d*x^2])], -(((b*c - a*d)*e)/(
a*(d*e - c*f)))])/(2*f*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[e + f*x^2
]) + (b*Sqrt[e]*(d*e - c*f)*Sqrt[c + d*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a + b*
x^2))]*EllipticF[ArcSin[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])], ((b
*c - a*d)*e)/(c*(b*e - a*f))])/(2*d*f*Sqrt[b*e - a*f]*Sqrt[(a*(c + d*x^2))/
(c*(a + b*x^2))]*Sqrt[e + f*x^2]) - (c*Sqrt[e]*(b*d*e - b*c*f - a*d*f)*Sqrt
[a + b*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))]*EllipticPi[(d*e)/(d*e - c
*f), ArcSin[(Sqrt[d*e - c*f]*x)/(Sqrt[e]*Sqrt[c + d*x^2])], -(((b*c - a*d)*
e)/(a*(d*e - c*f)))])/(2*a*d*f*Sqrt[d*e - c*f]*Sqrt[(c*(a + b*x^2))/(a*(c +
 d*x^2))]*Sqrt[e + f*x^2])
```


Rule 555

```
Int[(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2])/Sqrt[(e_) + (f_.)
*(x_)^2], x_Symbol] :> Simp[(d*x*Sqrt[a + b*x^2]*Sqrt[e + f*x^2])/(2*f*Sqrt
[c + d*x^2]), x] + (-Dist[(c*(d*e - c*f))/(2*f), Int[Sqrt[a + b*x^2]/((c +
```

```
d*x^2)^(3/2)*Sqrt[e + f*x^2]), x], x] + Dist[(b*c*(d*e - c*f))/(2*d*f), Int
[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x] - Dist[(b*d*e
- b*c*f - a*d*f)/(2*d*f), Int[Sqrt[c + d*x^2]/(Sqrt[a + b*x^2]*Sqrt[e + f*x
^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[(d*e - c*f)/c]
```

Rule 554

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)^(3/2)*Sqrt[(e_) + (f_.
)*(x_)^2]), x_Symbol] :> Dist[(Sqrt[c + d*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a +
 b*x^2))])/(a*Sqrt[e + f*x^2]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]), Subst
[Int[Sqrt[1 - ((b*c - a*d)*x^2)/c]/Sqrt[1 - ((b*e - a*f)*x^2)/e], x], x, x/
Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 552

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.
)*(x_)^2]), x_Symbol] :> Dist[(Sqrt[c + d*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a +
 b*x^2))])/(c*Sqrt[e + f*x^2]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]), Subst
[Int[1/(Sqrt[1 - ((b*c - a*d)*x^2)/c]*Sqrt[1 - ((b*e - a*f)*x^2)/e]), x], x
, x/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 553

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/(Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Dist[(a*Sqrt[c + d*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a +
 b*x^2))])/(c*Sqrt[e + f*x^2]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]), Subst
[Int[1/((1 - b*x^2)*Sqrt[1 - ((b*c - a*d)*x^2)/c]*Sqrt[1 - ((b*e - a*f)*x^2
)/e]), x], x, x/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx = \frac{dx\sqrt{a+bx^2}\sqrt{e+fx^2}}{2f\sqrt{c+dx^2}} - \frac{(c(de-cf))\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}}\,dx}{2f} + \frac{(bc(de-cf))\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}}{2df}$$

$$= \frac{dx\sqrt{a+bx^2}\sqrt{e+fx^2}}{2f\sqrt{c+dx^2}} + \frac{\left(b(de-cf)\sqrt{c+dx^2}\sqrt{\frac{a\left(e+fx^2\right)}{e\left(a+bx^2\right)}}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-\frac{(bc-ad)x^2}{c}}\sqrt{1-\frac{(be-af)x^2}{e}}}\,dx, x\right.}{2df\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}\sqrt{e+fx^2}}$$

$$= \frac{dx\sqrt{a+bx^2}\sqrt{e+fx^2}}{2f\sqrt{c+dx^2}} - \frac{\sqrt{e}\sqrt{de-cf}\sqrt{a+bx^2}\sqrt{\frac{c\left(e+fx^2\right)}{e\left(c+dx^2\right)}}E\left(\sin^{-1}\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{c+dx^2}}\right)|-\frac{(bc-ad)e}{a(de-cf)}\right)}{2f\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}\sqrt{e+fx^2}} + \frac{b\sqrt{\ldots}}{\ldots}$$

Mathematica [A] time = 1.35082, size = 503, normalized size = 0.92

$$\frac{\frac{\sqrt{e+fx^2}(be-2af)(de-cf)\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}\mathrm{EllipticF}\left(\sin^{-1}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right),\frac{bce-ade}{bce-acf}\right)}{\sqrt{e}f^2\sqrt{be-af}\sqrt{\frac{a\left(e+fx^2\right)}{e\left(a+bx^2\right)}}} + \frac{e\sqrt{a+bx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}(adf+bcf-bde)\Pi\left(\frac{af}{af-be};\sin^{-1}\left(\frac{\sqrt{af-bex}}{\sqrt{a}\sqrt{fx^2+e}}\right)|\frac{ade-acf}{bce-acf}\right)}{\sqrt{a}f^2\sqrt{af-be}\sqrt{\frac{e\left(a+bx^2\right)}{a\left(e+fx^2\right)}}} + \frac{x\sqrt{a+bx\ldots}}{\sqrt{e\ldots}}}{2\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/Sqrt[e + f*x^2],x]
```

```
[Out] ((x*Sqrt[a + b*x^2]*(c + d*x^2))/Sqrt[e + f*x^2] - (Sqrt[c]*Sqrt[-(d*e) + c
*f]*Sqrt[a + b*x^2]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*EllipticE[ArcSin[
(Sqrt[-(d*e) + c*f]*x)/(Sqrt[c]*Sqrt[e + f*x^2])], (b*c*e - a*c*f)/(a*d*e -
 a*c*f)])/(f*Sqrt[(e*(a + b*x^2))/(a*(e + f*x^2))]) + ((b*e - 2*a*f)*(d*e -
 c*f)*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]*Sqrt[e + f*x^2]*EllipticF[ArcSi
n[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])], (b*c*e - a*d*e)/(b*c*e -
a*c*f)])/(Sqrt[e]*f^2*Sqrt[b*e - a*f]*Sqrt[(a*(e + f*x^2))/(e*(a + b*x^2))]
) + (e*(-(b*d*e) + b*c*f + a*d*f)*Sqrt[a + b*x^2]*Sqrt[(e*(c + d*x^2))/(c*(
e + f*x^2))]*EllipticPi[(a*f)/(-(b*e) + a*f), ArcSin[(Sqrt[-(b*e) + a*f]*x)
/(Sqrt[a]*Sqrt[e + f*x^2])], (a*d*e - a*c*f)/(b*c*e - a*c*f)])/(Sqrt[a]*f^2
*Sqrt[-(b*e) + a*f]*Sqrt[(e*(a + b*x^2))/(a*(e + f*x^2))]))/(2*Sqrt[c + d*x
^2])
```

Maple [F] time = 0.066, size = 0, normalized size = 0.

$$\int \sqrt{bx^2+a}\sqrt{dx^2+c}\frac{1}{\sqrt{fx^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

```
[Out] int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2 + a}\sqrt{dx^2 + c}}{\sqrt{fx^2 + e}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/sqrt(f*x^2 + e), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="fri
cas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + bx^2}\sqrt{c + dx^2}}{\sqrt{e + fx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x**2)*sqrt(c + d*x**2)/sqrt(e + f*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2 + a}\sqrt{dx^2 + c}}{\sqrt{fx^2 + e}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/sqrt(f*x^2 + e), x)
```

3.105 $\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}\sqrt{e+fx^2}}\,dx$

Optimal. Leaf size=163

$$\frac{c\sqrt{e}\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\Pi\left(\frac{de}{de-cf};\sin^{-1}\left(\frac{\sqrt{de-cf}\,x}{\sqrt{e}\sqrt{dx^2+c}}\right)|-\frac{(bc-ad)e}{a(de-cf)}\right)}{a\sqrt{e+fx^2}\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

```
[Out] (c*Sqrt[e]*Sqrt[a + b*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))]*EllipticPi
[(d*e)/(d*e - c*f), ArcSin[(Sqrt[d*e - c*f]*x)/(Sqrt[e]*Sqrt[c + d*x^2])],
-(((b*c - a*d)*e)/(a*(d*e - c*f)))])/(a*Sqrt[d*e - c*f]*Sqrt[(c*(a + b*x^2)
)/(a*(c + d*x^2))]*Sqrt[e + f*x^2])
```

Rubi [A] time = 0.133839, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {553, 537}

$$\frac{c\sqrt{e}\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\Pi\left(\frac{de}{de-cf};\sin^{-1}\left(\frac{\sqrt{de-cf}\,x}{\sqrt{e}\sqrt{dx^2+c}}\right)|-\frac{(bc-ad)e}{a(de-cf)}\right)}{a\sqrt{e+fx^2}\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^2]/(Sqrt[a + b*x^2]*Sqrt[e + f*x^2]),x]
```

```
[Out] (c*Sqrt[e]*Sqrt[a + b*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))]*EllipticPi
[(d*e)/(d*e - c*f), ArcSin[(Sqrt[d*e - c*f]*x)/(Sqrt[e]*Sqrt[c + d*x^2])],
-(((b*c - a*d)*e)/(a*(d*e - c*f)))])/(a*Sqrt[d*e - c*f]*Sqrt[(c*(a + b*x^2)
)/(a*(c + d*x^2))]*Sqrt[e + f*x^2])
```

Rule 553

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/(Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Dist[(a*Sqrt[c + d*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a +
 b*x^2))])/(c*Sqrt[e + f*x^2]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]), Subst
[Int[1/((1 - b*x^2)*Sqrt[1 - ((b*c - a*d)*x^2)/c]*Sqrt[1 - ((b*e - a*f)*x^2
)/e]), x], x, x/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}\sqrt{e+fx^2}}\,dx = \frac{\left(c\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\right)\text{Subst}\left(\int \frac{1}{(1-dx^2)\sqrt{1-\frac{(-bc+ad)x^2}{a}}\sqrt{1-\frac{(de-cf)x^2}{e}}}\,dx, x, \frac{x}{\sqrt{c+dx^2}}\right)}{a\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$

$$= \frac{c\sqrt{e}\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\Pi\left(\frac{de}{de-cf};\sin^{-1}\left(\frac{\sqrt{de-cf}\,x}{\sqrt{e}\sqrt{c+dx^2}}\right)|-\frac{(bc-ad)e}{a(de-cf)}\right)}{a\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$

Mathematica [A] time = 0.100159, size = 162, normalized size = 0.99

$$\frac{c\sqrt{e}\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\Pi\left(\frac{de}{de-cf};\sin^{-1}\left(\frac{\sqrt{de-cf}\,x}{\sqrt{e}\sqrt{dx^2+c}}\right)|\frac{(ad-bc)e}{a(de-cf)}\right)}{a\sqrt{e+fx^2}\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^2]/(Sqrt[a + b*x^2]*Sqrt[e + f*x^2]),x]
```

```
[Out] (c*Sqrt[e]*Sqrt[a + b*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))]*EllipticPi
[(d*e)/(d*e - c*f), ArcSin[(Sqrt[d*e - c*f]*x)/(Sqrt[e]*Sqrt[c + d*x^2])],
((-(b*c) + a*d)*e)/(a*(d*e - c*f))])/(a*Sqrt[d*e - c*f]*Sqrt[(c*(a + b*x^2)
)/(a*(c + d*x^2))]*Sqrt[e + f*x^2])
```

Maple [F] time = 0.062, size = 0, normalized size = 0.

$$\int \sqrt{dx^2+c}\frac{1}{\sqrt{bx^2+a}}\frac{1}{\sqrt{fx^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)/(b*x^2+a)^(1/2)/(f*x^2+e)^(1/2),x)
```

```
[Out] int((d*x^2+c)^(1/2)/(b*x^2+a)^(1/2)/(f*x^2+e)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\sqrt{fx^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(d*x^2 + c)/(sqrt(b*x^2 + a)*sqrt(f*x^2 + e)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="fri
cas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}\sqrt{e+fx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)/(b*x**2+a)**(1/2)/(f*x**2+e)**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*x**2)/(sqrt(a + b*x**2)*sqrt(e + f*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\sqrt{fx^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(d*x^2 + c)/(sqrt(b*x^2 + a)*sqrt(f*x^2 + e)), x)
```

3.106 $\int \frac{\sqrt{c+dx^2}}{\left(a+bx^2\right)^{3/2}\sqrt{e+fx^2}}\,dx$

Optimal. Leaf size=148

$$\frac{\sqrt{e}\sqrt{c+dx^2}\sqrt{\frac{a\left(e+fx^2\right)}{e\left(a+bx^2\right)}}E\left(\sin^{-1}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)|\frac{(bc-ad)e}{c(be-af)}\right)}{a\sqrt{e+fx^2}\sqrt{be-af}\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}$$

```
[Out] (Sqrt[e]*Sqrt[c + d*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a + b*x^2))]*EllipticE[Ar
cSin[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])], ((b*c - a*d)*e)/(c*(b*
e - a*f))])/(a*Sqrt[b*e - a*f]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]*Sqrt[e
 + f*x^2])
```

Rubi [A] time = 0.0846888, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {554, 424}

$$\frac{\sqrt{e}\sqrt{c+dx^2}\sqrt{\frac{a\left(e+fx^2\right)}{e\left(a+bx^2\right)}}E\left(\sin^{-1}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)|\frac{(bc-ad)e}{c(be-af)}\right)}{a\sqrt{e+fx^2}\sqrt{be-af}\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^2]/((a + b*x^2)^(3/2)*Sqrt[e + f*x^2]),x]
```

```
[Out] (Sqrt[e]*Sqrt[c + d*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a + b*x^2))]*EllipticE[Ar
cSin[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])], ((b*c - a*d)*e)/(c*(b*
e - a*f))])/(a*Sqrt[b*e - a*f]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]*Sqrt[e
 + f*x^2])
```

Rule 554

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)^(3/2)*Sqrt[(e_) + (f_.
)*(x_)^2]), x_Symbol] :> Dist[(Sqrt[c + d*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a +
 b*x^2))])/(a*Sqrt[e + f*x^2]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]), Subst
[Int[Sqrt[1 - ((b*c - a*d)*x^2)/c]/Sqrt[1 - ((b*e - a*f)*x^2)/e], x], x, x/
Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^2}}{\left(a+bx^2\right)^{3/2}\sqrt{e+fx^2}}\,dx = \frac{\left(\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\right)\text{Subst}\left(\int \frac{\sqrt{1-\frac{(bc-ad)x^2}{c}}}{\sqrt{1-\frac{(be-af)x^2}{e}}}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{a\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

$$= \frac{\sqrt{e}\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}E\left(\sin^{-1}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{a+bx^2}}\right)|\frac{(bc-ad)e}{c(be-af)}\right)}{a\sqrt{be-af}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

Mathematica [A] time = 0.081812, size = 148, normalized size = 1.

$$\frac{\sqrt{e}\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}E\left(\sin^{-1}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)|\frac{(bc-ad)e}{c(be-af)}\right)}{a\sqrt{e+fx^2}\sqrt{be-af}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^2]/((a + b*x^2)^(3/2)*Sqrt[e + f*x^2]),x]
```

```
[Out] (Sqrt[e]*Sqrt[c + d*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a + b*x^2))]*EllipticE[Ar
cSin[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])], ((b*c - a*d)*e)/(c*(b*
e - a*f))])/(a*Sqrt[b*e - a*f]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]*Sqrt[e
 + f*x^2])
```

Maple [F] time = 0.076, size = 0, normalized size = 0.

$$\int \sqrt{dx^2+c}\left(bx^2+a\right)^{-\frac{3}{2}}\frac{1}{\sqrt{fx^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)/(b*x^2+a)^(3/2)/(f*x^2+e)^(1/2),x)
```

```
[Out] int((d*x^2+c)^(1/2)/(b*x^2+a)^(3/2)/(f*x^2+e)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2+c}}{\left(bx^2+a\right)^{\frac{3}{2}}\sqrt{fx^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(3/2)/(f*x^2+e)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(d*x^2 + c)/((b*x^2 + a)^(3/2)*sqrt(f*x^2 + e)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{fx^2+e}}{b^2fx^6+\left(b^2e+2\,abf\right)x^4+a^2e+\left(2\,abe+a^2f\right)x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(3/2)/(f*x^2+e)^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b^2*f*x^6 + (b^2*
e + 2*a*b*f)*x^4 + a^2*e + (2*a*b*e + a^2*f)*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+dx^2}}{\left(a+bx^2\right)^{\frac{3}{2}}\sqrt{e+fx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)/(b*x**2+a)**(3/2)/(f*x**2+e)**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*x**2)/((a + b*x**2)**(3/2)*sqrt(e + f*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2+c}}{\left(bx^2+a\right)^{\frac{3}{2}}\sqrt{fx^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(3/2)/(f*x^2+e)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(d*x^2 + c)/((b*x^2 + a)^(3/2)*sqrt(f*x^2 + e)), x)
```

3.107 $\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx$

Optimal. Leaf size=36

$$\text{Unintegrable}\left(\frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}},x\right)$$

```
[Out] Defer[Int][((a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(e + f*x^2)^(3/2), x]
```

Rubi [A] time = 0.0611418, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(e + f*x^2)^(3/2),x]
```

```
[Out] Defer[Int][((a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(e + f*x^2)^(3/2), x]
```

Rubi steps

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx$$

Mathematica [A] time = 0.755644, size = 0, normalized size = 0.

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(e + f*x^2)^(3/2),x]
```

```
[Out] Integrate[((a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(e + f*x^2)^(3/2), x]
```

Maple [A] time = 0.073, size = 0, normalized size = 0.

$$\int (bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}\,(fx^2+e)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

```
[Out] int((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{3}{2}}\sqrt{dx^2+c}}{\left(fx^2+e\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="max
ima")
```

```
[Out] integrate((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)/(f*x^2 + e)^(3/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="fri
cas")
```

```
[Out] Timed out
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^2\right)^{\frac{3}{2}}\sqrt{c+dx^2}}{\left(e+fx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)*(d*x**2+c)**(1/2)/(f*x**2+e)**(3/2),x)
```

```
[Out] Integral((a + b*x**2)**(3/2)*sqrt(c + d*x**2)/(e + f*x**2)**(3/2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{3}{2}}\sqrt{dx^2+c}}{\left(fx^2+e\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="gia
c")
```

```
[Out] integrate((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)/(f*x^2 + e)^(3/2), x)
```

3.108 $\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx$

Optimal. Leaf size=484

$$-\frac{c^{3/2}\sqrt{a+bx^2}(be-af)\text{EllipticF}\left(\tan^{-1}\left(\frac{x\sqrt{de-cf}}{\sqrt{c}\sqrt{e+fx^2}}\right),-\frac{e(bc-ad)}{a(de-cf)}\right)}{aef\sqrt{c+dx^2}\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{x\sqrt{a+bx^2}(de-cf)}{ef\sqrt{c+dx^2}\sqrt{e+fx^2}}+\frac{\sqrt{c}\sqrt{a+bx^2}\sqrt{de-cf}E\left(\tan\right.}{ef\sqrt{c+dx^2}}$$

```
[Out] -(((d*e - c*f)*x*Sqrt[a + b*x^2])/(e*f*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])) +
(Sqrt[c]*Sqrt[d*e - c*f]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d*e - c*f]*
x)/(Sqrt[c]*Sqrt[e + f*x^2])], -(((b*c - a*d)*e)/(a*(d*e - c*f)))])/(e*f*Sq
rt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (c^(3/2)*(b*e - a*f)
*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d*e - c*f]*x)/(Sqrt[c]*Sqrt[e + f*x
^2])], -(((b*c - a*d)*e)/(a*(d*e - c*f)))])/(a*e*f*Sqrt[d*e - c*f]*Sqrt[(c*
(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (b*c*Sqrt[e]*Sqrt[a + b*x^
2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))]*EllipticPi[(d*e)/(d*e - c*f), ArcS
in[(Sqrt[d*e - c*f]*x)/(Sqrt[e]*Sqrt[c + d*x^2])], -(((b*c - a*d)*e)/(a*(d*
e - c*f)))])/(a*f*Sqrt[d*e - c*f]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqr
t[e + f*x^2])
```

Rubi [A] time = 0.753475, antiderivative size = 484, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {557, 553, 537, 554, 422, 418, 492, 411}

$$-\frac{c^{3/2}\sqrt{a+bx^2}(be-af)F\left(\tan^{-1}\left(\frac{\sqrt{de-cf}x}{\sqrt{c}\sqrt{fx^2+e}}\right)|-\frac{(bc-ad)e}{a(de-cf)}\right)}{aef\sqrt{c+dx^2}\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{x\sqrt{a+bx^2}(de-cf)}{ef\sqrt{c+dx^2}\sqrt{e+fx^2}}+\frac{\sqrt{c}\sqrt{a+bx^2}\sqrt{de-cf}E\left(\tan^{-1}\left(\frac{\sqrt{d}}{\sqrt{c}}\right.\right.}{ef\sqrt{c+dx^2}\sqrt{\frac{c(a+b}{a(c+d}}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(e + f*x^2)^(3/2),x]
```


```
[Out] -(((d*e - c*f)*x*Sqrt[a + b*x^2])/(e*f*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])) +
(Sqrt[c]*Sqrt[d*e - c*f]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d*e - c*f]*
x)/(Sqrt[c]*Sqrt[e + f*x^2])], -(((b*c - a*d)*e)/(a*(d*e - c*f)))])/(e*f*Sq
rt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (c^(3/2)*(b*e - a*f)
*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d*e - c*f]*x)/(Sqrt[c]*Sqrt[e + f*x
^2])], -(((b*c - a*d)*e)/(a*(d*e - c*f)))])/(a*e*f*Sqrt[d*e - c*f]*Sqrt[(c*
(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (b*c*Sqrt[e]*Sqrt[a + b*x^
2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))]*EllipticPi[(d*e)/(d*e - c*f), ArcS
in[(Sqrt[d*e - c*f]*x)/(Sqrt[e]*Sqrt[c + d*x^2])], -(((b*c - a*d)*e)/(a*(d*
e - c*f)))])/(a*f*Sqrt[d*e - c*f]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqr
t[e + f*x^2])
```


Rule *557*

```
Int[(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2])/((e_) + (f_.)*(x_
)^2)^(3/2), x_Symbol] :> Dist[b/f, Int[Sqrt[c + d*x^2]/(Sqrt[a + b*x^2]*Sqr
t[e + f*x^2]), x], x] - Dist[(b*e - a*f)/f, Int[Sqrt[c + d*x^2]/(Sqrt[a + b
*x^2]*(e + f*x^2)^(3/2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 553

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/(Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Dist[(a*Sqrt[c + d*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a +
 b*x^2))])/(c*Sqrt[e + f*x^2]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]), Subst
[Int[1/((1 - b*x^2)*Sqrt[1 - ((b*c - a*d)*x^2)/c]*Sqrt[1 - ((b*e - a*f)*x^2
)/e]), x], x, x/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 554

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)^(3/2)*Sqrt[(e_) + (f_.
)*(x_)^2]), x_Symbol] :> Dist[(Sqrt[c + d*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a +
 b*x^2))])/(a*Sqrt[e + f*x^2]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]), Subst
[Int[Sqrt[1 - ((b*c - a*d)*x^2)/c]/Sqrt[1 - ((b*e - a*f)*x^2)/e], x], x, x/
Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{\left(e+fx^2\right)^{3/2}}\,dx = \frac{b\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}\sqrt{e+fx^2}}\,dx}{f} - \frac{(be-af)\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}\left(e+fx^2\right)^{3/2}}\,dx}{f}$$

$$= -\frac{\left((be-af)\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}\right)\text{Subst}\left(\int \frac{\sqrt{1-\frac{(-de+cf)x^2}{c}}}{\sqrt{1-\frac{(-be+af)x^2}{a}}}\,dx, x, \frac{x}{\sqrt{e+fx^2}}\right)}{ef\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}} + \frac{\left(bc\sqrt{a+bx^2}\sqrt{\frac{c(e+\cdots}{e(c-\cdots}}\right.}{\cdots}$$

$$= \frac{bc\sqrt{e}\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\Pi\left(\frac{de}{de-cf};\sin^{-1}\left(\frac{\sqrt{de-cfx}}{\sqrt{e}\sqrt{c+dx^2}}\right)|-\frac{(bc-ad)e}{a(de-cf)}\right)}{af\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}} - \frac{\left((be-af)\sqrt{c+dx^2}\sqrt{\frac{e(a\cdots}{a(e\cdots}}\right.}{\cdots}$$

$$= -\frac{(de-cf)x\sqrt{a+bx^2}}{ef\sqrt{c+dx^2}\sqrt{e+fx^2}} - \frac{c^{3/2}(be-af)\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{de-cfx}}{\sqrt{c}\sqrt{e+fx^2}}\right)|-\frac{(bc-ad)e}{a(de-cf)}\right)}{aef\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} + \frac{bc\sqrt{e}\sqrt{\cdots}}{\cdots}$$

$$= -\frac{(de-cf)x\sqrt{a+bx^2}}{ef\sqrt{c+dx^2}\sqrt{e+fx^2}} + \frac{\sqrt{c}\sqrt{de-cf}\sqrt{a+bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{de-cfx}}{\sqrt{c}\sqrt{e+fx^2}}\right)|-\frac{(bc-ad)e}{a(de-cf)}\right)}{ef\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} - \frac{c^{3/2}(be\cdots}{\cdots}$$

Mathematica [F] time = 0.66681, size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{\left(e+fx^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(e + f*x^2)^(3/2),x]
```

```
[Out] Integrate[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(e + f*x^2)^(3/2), x]
```

Maple [F] time = 0.06, size = 0, normalized size = 0.

$$\int \sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

```
[Out] int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{\left(fx^2+e\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(f*x^2 + e)^(3/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="fri
cas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{\left(e+fx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)/(f*x**2+e)**(3/2),x)
```

```
[Out] Integral(sqrt(a + b*x**2)*sqrt(c + d*x**2)/(e + f*x**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{\left(fx^2+e\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(f*x^2 + e)^(3/2), x)
```

3.109 $\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}\left(e+fx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=319

$$\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\tan^{-1}\left(\frac{x\sqrt{de-cf}}{\sqrt{c}\sqrt{e+fx^2}}\right),-\frac{e(bc-ad)}{a(de-cf)}\right)}{ae\sqrt{c+dx^2}\sqrt{de-cf}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\frac{x\sqrt{a+bx^2}(de-cf)}{e\sqrt{c+dx^2}\sqrt{e+fx^2}(be-af)}-\frac{\sqrt{c}\sqrt{a+bx^2}\sqrt{de-cf}\,E\left(\tan^{-1}\right.}{e\sqrt{c+dx^2}(be-af}$$

```
[Out] ((d*e - c*f)*x*Sqrt[a + b*x^2])/(e*(b*e - a*f)*Sqrt[c + d*x^2]*Sqrt[e + f*x
^2]) - (Sqrt[c]*Sqrt[d*e - c*f]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d*e
- c*f]*x)/(Sqrt[c]*Sqrt[e + f*x^2])], -(((b*c - a*d)*e)/(a*(d*e - c*f)))])/
(e*(b*e - a*f)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (c^
(3/2)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d*e - c*f]*x)/(Sqrt[c]*Sqrt[e
+ f*x^2])], -(((b*c - a*d)*e)/(a*(d*e - c*f)))])/(a*e*Sqrt[d*e - c*f]*Sqrt[
(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.46667, antiderivative size = 319, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {554, 422, 418, 492, 411}

$$\frac{c^{3/2}\sqrt{a+bx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{de-cf}\,x}{\sqrt{c}\sqrt{fx^2+e}}\right)|-\frac{(bc-ad)e}{a(de-cf)}\right)}{ae\sqrt{c+dx^2}\sqrt{de-cf}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\frac{x\sqrt{a+bx^2}(de-cf)}{e\sqrt{c+dx^2}\sqrt{e+fx^2}(be-af)}-\frac{\sqrt{c}\sqrt{a+bx^2}\sqrt{de-cf}\,E\left(\tan^{-1}\left(\frac{\sqrt{de-c}}{\sqrt{c}\sqrt{fx^2}}\right.\right.}{e\sqrt{c+dx^2}(be-af)\sqrt{\frac{c(a+b}{a(c+}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^2]/(Sqrt[a + b*x^2]*(e + f*x^2)^(3/2)),x]
```

```
[Out] ((d*e - c*f)*x*Sqrt[a + b*x^2])/(e*(b*e - a*f)*Sqrt[c + d*x^2]*Sqrt[e + f*x
^2]) - (Sqrt[c]*Sqrt[d*e - c*f]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d*e
- c*f]*x)/(Sqrt[c]*Sqrt[e + f*x^2])], -(((b*c - a*d)*e)/(a*(d*e - c*f)))])/
(e*(b*e - a*f)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (c^
(3/2)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d*e - c*f]*x)/(Sqrt[c]*Sqrt[e
+ f*x^2])], -(((b*c - a*d)*e)/(a*(d*e - c*f)))])/(a*e*Sqrt[d*e - c*f]*Sqrt[
(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rule 554

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)^(3/2)*Sqrt[(e_) + (f_.
)*(x_)^2]), x_Symbol] :> Dist[(Sqrt[c + d*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a +
 b*x^2))])/(a*Sqrt[e + f*x^2]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]), Subst
[Int[Sqrt[1 - ((b*c - a*d)*x^2)/c]/Sqrt[1 - ((b*e - a*f)*x^2)/e], x], x, x/
Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}\left(e+fx^2\right)^{3/2}}\,dx = \frac{\left(\sqrt{c+dx^2}\sqrt{\frac{e\left(a+bx^2\right)}{a\left(e+fx^2\right)}}\right)\text{Subst}\left(\int \frac{\sqrt{1-\frac{(-de+cf)x^2}{c}}}{\sqrt{1-\frac{(-be+af)x^2}{a}}}\,dx, x, \frac{x}{\sqrt{e+fx^2}}\right)}{e\sqrt{a+bx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{\frac{e\left(a+bx^2\right)}{a\left(e+fx^2\right)}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{(-be+af)x^2}{a}}\sqrt{1-\frac{(-de+cf)x^2}{c}}}\,dx, x, \frac{x}{\sqrt{e+fx^2}}\right)}{e\sqrt{a+bx^2}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}} - \frac{\left((-de+cf)\sqrt{c+\cdots}\right.}{\cdots}$$

$$= \frac{(de-cf)x\sqrt{a+bx^2}}{e(be-af)\sqrt{c+dx^2}\sqrt{e+fx^2}} + \frac{c^{3/2}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{de-cf}x}{\sqrt{c}\sqrt{e+fx^2}}\right)|-\frac{(bc-ad)e}{a(de-cf)}\right)}{ae\sqrt{de-cf}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}\sqrt{c+dx^2}} + \frac{\left(a(-de+\cdots\right.}{\cdots}$$

$$= \frac{(de-cf)x\sqrt{a+bx^2}}{e(be-af)\sqrt{c+dx^2}\sqrt{e+fx^2}} - \frac{\sqrt{c}\sqrt{de-cf}\sqrt{a+bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{de-cf}x}{\sqrt{c}\sqrt{e+fx^2}}\right)|-\frac{(bc-ad)e}{a(de-cf)}\right)}{e(be-af)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}\sqrt{c+dx^2}} + \frac{c^{\cdots}}{\cdots}$$

Mathematica [A] time = 0.0987182, size = 148, normalized size = 0.46

$$\frac{\sqrt{a}\sqrt{c+dx^2}\sqrt{\frac{e\left(a+bx^2\right)}{a\left(e+fx^2\right)}}E\left(\sin^{-1}\left(\frac{\sqrt{af-be}x}{\sqrt{a}\sqrt{fx^2+e}}\right)|\frac{a(cf-de)}{c(af-be)}\right)}{e\sqrt{a+bx^2}\sqrt{af-be}\sqrt{\frac{e\left(c+dx^2\right)}{c\left(e+fx^2\right)}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^2]/(Sqrt[a + b*x^2]*(e + f*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[a]*Sqrt[c + d*x^2]*Sqrt[(e*(a + b*x^2))/(a*(e + f*x^2))]*EllipticE[Ar
cSin[(Sqrt[-(b*e) + a*f]*x)/(Sqrt[a]*Sqrt[e + f*x^2])], (a*(-(d*e) + c*f))/
(c*(-(b*e) + a*f))])/(e*Sqrt[-(b*e) + a*f]*Sqrt[a + b*x^2]*Sqrt[(e*(c + d*x
^2))/(c*(e + f*x^2))])
```

Maple [F] time = 0.069, size = 0, normalized size = 0.

$$\int \sqrt{dx^2+c}\frac{1}{\sqrt{bx^2+a}}\left(fx^2+e\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)/(b*x^2+a)^(1/2)/(f*x^2+e)^(3/2),x)
```

```
[Out] int((d*x^2+c)^(1/2)/(b*x^2+a)^(1/2)/(f*x^2+e)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(d*x^2 + c)/(sqrt(b*x^2 + a)*(f*x^2 + e)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{fx^2+e}}{bf^2x^6+\left(2\,bef+af^2\right)x^4+ae^2+\left(be^2+2\,aef\right)x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="fri
cas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b*f^2*x^6 + (2*b*
e*f + a*f^2)*x^4 + a*e^2 + (b*e^2 + 2*a*e*f)*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}\left(e+fx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)/(b*x**2+a)**(1/2)/(f*x**2+e)**(3/2),x)
```

```
[Out] Integral(sqrt(c + d*x**2)/(sqrt(a + b*x**2)*(e + f*x**2)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(d*x^2 + c)/(sqrt(b*x^2 + a)*(f*x^2 + e)^(3/2)), x)
```

3.110 $\int \frac{\sqrt{c+dx^2}}{\left(a+bx^2\right)^{3/2}\left(e+fx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=36

$$\text{Unintegrable}\left(\frac{\sqrt{c+dx^2}}{\left(a+bx^2\right)^{3/2}\left(e+fx^2\right)^{3/2}},x\right)$$

```
[Out] Defer[Int][Sqrt[c + d*x^2]/((a + b*x^2)^(3/2)*(e + f*x^2)^(3/2)), x]
```

Rubi [A] time = 0.0600382, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt{c+dx^2}}{\left(a+bx^2\right)^{3/2}\left(e+fx^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[c + d*x^2]/((a + b*x^2)^(3/2)*(e + f*x^2)^(3/2)),x]
```

```
[Out] Defer[Int][Sqrt[c + d*x^2]/((a + b*x^2)^(3/2)*(e + f*x^2)^(3/2)), x]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^2}}{\left(a+bx^2\right)^{3/2}\left(e+fx^2\right)^{3/2}}\,dx=\int \frac{\sqrt{c+dx^2}}{\left(a+bx^2\right)^{3/2}\left(e+fx^2\right)^{3/2}}\,dx$$

Mathematica [A] time = 0.967956, size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+dx^2}}{\left(a+bx^2\right)^{3/2}\left(e+fx^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[c + d*x^2]/((a + b*x^2)^(3/2)*(e + f*x^2)^(3/2)),x]
```

```
[Out] Integrate[Sqrt[c + d*x^2]/((a + b*x^2)^(3/2)*(e + f*x^2)^(3/2)), x]
```

Maple [A] time = 0.069, size = 0, normalized size = 0.

$$\int \sqrt{dx^2+c}\left(bx^2+a\right)^{-\frac{3}{2}}\left(fx^2+e\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)/(b*x^2+a)^(3/2)/(f*x^2+e)^(3/2),x)
```

```
[Out] int((d*x^2+c)^(1/2)/(b*x^2+a)^(3/2)/(f*x^2+e)^(3/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2+c}}{\left(bx^2+a\right)^{\frac{3}{2}}\left(fx^2+e\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(3/2)/(f*x^2+e)^(3/2),x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(d*x^2 + c)/((b*x^2 + a)^(3/2)*(f*x^2 + e)^(3/2)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{fx^2+e}}{b^2f^2x^8+2\left(b^2ef+abf^2\right)x^6+\left(b^2e^2+4\,abef+a^2f^2\right)x^4+a^2e^2+2\left(abe^2+a^2ef\right)x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(3/2)/(f*x^2+e)^(3/2),x, algorithm="fri
cas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b^2*f^2*x^8 + 2*(
b^2*e*f + a*b*f^2)*x^6 + (b^2*e^2 + 4*a*b*e*f + a^2*f^2)*x^4 + a^2*e^2 + 2*
(a*b*e^2 + a^2*e*f)*x^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+dx^2}}{\left(a+bx^2\right)^{\frac{3}{2}}\left(e+fx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)/(b*x**2+a)**(3/2)/(f*x**2+e)**(3/2),x)
```

```
[Out] Integral(sqrt(c + d*x**2)/((a + b*x**2)**(3/2)*(e + f*x**2)**(3/2)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2+c}}{\left(bx^2+a\right)^{\frac{3}{2}}\left(fx^2+e\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(3/2)/(f*x^2+e)^(3/2),x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(d*x^2 + c)/((b*x^2 + a)^(3/2)*(f*x^2 + e)^(3/2)), x)
```

3.111 $\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=541

$$\frac{\sqrt{e}\sqrt{c+dx^2}(bc-ad)(2be-af)\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right),\frac{e(bc-ad)}{c(be-af)}\right)}{2b^2c\sqrt{e+fx^2}\sqrt{be-af}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{a\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}(adf-b(cf+de))}{2b^2\sqrt{c}\sqrt{e+fx^2}\sqrt{bc}}$$

```
[Out] (x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(2*Sqrt[a + b*x^2]) - (Sqrt[c]*Sqrt[b*c
 - a*d]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]*Sqrt[e + f*x^2]*EllipticE[Arc
Sin[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])], (c*(b*e - a*f))/((b*c -
 a*d)*e)])/(2*b*Sqrt[c + d*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a + b*x^2))]) + ((
b*c - a*d)*Sqrt[e]*(2*b*e - a*f)*Sqrt[c + d*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a
 + b*x^2))]*EllipticF[ArcSin[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])]
, ((b*c - a*d)*e)/(c*(b*e - a*f))])/(2*b^2*c*Sqrt[b*e - a*f]*Sqrt[(a*(c + d
*x^2))/(c*(a + b*x^2))]*Sqrt[e + f*x^2]) - (a*(a*d*f - b*(d*e + c*f))*Sqrt[
c + d*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a + b*x^2))]*EllipticPi[(b*c)/(b*c - a*
d), ArcSin[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])], (c*(b*e - a*f))/
((b*c - a*d)*e)])/(2*b^2*Sqrt[c]*Sqrt[b*c - a*d]*Sqrt[(a*(c + d*x^2))/(c*(a
 + b*x^2))]*Sqrt[e + f*x^2])
```

Rubi [A] time = 0.436035, antiderivative size = 541, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.206, Rules used = {556, 554, 424, 552, 419, 553, 537}

$$\frac{\sqrt{e}\sqrt{c+dx^2}(bc-ad)(2be-af)\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}F\left(\sin^{-1}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)|\frac{(bc-ad)e}{c(be-af)}\right)}{2b^2c\sqrt{e+fx^2}\sqrt{be-af}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{a\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}(adf-b(cf+de))\Pi\left(\frac{bc}{bc-ad};\right.}{2b^2\sqrt{c}\sqrt{e+fx^2}\sqrt{bc-ad}\sqrt{\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/Sqrt[a + b*x^2],x]
```

```
[Out] (x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(2*Sqrt[a + b*x^2]) - (Sqrt[c]*Sqrt[b*c
 - a*d]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]*Sqrt[e + f*x^2]*EllipticE[Arc
Sin[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])], (c*(b*e - a*f))/((b*c -
 a*d)*e)])/(2*b*Sqrt[c + d*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a + b*x^2))]) + ((
b*c - a*d)*Sqrt[e]*(2*b*e - a*f)*Sqrt[c + d*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a
 + b*x^2))]*EllipticF[ArcSin[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])]
, ((b*c - a*d)*e)/(c*(b*e - a*f))])/(2*b^2*c*Sqrt[b*e - a*f]*Sqrt[(a*(c + d
*x^2))/(c*(a + b*x^2))]*Sqrt[e + f*x^2]) - (a*(a*d*f - b*(d*e + c*f))*Sqrt[
c + d*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a + b*x^2))]*EllipticPi[(b*c)/(b*c - a*
d), ArcSin[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])], (c*(b*e - a*f))/
((b*c - a*d)*e)])/(2*b^2*Sqrt[c]*Sqrt[b*c - a*d]*Sqrt[(a*(c + d*x^2))/(c*(a
 + b*x^2))]*Sqrt[e + f*x^2])
```

Rule 556

```
Int[(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2])/Sqrt[(e_) + (f_.)
*(x_)^2], x_Symbol] :> Simp[(x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(2*Sqrt[e +
 f*x^2]), x] + (Dist[(e*(b*e - a*f))/(2*f), Int[Sqrt[c + d*x^2]/(Sqrt[a + b
```

```
*x^2]*(e + f*x^2)^(3/2)), x], x] + Dist[((b*e - a*f)*(d*e - 2*c*f))/(2*f^2)
, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x] - Dist[(b
*d*e - b*c*f - a*d*f)/(2*f^2), Int[Sqrt[e + f*x^2]/(Sqrt[a + b*x^2]*Sqrt[c
+ d*x^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NegQ[(d*e - c*f)/c]
```

Rule 554

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)^(3/2)*Sqrt[(e_) + (f_.
)*(x_)^2]), x_Symbol] :> Dist[(Sqrt[c + d*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a +
 b*x^2))])/(a*Sqrt[e + f*x^2]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]), Subst
[Int[Sqrt[1 - ((b*c - a*d)*x^2)/c]/Sqrt[1 - ((b*e - a*f)*x^2)/e], x], x, x/
Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 552

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.
)*(x_)^2]), x_Symbol] :> Dist[(Sqrt[c + d*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a +
 b*x^2))])/(c*Sqrt[e + f*x^2]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]), Subst
[Int[1/(Sqrt[1 - ((b*c - a*d)*x^2)/c]*Sqrt[1 - ((b*e - a*f)*x^2)/e]), x], x
, x/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 553

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/(Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Dist[(a*Sqrt[c + d*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a +
 b*x^2))])/(c*Sqrt[e + f*x^2]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]), Subst
[Int[1/((1 - b*x^2)*Sqrt[1 - ((b*c - a*d)*x^2)/c]*Sqrt[1 - ((b*e - a*f)*x^2
)/e]), x], x, x/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{\sqrt{a+bx^2}}\,dx = \frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2\sqrt{a+bx^2}} - \frac{(a(bc-ad))\int \frac{\sqrt{e+fx^2}}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx}{2b} + \frac{((bc-ad)(2be-af))\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx}}}{2b^2}$$

$$= \frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2\sqrt{a+bx^2}} - \frac{(bc-ad)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}\,\text{Subst}\left(\int \frac{\sqrt{1-\frac{(be-af)x^2}{e}}}{\sqrt{1-\frac{(bc-ad)x^2}{c}}}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{2b\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}} + \cdots$$

$$= \frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2\sqrt{a+bx^2}} - \frac{\sqrt{c}\sqrt{bc-ad}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}E\left(\sin^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)|\frac{c(be-af)}{(bc-ad)e}\right)}{2b\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}} + \frac{(bc-\cdots}{\cdots}$$

Mathematica [A] time = 1.62308, size = 512, normalized size = 0.95

$$\frac{\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\left(\sqrt{e}\sqrt{a+bx^2}\sqrt{bc-ad}(2bc-ad)\sqrt{be-af}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\text{EllipticF}\left(\sin^{-1}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right),\frac{bce-ade}{bce-acf}\right)+b^2cx\left(e+\cdots\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]*(b^2*c*Sqrt[b*c - a*
d]*x*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]*(e + f*x^2) - b*c*Sqrt[b*c - a*d
]*Sqrt[e]*Sqrt[b*e - a*f]*Sqrt[a + b*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a + b*x^
2))]*EllipticE[ArcSin[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])], (b*c*
e - a*d*e)/(b*c*e - a*c*f)] + Sqrt[b*c - a*d]*(2*b*c - a*d)*Sqrt[e]*Sqrt[b*
e - a*f]*Sqrt[a + b*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a + b*x^2))]*EllipticF[Ar
cSin[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])], (b*c*e - a*d*e)/(b*c*e
 - a*c*f)] - a*Sqrt[c]*(a*d*f - b*(d*e + c*f))*Sqrt[a + b*x^2]*Sqrt[(a*(e +
 f*x^2))/(e*(a + b*x^2))]*EllipticPi[(b*c)/(b*c - a*d), ArcSin[(Sqrt[b*c -
a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])], (b*c*e - a*c*f)/(b*c*e - a*d*e)]))/(2*a
*b^2*Sqrt[b*c - a*d]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Maple [F] time = 0.059, size = 0, normalized size = 0.

$$\int \sqrt{dx^2+c}\sqrt{fx^2+e}\frac{1}{\sqrt{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^(1/2),x)
```

```
[Out] int((d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2 + c}\sqrt{fx^2 + e}}{\sqrt{bx^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/sqrt(b*x^2 + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="fri
cas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c + dx^2}\sqrt{e + fx^2}}{\sqrt{a + bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)*(f*x**2+e)**(1/2)/(b*x**2+a)**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*x**2)*sqrt(e + f*x**2)/sqrt(a + b*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2 + c}\sqrt{fx^2 + e}}{\sqrt{bx^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/sqrt(b*x^2 + a), x)
```

3.112 $\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$

Optimal. Leaf size=36

$$\text{Unintegrable}\left(\frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}},x\right)$$

```
[Out] Defer[Int][(a + b*x^2)^(3/2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x]
```

Rubi [A] time = 0.0559183, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*x^2)^(3/2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

```
[Out] Defer[Int][(a + b*x^2)^(3/2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x]
```

Rubi steps

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

Mathematica [A] time = 0.170321, size = 0, normalized size = 0.

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*x^2)^(3/2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

```
[Out] Integrate[(a + b*x^2)^(3/2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x]
```

Maple [A] time = 0.068, size = 0, normalized size = 0.

$$\int (bx^2+a)^{\frac{3}{2}}\frac{1}{\sqrt{dx^2+c}}\frac{1}{\sqrt{fx^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

```
[Out] int((b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{3}{2}}}{\sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate((b*x^2 + a)^(3/2)/(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="fri
cas")
```

```
[Out] Timed out
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^2\right)^{\frac{3}{2}}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)**(3/2)/(sqrt(c + d*x**2)*sqrt(e + f*x**2)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{3}{2}}}{\sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate((b*x^2 + a)^(3/2)/(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

3.113 $\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$

Optimal. Leaf size=159

$$\frac{a\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\Pi\left(\frac{bc}{bc-ad};\sin^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)|\frac{c(be-af)}{(bc-ad)e}\right)}{\sqrt{c}\sqrt{e+fx^2}\sqrt{bc-ad}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

```
[Out] (a*Sqrt[c + d*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a + b*x^2))]*EllipticPi[(b*c)/(
b*c - a*d), ArcSin[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])], (c*(b*e
- a*f))/((b*c - a*d)*e)])/(Sqrt[c]*Sqrt[b*c - a*d]*Sqrt[(a*(c + d*x^2))/(c*
(a + b*x^2))]*Sqrt[e + f*x^2])
```

Rubi [A] time = 0.135229, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {553, 537}

$$\frac{a\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\Pi\left(\frac{bc}{bc-ad};\sin^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)|\frac{c(be-af)}{(bc-ad)e}\right)}{\sqrt{c}\sqrt{e+fx^2}\sqrt{bc-ad}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

```
[Out] (a*Sqrt[c + d*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a + b*x^2))]*EllipticPi[(b*c)/(
b*c - a*d), ArcSin[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])], (c*(b*e
- a*f))/((b*c - a*d)*e)])/(Sqrt[c]*Sqrt[b*c - a*d]*Sqrt[(a*(c + d*x^2))/(c*
(a + b*x^2))]*Sqrt[e + f*x^2])
```

Rule 553

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/(Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Dist[(a*Sqrt[c + d*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a +
 b*x^2))])/(c*Sqrt[e + f*x^2]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]), Subst
[Int[1/((1 - b*x^2)*Sqrt[1 - ((b*c - a*d)*x^2)/c]*Sqrt[1 - ((b*e - a*f)*x^2
)/e]), x], x, x/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \frac{\left(a\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\right)\text{Subst}\left(\int \frac{1}{(1-bx^2)\sqrt{1-\frac{(bc-ad)x^2}{c}}\sqrt{1-\frac{(be-af)x^2}{e}}}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{c\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

$$= \frac{a\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\Pi\left(\frac{bc}{bc-ad};\sin^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)|\frac{c(be-af)}{(bc-ad)e}\right)}{\sqrt{c}\sqrt{bc-ad}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

Mathematica [A] time = 0.0917638, size = 159, normalized size = 1.

$$\frac{a\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\Pi\left(\frac{bc}{bc-ad};\sin^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)|\frac{c(be-af)}{(bc-ad)e}\right)}{\sqrt{c}\sqrt{e+fx^2}\sqrt{bc-ad}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

```
[Out] (a*Sqrt[c + d*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a + b*x^2))]*EllipticPi[(b*c)/(
b*c - a*d), ArcSin[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])], (c*(b*e
- a*f))/((b*c - a*d)*e)])/(Sqrt[c]*Sqrt[b*c - a*d]*Sqrt[(a*(c + d*x^2))/(c*
(a + b*x^2))]*Sqrt[e + f*x^2])
```

Maple [F] time = 0.062, size = 0, normalized size = 0.

$$\int \sqrt{bx^2+a}\frac{1}{\sqrt{dx^2+c}}\frac{1}{\sqrt{fx^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

```
[Out] int((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(b*x^2 + a)/(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="fri
cas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + bx^2}}{\sqrt{c + dx^2}\sqrt{e + fx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x**2)/(sqrt(c + d*x**2)*sqrt(e + f*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2 + a}}{\sqrt{dx^2 + c}\sqrt{fx^2 + e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(b*x^2 + a)/(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

3.114 $\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$

Optimal. Leaf size=148

$$\frac{\sqrt{e}\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right),\frac{e(bc-ad)}{c(be-af)}\right)}{c\sqrt{e+fx^2}\sqrt{be-af}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

```
[Out] (Sqrt[e]*Sqrt[c + d*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a + b*x^2))]*EllipticF[Ar
cSin[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])], ((b*c - a*d)*e)/(c*(b*
e - a*f))])/(c*Sqrt[b*e - a*f]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]*Sqrt[e
 + f*x^2])
```

Rubi [A] time = 0.0799099, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {552, 419}

$$\frac{\sqrt{e}\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}F\left(\sin^{-1}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)|\frac{(bc-ad)e}{c(be-af)}\right)}{c\sqrt{e+fx^2}\sqrt{be-af}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

```
[Out] (Sqrt[e]*Sqrt[c + d*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a + b*x^2))]*EllipticF[Ar
cSin[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])], ((b*c - a*d)*e)/(c*(b*
e - a*f))])/(c*Sqrt[b*e - a*f]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]*Sqrt[e
 + f*x^2])
```

Rule 552

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.
)*(x_)^2]), x_Symbol] :> Dist[(Sqrt[c + d*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a +
 b*x^2))])/(c*Sqrt[e + f*x^2]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]), Subst
[Int[1/(Sqrt[1 - ((b*c - a*d)*x^2)/c]*Sqrt[1 - ((b*e - a*f)*x^2)/e]), x], x
, x/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \frac{\left(\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{(bc-ad)x^2}{c}}\sqrt{1-\frac{(be-af)x^2}{e}}}dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{c\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

$$= \frac{\sqrt{e}\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}F\left(\sin^{-1}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{a+bx^2}}\right)|\frac{(bc-ad)e}{c(be-af)}\right)}{c\sqrt{be-af}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

Mathematica [A] time = 0.0830293, size = 148, normalized size = 1.

$$\frac{\sqrt{e}\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\text{EllipticF}\left(\sin^{-1}\left(\frac{x\sqrt{be-af}}{\sqrt{e}\sqrt{a+bx^2}}\right), \frac{e(bc-ad)}{c(be-af)}\right)}{c\sqrt{e+fx^2}\sqrt{be-af}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

```
[Out] (Sqrt[e]*Sqrt[c + d*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a + b*x^2))]*EllipticF[Ar
cSin[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])], ((b*c - a*d)*e)/(c*(b*
e - a*f))])/(c*Sqrt[b*e - a*f]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]*Sqrt[e
 + f*x^2])
```

Maple [F] time = 0.067, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^2+a}}\frac{1}{\sqrt{dx^2+c}}\frac{1}{\sqrt{fx^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

```
[Out] int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="m
axima")
```

```
[Out] integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{fx^2+e}}{bdfx^6+\left(bde+(bc+ad)f\right)x^4+ace+\left(acf+(bc+ad)e\right)x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="f
ricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b*d*f*x^6 + (b*d*
e + (b*c + a*d)*f)*x^4 + a*c*e + (a*c*f + (b*c + a*d)*e)*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*x**2)*sqrt(c + d*x**2)*sqrt(e + f*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

3.115 $\int \frac{1}{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$

Optimal. Leaf size=36

$$\text{Unintegrable}\left(\frac{1}{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}\sqrt{e+fx^2}},x\right)$$

```
[Out] Defer[Int][1/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x]
```

Rubi [A] time = 0.0552669, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

```
[Out] Defer[Int][1/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx=\int \frac{1}{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

Mathematica [A] time = 0.873079, size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

```
[Out] Integrate[1/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x]
```

Maple [A] time = 0.07, size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^{-\frac{3}{2}}\frac{1}{\sqrt{dx^2+c}}\frac{1}{\sqrt{fx^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

```
[Out] int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{3}{2}}\sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="m
axima")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{fx^2+e}}{b^2dfx^8+\left(b^2de+\left(b^2c+2\,abd\right)f\right)x^6+\left(\left(b^2c+2\,abd\right)e+\left(2\,abc+a^2d\right)f\right)x^4+a^2ce+\left(a^2cf+\left(2\,abc+a^2d\right)e\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="f
ricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b^2*d*f*x^8 + (b^
2*d*e + (b^2*c + 2*a*b*d)*f)*x^6 + ((b^2*c + 2*a*b*d)*e + (2*a*b*c + a^2*d)
*f)*x^4 + a^2*c*e + (a^2*c*f + (2*a*b*c + a^2*d)*e)*x^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+bx^2\right)^{\frac{3}{2}}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x**2)**(3/2)*sqrt(c + d*x**2)*sqrt(e + f*x**2)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{3}{2}}\sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
```

```
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
```

```
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do
   not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
```

```
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
```

```
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
```

```
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
```

```
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len
    (flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                         #since this estimate of leaf count is bit lower than
```

```
                      #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
    hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

```

```
def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
```

```
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): # is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, " leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=", expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```